UNIVERSIDADE DE ÉVORA

DEPARTAMENTO DE ECONOMIA

DOCUMENTO DE TRABALHO Nº **2003/01**

January

Constraints on Dematerialisation and Allocation of Natural Capital along a Sustainable Growth Path

(1 st Draft)

José Manuel Madeira Belbute
Department of Economics, University of Évora

João Rodrigues
PhD Student, IDMEC, IST (joaofrodrigues@hotmail.com)

Tiago Domingos
IDMEC, Institute of Mechanical Engineering, IST

Pedro Conceição
IN+, Center for Innovation, Technology and Policy Research, Instituto Superior Técnico

UNIVERSIDADE DE ÉVORA
DEPARTAMENTO DE ECONOMIA
Largo dos Colegiais, 2 – 7000-803 Évora – Portugal
Tel.: +351 266 740 894 Fax: +351 266 742 494
www.decon.uevora.pt wp.economia@uevora.pt

Resumo/ Abstract:

This paper extends the neoclassical growth model with natural capital by introducing two new concepts: allocation of natural capital and materialization.
We consider that anthropogenic environmental impact is correlated with the throughput of the economy (materialisation). Materialisation is the material throughput per unit of economic activity. We capture the effect of the reduction of this throughput – dematerialisation – in the elasticities of materialisation and aggregate environmental impact.
In our framework the fraction of natural capital devoted to production does not provide direct environmental services nor does it contribute to ecosystem functioning namely affecting the carrying capacity of natural capital.
We analyse an optimal sustainable growth path, in the context of exogenous technological change. Our main conclusion is that the ratio of dematerialisation elasticities must equal the inverse of the share of natural capital in order to assure unbounded economic growth with constant natural capital.

Palavras-chave/Keyword: Economic growth, environmental concerns, sustainability constraints, materialisation and allocation of natural capital.

Classificação JEL/JEL Classification: C62, O40, Q01

1 Introduction

Sustainable development a top concern among economists and natural scientists, as well as among development agencies and the general public. Neoclassical growth theory has tried to address this problem (Solow, 1974; Aghion and Howitt, 1998) but it has always been greeted with some scepticism due to its tenuous biophysical rigour[1]. The aim of this paper is to contribute to a more satisfactory depiction of economic-environmental interaction in the neoclassical growth theory framework .

We start with an extension of Belbute (1999). In the original model built and natural capital are used as inputs in production, with natural capital subject to logistic regeneration. In Rodrigues et al. (2002) a first extension was presented where, besides dynamic environmental impact (that reduces available natural capital), society causes a structural interference on the natural system that diminishes the carrying capacity of natural capital. Now, with the purpose of making dynamic and structural human-Nature interactions endogenous we introduce two potentially powerful concepts: dematerialization and allocation of natural capital.

Dematerialization is a concept from industrial ecology. Industrial ecology studies material flows within society and how the productive process can be optimised in order to minimise resource waste and pollution. According to industrial ecology, the problems of resource exhaustion and pollution (inputs and outputs of the production process) can both be assigned to the material throughput of the economy (Hinterberger et al., 1997) - which we define as its degree of materialisation. If the material throughput per unit of wealth decreases fast enough (the process of *de*materialisation) then it is possible to reconcile the ecological economic requirement for a non-increasing material economy (Constanza et al., 1997a) and conventional economic wisdom that growth is good. We explain this process of dematerialisation through technological change (new technologies may be resource saving) and the composition effect (less materialised sectors of society may grow faster than average).

Human society depends on a variety of ecosystem services, most of which are invisible and unrewarded (Daily, 1997). That would not be relevant to economic analysis, were not for the fact that the extent of human dominion of the biosphere is threatening ecosystem functioning (Vitousek et al., 1997). The way we found to introduce this idea in economic modelling was to consider that natural capital is either free or enslaved to production (England, 1998b; 2000). Free natural capital provides direct environmental services to society (Belbute, 1999) and contributes to ecological functioning while enslaved natural capital fuels the productive process but is unable to perform any of those two functions.

The structure of the paper is as follows. In sections 2 and 3 a general neoclassical growth model is presented (section 2 focus on the biophysical aspect and section 3 on technology and consumer behavior). In section 4 an analysis of the model is presented, focusing on the constraints that arise along a sustainable growth path. In section 5 conclusions close the paper.

[1] For critiques of neoclassical economics see: Blaug (1991) on methodological aspects; Nelson (1997) on policy implications; Hall (2000) on the biophysical basis and Cabeza Gutés (1996) on the assumptions of growth theory.

2 Role and dynamics of natural capital

2.1 Role of natural capital

Natural capital is the aggregation of all environmental assets, and is used by society for three broadly defined purposes: (1) environmental services, (2) resource uptake and (3) waste refusal (Dunlap, 1993; England, 1998b).

Regarding environmental services Georgescu-Roegen (1971) called Nature "the silent companion of man" to draw attention to the fact that Nature works as a fund (i.e., it produces a service and is not consumed) performing a diversity of functions as maintaining soil fertility, climate control or natural beauty[2]. The spatial and temporal scales of ecosystem functioning vary greatly and there is presently great uncertainty regarding the true extent of societal dependence on natural ecosystems (Daily, 1997; Levin, 1999).

The economic process needs not only environmental services but also a material and energy flow of low entropy. It is common to distinguish between renewable and non-renewable resources, but the distinction is arbitrary, i.e., it depends on the temporal and spatial scales considered. Most resources used by humans are, to a great extent, a result of ecosystem processes, so we will assume they will behave in a way similar to conventional population or ecosystem dynamics (Gurney and Nisbet, 1998).

At the other end of the economic process, the refusal of high entropy residuals is unavoidable, both in the production process and during consumption. Nature receives what society no longer wants and its assimilation capacity is subject to critical loads and specific degradation rates[3]. Aghion and Howitt (1998) consider that the rate at which the environment assimilates pollution increases as the pollution load increases, ending abruptly as a critical load is reached. This is highly unrealistic, and a sort of logistic behaviour is instead to be expected (Belbute, 1998b).

Pollution (outflow of the production process) and natural resources (inflow to production) are, from an ecological point of view, disturbances that can be grouped into natural capital depletion. Natural capital is the provider and the absorber of flows, not the flows themselves. Environmental amenities are used without being consumed but human action does interfere with them.

2.2 Dynamics of the natural system

Natural capital must obey the balance equation:

$$\frac{dN}{dt} = R(N) - P(Y)\,. \tag{1}$$

R(N) is natural regeneration, which depends on the stock of natural capital, and *P(Y)* is a throughput disturbance, that includes the negative effects of resource depletion and pollution and depends on the level of economic activity (output). We will discuss

---

[2] Kaufmann (1995) considers only climate control as the fund action of natural systems while van den Bergh and Hofkes (1997) do not consider the function action of natural capital, in the framework of a neoclassical growth model. Belbute (1998a) considers that environmental services affect utility but not the productive process.

[3] England (2000) does not consider assimilation as a role of natural capital.

P(Y) further ahead, in section 2.2.3, and now we will focus on the endogenous dynamics of natural capital.

In some models (Aghion and Howitt, 1998, for pollution, and Kolstad and Tolman, 1997, for climate change), regeneration is considered to be linear. Following Belbute (1998a), we consider that regeneration, *R(N)*, should be of logistic form, decreasing both as the system increases to its carrying capacity, *CC*, and decreases to zero. An explicit functional form is[4]:

$$R(N) = r\,N\,(CC - N). \tag{2}$$

Considering a constant carrying capacity is the same as saying that all environmental impact is reversible, because once the disturbance has ceased (if *P(Y)* becomes zero), no matter how harsh the disturbance has been, the system will always return to the original steady-state (the carrying capacity). A way to overcome this problem is to consider a changing carrying capacity. Following an analogy from population dynamics, an equation for CC, originally presented by Cohen (1995), is adapted to the context of natural capital (Rodrigues et al., 2002):

$$\frac{dCC}{dt} = \frac{l}{N}\frac{dN}{dt} - dist\ . \tag{3}$$

If we accept that most natural capital services are in fact a result of ecosystem dynamics, we can identify the time scale of natural capital dynamics with that of ecological succession. This dynamic equation describes a positive effect on the increase of the carrying capacity originated by an increase in the stock of natural capital, $dCC/dt \propto dN/dt$. A mechanistic basis for this behaviour is increasing returns to scale due to specialisation: the development of the ecosystem gives birth to the appearance of new ecological niches and an increase in ecosystem functioning. Yet, there must exist a “vanishing rate of opportunities” or a decreasing marginal benefit on ecosystem functioning due to natural capital increase, hence the term *l/N*. As *N* rises, an extra increase of *N* will be reflected in a smaller increase of *CC*[5].

The term $dist > 0$ reflects a structural interference caused by human action that disturbs the natural system not because of the consumption or the release of flows (that effect is captured in *P(Y)*) but because of human disturbance on ecosystem structure and functioning. For example, in the timber exploitation of a forested area, there is a negative dynamic (associated with flows, and occurring only during the exploitation time) impact due to pure timber extraction, erosion due to water runoff while the soil is uncovered, soil compaction, noise and other forms of pollution disturbing local populations, etc. There is also an impact associated with human action that does not cease immediately when human action ceases, and must therefore be reflected as a decrease of carrying capacity. This structural interference may be habitat fragmentation due to road construction, the removal of native species or the introduction of exotic species, waterline diversion and interference with the hydrological regime. Notice that in our model, in the long run the natural system may return to the previous, to a larger or to a smaller steady state of natural capital. All depends on the duration and relative intensity of *P(Y)* and *dist*.

[4] Our definition of specific growth rate, *r*, is slightly different from usual. The mathematical properties of the logistic are given in Belbute (1998b) and applied in a bioeconomic context in Clark (1976). For a critique of the ubiquity and applicability of the logistic equation, see Peters (1991).

[5] Inversely, the marginal benefit of an increase in *N* upon *CC* will rise to infinity as *N* approaches zero. This is unrealistic but it should not distress us because we will make our analysis along a sustainable growth path (hopefuly) with *N* far away from zero.

2.3 Allocation of natural capital

England (2000) advanced the stimulating insight that ecological services are provided by the fraction of land not occupied by Mankind. Even though purely geographical space does not correspond to ecological space we can conceptualise that natural capital is in fact composed by a fraction used for productive processes ("the biological slaves of Mankind") and a fraction of "free" natural capital. Modern agriculture and modern land occupation are in general highly disruptive of ecosystem function (Odum, 1969). If we think of an intensity of use of land rather than an absolute dichotomy between allocated forms of natural capital, we can consider that from the total stock of natural capital, *N*, society may choose to use for productive purposes a certain fraction, *uN*. To the remaining part *(1 – u)N* we call "free natural capital" which provides direct environmental services that directly affects human well-being..[6]

If we consider that only free natural capital contributes to ecological services then we can model structural interference as:

$$\frac{dCC}{dt} = \frac{l}{(1-u)N}\frac{d(1-u)N}{dt} = \frac{l}{N}\frac{dN}{dt} - \frac{l}{1-u}\frac{du}{dt} \tag{4}$$

Comparing to equation (3), it is immediately clear that structural interference term, *dist*, is related to the allocation of natural capital to production. This makes clear that structural interference is caused by a changing allocation of natural capital.

We consider the allocation of natural capital instantaneous, costless and reversible. By this we mean that we are ignoring the process of allocation, which usually involves built capital, labour and time. For simplicity, we consider that abandoned natural capital will behave as free, albeit impoverished natural capital (due to its reduced carrying capacity). This is justified by examples of abandoned territories that reverted to their previous natural state (e.g., forest recovery in Yucatan or Sweden). However, for some types of structural interference caused by human appropriation (e.g. lake acidification, biodiversity loss, severe soil erosion) the time scales and the possibility of return to different equilibria may be such that structural interference may effectively be considered as ultimately irreversible. We will ignore these problems.

2.4 The problem of aggregation and valuation

We are assuming substitutability among the different functions of natural capital, which is at least debatable, as is the aggregation of any kind of capital itself, with the special handicap of the extremely diverse dynamics in this case. Dunlap (1993) proposes that competition among the three functions should be considered as well as a carrying capacity for them as a whole – Nature's ability to tolerate Man's demands. We take a different approach. We aggregate all functions as natural capital so competition among functions is only addressed if we add the competing functions – the burden is passed on to the empirical aggregation work. Unfortunately this problem seems to be pervasive: a forest is a stock of timber as much as a life support for biodiversity. Comprehensive listing of all the functions played by natural capital for human use is necessary.

---

[6] Endre and Radkes (1998) present a growth model to study the effect of the allocation of land use between agriculture and forest where only the latter enters a logistic regeneration function for natural capital. Even though the modelling options are different, we are modelling the same phenomenon.

In fact, the valuation and aggregation of capital is even today rather controversial (van den Bergh and Verbruggen, 1998). Harte (1995) claims that ecosystems are dynamic entities and therefore it is meaningless to talk about a "stock" of natural capital. According to Kaufmann (1995), natural capital should be valued by the goods and services it provides to humans measured in terms of their opportunity costs, therefore depending on human tastes and technological abilities. Hinterberger (1997) points out that a rise in the prices of natural assets may increase the value of natural capital even in the case of severe depletion. Ecosystem and biophysical cycles behave independently of human choices. So we face a conceptual dilemma, regarding the valuation of natural capital: economic when it is used for human purposes, biophysical for matters of endogenous dynamics. An important result of the valuation of ecosystem services (Constanza et al., 1997b) was the finding of a strong correlation between value and primary productivity for most ecosystems (Constanza et al., 1998b). This result is important because it suggests that biophysical and economic valuation may, in many aspects, coincide.

Regarding empirical assessment, the several parameters alluded so far may be estimated even without a precise quantification of natural capital. Wackernagel and Rees (1997) refer to the ecological footprint as the “appropriated carrying capacity”, or the ecological space required for the economy or population; this may be an indicator of uN[7] (measured in units of area). Vitousek (1997) estimated that Man appropriates about 40 % of Earth’s liquid primary production; this may a rough estimate of the fraction u. Energy per unit of wealth or total embodied primary solar energy (Hall, 2000), can be an estimate of uN/Y, the ratio of enslaved natural capital to output. Purely ecological indicators can help determine the state of the natural system. Ascendancy (Ulanowicz, 1986) can be a measure of the degree of organisation of a system, CC. If life is a manifestation of the second law of thermodynamics (Schneider and Kay, 1994), net dissipated solar radiation can be a measure of a system's distance to its climax, N/CC. These several measurement methods may, in principle, be used for an empirical testing of our model.

3 Technology and consumer behaviour

3.1 Built capital and knowledge

According to Georgescu-Roegen (1971) and England (1998b), fund agents act on the process, not being consumed (though they can be damaged) and thus keeping their identity. Flows are changed in the process, the inflow being different from the outflow. A process occurs within definite space and time boundaries. Energy will be dissipated in any real process; mass will be conserved but probably some amount of it will not enter the final product, thus being disposed of as waste. England discusses the addition to this thermodynamic (thus static) framework of the ecological view of Faber et al. (1994), who stress that organisms are funds that interact with each other, performing services.

Conventional production factors are funds. Flows are usually not considered in aggregate models (they are referred to as intermediate goods). In our model we will use K *for* built capital and A for intellectual capital as human production factor (Rebelo, 1991). We consider K as an extensive, physically measurable property and A as an intensive, efficiency related property[8]. We are not concerned with aggregation at this point. We are also considering a no population growth scenario so

---

[7] For a criticism of the ecological footprint, see van den Bergh and Verbruggen (1998).

[8] Related formulations can be found in Serageldin and Steer (1994), Ayres et al. (1998) and van den Bergh (1998).

our model can be understood either as representing the whole society or a representative agent.

We will refer to *A* as knowledge or intellectual capital (Aghion and Howitt, 1998), encompassing human capital (Lucas, 1988) and institutions (North, 1990). Endogenous growth theory in the 90's has placed great emphasis on the dynamics of *A*, but in the present paper our focus is elsewhere. So we will consider a constant exogenous rate of knowledge creation, *g*:

$$\frac{\dot{A}}{A} = g\ . \tag{5}$$

Built capital includes several types of tools and equipment of the private sector and society's infrastructures. As usual, we assume that this stock depreciates at a constant rate, $\delta > 0$ but it may be increased by gross investment (the fraction of production, Y, that is not consumed, C) so that the net increase in the stock of physical capital at any point in time can given by :

$$dK/dt = Y - C - \delta K. \tag{6}$$

It is common in models that deal with environmental issues to consider pollution abatement to reduce available output for consumption and investment (Andreoni and Levinson, 2001; Lieb, 2001; Belbute, 1999; Aghion and Howitt, 1998). We challenge this traditional view and argue that environmental expenditure consists of pure pollution abatement, restoration effort and environmental innovation. Money spent on pollution abatement and restoration is conventional consumption (??), the fact that it is unwanted consumption makes it a defensive expenditure that can be grouped with money spent on lawyers, policemen, clothes, fuel to be wasted on traffic jams and the like. It is in the relation between consumption and utility that such expenditure can be discriminated[9]. As for environmental innovation, it should be considered only if we were considering endogenous technological change. We discuss this issue further ahead, in section 2.2.3.

3.2 Substitutability between man-made and natural capital

The degree of substitutability between natural and built capital is important because it affects the choice of the specific form for the production function. Following the idea that they are complements (mainstream economics), Cobb-Douglas (Solow, 1974), AK (Belbute, 1999) or Schumpeterian (Aghion and Howitt, 1998) production functions have been used. On the other hand, England (2000) presents a growth model with natural and built capital as perfect substitutes (ecological economics).

We will discuss how our model addresses three of the criticisms posed by ecological economists against a high degree of substitutability: the existence of viability thresholds, embodiment concerns and indirect resource use. Keil (1998) presents a different but convincing criticism of these questions based on Georgescu-Roegen's production theoretical approach.

If there is a critical value of natural capital below which human economy cannot thrive, then the substitutability between man-made and natural capital can only be marginal. In Daly's (1997) example, we can survive an increase in the size of the ozone layer by buying more sunglasses, but if the ozone layer were to disappear completely, it would not be feasible to supply all living beings with sunglasses. In our model these criticality effects are captured in the dynamics of the carrying capacity.

---

[9] See Constanza et al. (1997a) or England (1998a) for a discussion on sustainable welfare indicators.

We consider that structural interference (with the specific functional form of natural capital allocation) causes a loss of natural capital's carrying capacity. If this interference is strong enough, the natural system may collapse, entailing the collapse of economic subsystem.

Embodiment concerns arise because built capital is, from the physical point of view, transformed natural capital. Because of the inevitability of thermodynamic inefficiency, even if some degree of substitutability exists, it must be bounded (Kaufmann, 1995). In our model the problem of embodiment is captured by the joint dynamics of physical capital accumulation and environmental impact. For capital to be accumulated it must first be produced and production causes environmental damage.

Regarding the problem of indirect resource use, van den Bergh (1998) hints, in footnote 6, that in aggregate models isoquants should be backward bending. The explanation is that at the macro level production factors are never primary inputs, but instead, the use of a given input requires the use of all other inputs. Stern (1997) discusses the problem of indirect resource use of energy in economic production. We consider the problem of indirect resource use by taking aggregate environmental impact, P, as a production function, i.e., $P=P(Y)$, rather than as a function of the fraction of natural capital allocated to production, i.e. $P=P(uN)$. This way, an increase in production will entail a decrease in available natural capital for further production.

Because we can address these criticisms outside the production function, we can consider built and natural capital as imperfect substitutes. Let the production function be continuous, concave, class C^2, positive and unbounded and let both inputs, allocated natural capital, uN and built capital K, be essential for production (Belbute, 1998b; Solow, 1974). Knowledge, A, is a scale factor with exogenous dynamics. We will use the Cobb-Douglas functional form:

$$Y = AK^{\alpha}(uN)^{1-\alpha} \tag{7}$$

Output, Y, is homogeneous of first degree and it possesses elasticity α in respect to built capital and elasticity $1 - \alpha$ in respect to natural capital allocated to production.

3.3 Materialisation

According to the Environmental Kuznets Hypothesis (EKH), environmental problems and income should have an inverse-U relation, and so, from a certain point in time onward, environmental impact should decrease as the economy grows[10]. The factors that might explain the EKH are scale, composition and technological change (Grossman and Krueger, 1995; Torras and Boyce, 1998). As the economy grows, pollution and the demand for resources also grow (the scale effect), but if economic sectors with lower than average environmental impact grow above average (composition effect) and new cleaner technologies are invented (technological change), overall environmental impact may decrease.

A way to model these ideas is to recall the Ehrlich-Commoner debate and the IPAT equation (Dunlap, 1993). This long lasting debate questioned the relative merit of population and technology in environmental degradation and was eventually overcome in the IPAT equation: $I = P \cdot A \cdot T$, where I is environmental impact, P is population, A is affluence (GDP per capita) and T is technology (environmental impact per GDP). With this in mind we can start by saying that environmental impact is the product of market activity of the whole society and a coefficient that expresses the material throughput per unit of economic activity, $P = m \cdot Y$. Let us call

[10] See Andreoni and Levinson (2001) and Lieb (2001) for recent developments on the EKH.

materialisation to *m*. As innovation takes place and society learns to use resources better, we expect *m* to decrease, thus $\partial m/\partial A < 0$, where *A* stands for knowledge.

The process of dematerialization, i.e., the decrease in the environmental intensity of the economy, must comprehend the two dimensions of technological change (leading to a reduction of *m*) and composition. A way to capture both effects may be to use the functional forms $P = mY^n$ and $m = m_0 A^{-a}$, with positive *n*, *a* and m_0. The latter term is a scale factor, *a* is the elasticity of materialisation with respect to knowledge (capturing the environmental benefit of technological change) and *n* is the elasticity of pollution with respect to production (capturing the environmental impact of the composition effect). Thus:

$$P = m_0 A^{-a} Y^n \tag{8}$$

We will consider as a first approach that *a* and *n* are constant and exogenous. We suspect that structural change is demand controlled, with a more environmentally friendly society favouring the success of more environmentally benign economic activities. To consider that materialisation changes exogenously does not imply that environmental expenditure is irrelevant, on the contrary, we consider that *a* is probably a function of environmental expenditure, but at this stage we neglect this further complication.

We consider that environmental expenditure consists of three parcels with different properties: pollution abatement, restoration effort and environmentally biased innovation. We consider that pollution abatement, the expenditure usually considered in this context (Belbute, 1999; Aghion and Howitt, 1998; Andreoni and Levinson, 2001; Lieb, 2001), manifests in a reduction of the flow of pollution released to the natural environment. Therefore the flow of pollution being abated cannot exceed the flow of pollution being generated. Restoration effort accelerates natural regeneration and is therefore limited by environmental quality (the ratio between natural capital and its carrying capacity). Environmentally biased innovation is the investment in purposefully natural capital saving technology.

The concept of materialization, though not common in growth theory, is important in Industrial Ecology. Hinterberger et al. (1997) argue that natural capital stock maintenance concerns should be dropped in favour of material flow accounting, for purposes of sustainability assessment. Material input per service provided (MIPS) is a measurement of material flows (whether embodied, consumed, lost due to inefficiency or rejected) involved in provision of an economic service, thus being a rather good proxy for materialisation, i.e. we expect $m \propto MIPS^n$. In biophysical terms there is no analogue to currency and therefore the aggregation of material flows always involve subjective valuation, which advises caution when dealing with flow accounting (Canas, 2002).

3.4 Utility function and environmental concerns

The driving force of the economic process is the pursuit of happiness. Producers seek profit and consumers seek access to goods and services. We consider in our model that a continuously overlapping succession of individuals will behave so as to maximise their utility function throughout their lives. Therefore, utility, *U(t)*, should be such that the integral of present-value utility between initial and infinite time, $\int_0^\infty e^-$

$^{\rho t}$ *U(t)dt*, is a maximum, where ρ is the discount rate[11]. We consider that utility is a function of consumption, *C(t)*, and of direct environmental services provided by free natural capital, *(1-u(t))N(t)*, valued by society's environmental concerns, ϕ. Following Belbute (1999), we consider that utility has constant and unitary intertemporal elasticity of substitution:

$$U(C, (1-u)N) = \ln C + \phi \cdot \ln((1-u)N). \qquad (9)$$

With this formulation, there is diminishing but ever positive marginal utility, the usual properties of the utility function are observed (Belbute 1998b). A strong but convenient underlying assumption is that consumption is independent of the environment upon which it takes place, i.e. the cross derivative $\partial^2 U / \partial C \partial((1-u)N)$ is zero.

ϕ is the elasticity of substitution between consumption and direct environmental services for constant utility. From eq. (9):

$$\phi = -\frac{dC}{C}\frac{(1-u)N}{d\big((1-u)N\big)}\bigg|_{U=const.} \qquad (10)$$

The bioeconomic meaning of ϕ is not so straightforward because it depends on whether we interpret the role of direct environmental services as hedonistic or materialistic (i.e, if they are immaterial "wants" or physiological "needs"). It has been suggested that environmental concerns rise with income (Martínez-Alier, 1995). On the other hand, the physiological needs of humans vary with the environment (mostly with Latitude, Parker, 2000), supporting the view of a materialistic role. Aghion and Howitt (1998) point out that in the last centuries humankind has successfully substituted many environmental services by economic ones, therefore replacing *N* by *C* in the Utility Function. If this is true, then the role of environmental concerns (whether materialistic or hedonistic) should be similar to that of consumption goods. An important question is what happens in the case of extreme depletion of natural capital. Considering the limit is the same as to talk about basic needs, which involve limits to substitutability (Stern, 1997) (...). It is reasonable to assume that as both *N* or *CC* tend to zero, no amount of the other good can compensate the loss of welfare, thus $\lim_{X \to 0} \frac{\partial U}{\partial X} = \infty;\ X = C, (1-u)N$. This property is verified by our specific functional form (eq. 9). We believe that for low levels of both *N* and *C*, we are talking about "needs" but in an affluent society or environment, the individual will satisfy his "wants". In this case ϕ may be not only dependent on the environment but also be able to change according to societal preferences. In our model we will consider constant ϕ.

3.5 Sustainability constraints

According to the Bruntland Report, Sustainable Development "*meets the needs of the present generation without compromising the ability of future generations to meet their own needs*". In this statement we find the concepts of intergenerational solidarity (long run optimisation), equity (social welfare optimisation) and efficiency (to balance both). The traditional concepts of weak (Cabeza Gutés, 1996), strong (van den Bergh and de Mooij, 1997; Constanza et al., 1998a) and sensible sustainability (Serageldin and Steer, 1994) are based on considerations about technology (the degree of

[11] We will consider a constant discount rate. See Azar and Holmberg (1995), Rabl (1996) and Hall (2000) for a discussion on intergenerational discounting and Bruce (1995) on the biological basis of discounting.

complementarity between built and natural capital) and ecosystem functioning (whether there are or not lower thresholds of ecological viability). We will not make use of these traditional concepts as operational tools because they take empirical facts as theoretical assumptions and because they do not accommodate the specificities of our model (dynamics for *CC*, materialisation and direct environmental services).

An operational definition of sustainability may consist in a set of two constraints: intergenerational equity and biophysical sustainability.

Intergenerational equity, an implicit assumption of traditional sustainability concepts, demands:

$$dU/dt \geq 0, \tag{11}$$

Or non-diminishing social welfare (here interpreted as utility). Because we consider $U = U(C, N)$ and $Y = Y(N, K)$, environmental degradation is reflected twofold upon U: through direct services provision and *via* $C(Y)$.

Biophysical sustainability imposes as a general constraint that the ecological system does not collapse. We will consider that natural capital, N, and its carrying capacity, CC, must both remain non-negative:

$$N > 0 \text{ and } CC > 0. \tag{12}$$

Regarding what a sustainable scenario may be, Daly (1977) proposed that an optimal size for the human economy exists and that a “steady state” economy should be reached. Endogenous growth theory allows growth to continue indefinitely if environmental concerns and innovation are taken into the picture, so that the “material” side of the economy ceases to grow while its intellectual side keeps on growing (Aghion and Howitt, 1998). For the moment, depletion of natural capital is still increasing (Vitousek, 1997), raising the suspicion that our current growth path is probably unsustainable.

4 An optimal sustainable growth path

4.1 General optimal sustainable growth path

The purpose of society is to maximise intertemporal utility subject a discount rate, ρ:

$$\int_0^\infty e^{-\rho t} U(C,(1-u)N;\phi)dt\,. \tag{13}$$

Instantaneous utility is given by eq. 9. Differentiating eq. (7) we obtain:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\left(\frac{\dot{N}}{N} + \frac{\dot{u}}{u}\right). \tag{14}$$

The exogenous dynamics of A are given by eq. (5), capital accumulation by eq. (6) and natural capital is subject to eqs. (1-4). The dynamics of P are given by eq. (8). Because knowledge is generated exogenously, society’s state variables are K, N and CC. Society’s control variables are C and u. The exogenous parameters of the model are ϕ, n, a, α, r, l, ρ, δ and g.

The Hamiltonian of the system is:

$$H = U(C,N,u) + \lambda_1\frac{dK}{dt} + \lambda_2\frac{dN}{dt} + \lambda_3\frac{dCC}{dt}\,. \tag{15}$$

According to Pontryiagine's maximum principle (Tu, 1996), along the optimal growth path:

$$\frac{\partial H}{\partial C} = 0 \quad \text{and} \quad \frac{\partial H}{\partial u} = 0 . \tag{16}$$

Euler's motion equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial K}; \quad \dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial N}; \quad \dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial CC} . \tag{17}$$

The solution is subject to the transversality constraints:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1 K = \lim_{t\to\infty} e^{-\rho t}\lambda_2 N = \lim_{t\to\infty} e^{-\rho t}\lambda_3 CC = 0 . \tag{18}$$

The controllability domain is given by:

$$u \in \left]0,1\right] \quad \text{and} \quad C(t) \le Y(t) . \tag{19}$$

The solution must obey the sustainability constraints given by eqs. (11) and (12) and is also subject to initial conditions (let $t_0 = 0$):

$$A(0) = A_0; \; K(0) = K_0; \; N(0) = N_0; \; CC(0) = CC_0; \; u(0) = u_0; \; C(0) = C_0 . \tag{20}$$

Let us make the transformation of variables $c = C/K$, $y = Y/K$ and $P = mY^n$. The general solution, obeying Pontryiagine's maximum principle, Euler's motion equations, the state equations and the objective integral can be simplified to the following dynamic system (there is an implicit equation for the shadow-price of built capital, because $\lambda_1 = 1/C$):

$$\frac{dU}{dt} = \frac{\dot{c}}{c} + \frac{\dot{K}}{K} + \phi\left(\frac{\dot{N}}{N} - \frac{\dot{u}}{1-u}\right); \tag{21}$$

$$\frac{\dot{K}}{K} = y - c - \delta \; ; \tag{22}$$

$$\frac{\dot{y}}{y} = g + (1-\alpha)\left(\frac{\dot{N}}{N} + \frac{\dot{u}}{u} - y + c + \delta\right); \tag{23}$$

$$\frac{\dot{c}}{c} = -(1-\alpha)y - n\alpha\lambda_2 Pc + c - \rho \; ; \tag{24}$$

$$\frac{dN}{dt} = rN(CC - N) - P ; \tag{25}$$

$$\frac{dCC}{dt} = l\left(\frac{\dot{N}}{N} - \frac{\dot{u}}{1-u}\right); \tag{26}$$

$$\frac{\dot{P}}{P} = -ag + n\left(g + \alpha\left(y - c - \delta\right) + (1-\alpha)\left(\frac{\dot{N}}{N} + \frac{\dot{u}}{u}\right)\right); \tag{27}$$

$$\frac{\lambda_3 l}{1-\alpha}\frac{\dot{u}}{1-u} = \frac{1-u}{u}\left(y - \lambda_2 nP\right) - \frac{\phi}{1-\alpha} ; \tag{28}$$

$$\frac{d\lambda_2}{dt} = \lambda_2\left(\rho - R_N + n(1-\alpha)\frac{P}{N}\right) - \left(\frac{\phi}{N} + (1-\alpha)\frac{y}{N}\right) + \lambda_3 l \frac{\dot{N}}{N^2}\,; \tag{29}$$

$$\frac{d\lambda_3}{dt} = \lambda_3\rho - \lambda_2 R_{CC}\,. \tag{30}$$

This is an autonomous optimal system of 8 equations (eqs. (21) and (22) are obtained by solving the other equations). We will focus on finding a particular solution that meets the double goal of biophysical steady state and unbounded balanced economic growth, i.e.

$$\frac{dN}{dt} = \frac{dCC}{dt} = \frac{dy}{dt} = \frac{dc}{dt} = 0\,.$$

4.2 Biophysical steady-state and economic balanced growth

If we search for such a solution eqs. (23-26) must be set to zero. From eqs. (25-26) we obtain that eq. (27-28) too must be zero. Thus N^*, CC^*, u^*, P^*, R_N^*, c^* and y^* are constants. Simplifying the system it is noticeable that some equations are degenerate[12] and thus there are only three constraints for six variables, yielding three degrees of freedom.

From eqs. (24) and (27) we obtain our first important result:

$$y^* - c^* - \delta = \frac{1}{1-\alpha} g = \frac{a}{n} g = zg\,; \tag{31}$$

A necessary condition for the existence of a steady state of eqs. (23-28) is that the ratio of dematerialisation elasticity, a/n, equals the inverse of the fraction of output of natural capital, $1/(1-\alpha)$. This is a strong constraint on the parameters of the model, indicating that the system is structurally unstable. If this technological constraint holds and society decides to consume a certain fraction of income so that eq. (31) holds, from eq. (22) and the definition of y and c that in the steady state (y^*, c^*, N^*, CC^*) society will experience balanced growth and the natural system will remain at steady state:

$$\frac{dU}{dt} = \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = zg \text{ with } z = \frac{a}{n} = \frac{1}{1-\alpha} \text{ and } \frac{dN}{dt} = \frac{dCC}{dt} = 0. \tag{32}$$

From eq. (25) we know that $P^* = R(N^*, CC^*)$. Recalling that $P = m_0 A^{-a} Y^n$ and that $Y = yK$ by definition, we can write:

$$y^{*n} = \left(\frac{A}{K^{1-\alpha}}\right)^a \frac{R(N^*, CC^*)}{m_0}\,. \tag{33}$$

The steady state sustainable level of output per stock of capital is proportional to the ratio $A/K^{1-\alpha}$ and to the regeneration of natural capital. Because $A/K^{1-\alpha}$ is constant (from eq. 33), the ratio of the initial stocks of A and K sets y for the whole growth trajectory (note that $R(N^*, CC^*)$ is also constant). Because of eq. (31), only if the $A/K^{1-\alpha}$ ratio and $R(N^*, CC^*)$ are such that $y^* > \delta + zg$ will it be possible to have positive consumption.

---

[12] Eq. (27) bears the same information as eq. (23), eq. (26) has no information and eqs. (28) and (24) must be manipulated to eliminate the term $n\lambda_2 P$.

Using eqs. (24) and (28) to eliminate the term $n\lambda_2 P$ and substituting c^* by y^* using eq. (31), we obtain a constraint on the steady state fraction of natural capital allocated to production in terms of the sustainable output per stock of capital:

$$\phi \frac{\alpha}{1-\alpha}\frac{u^*}{1-u^*} = \frac{(1-\alpha)y^* + \rho}{y^* - (\delta + zg)} + \alpha y^* - 1 . \qquad (34)$$

The term $u^*/(1-u^*)$ on the left side of eq. (34) is monotonous with u^* and its domain is R^+. The right side of eq. (34) is a function of y^*, it has a vertical asymptote at $y^* = \delta + zg$, in tends to αy^* as y* tends to infinity and has a a minimum at:

$$y^*_{min} = \delta + zg + \sqrt{\frac{1}{\alpha}\left(\rho + (1-\alpha)(\delta + zg)\right)} . \qquad (35)$$

Thus, u^* will be close to 1 both if y^* is near the lower threshold $\delta + zg$ and if y^* is very large but, for positive c^*, it will not be optimal and sustainable to allocate all natural capital to production. The closer y^* is to y^*_{min} (eq. 35) the smaller will u^* be. The higher ϕ is, the smaller will the optimal u^* be. A more interesting result is that if society is that the *less* impatient, that is, the lower ρ, the smaller will u^* be. If the discount rate is below a certain value ρ_{min} there will appear a region of y^* for which no positive u^* exists. Let ρ_{min} be such that $\beta_1(\rho_{min}) = 0$, where $\beta_1(\rho)$ is given by:

$$\beta_1 = (1 + \delta + zg)^2 - \frac{4}{\alpha}(\rho + \delta + zg). \qquad (36)$$

If $\rho > \rho_{min}$, society may allocate a stationary fraction of natural capital to production, u^*, according to eq. (34) and enjoy a sustainable steady growth path, where y^* is determined by eq. (33). If $\rho \le \rho_{min}$, society may only allocate $u^* \in \,]0, 1[$ if:

$$\delta + zg < y^* < 1 + \delta + zg - \sqrt{\beta_1(\rho)} \text{ or } 1 + \delta + zg + \sqrt{\beta_1} < y^* < +\infty . \qquad (37)$$

To check the transversality conditions, one must know the dynamics of the shadow prices, which simplify from eq. (32) (because $U_C = \lambda_1$, resulting from the first order maximum conditions) and from eqs. (29-30):

$$\frac{\dot{\lambda}_1}{\lambda_1} = -\frac{\dot{C}}{C} = -zg ; \qquad (38)$$

$$\frac{d\lambda_2}{dt} = \lambda_2(\rho - R_N) - \frac{\phi}{N^*}\frac{1}{1-u^*} ; \qquad (39)$$

$$\frac{d\lambda_3}{dt} = \lambda_3 \rho - \lambda_2 R_{CC} . \qquad (40)$$

The current value of the stock of built capital is $\lambda_1 K = 1/c^*$, which is constant so the transversality condition holds. Referring to natural capital and its carrying capacity, the analytical solutions to eqs. (39) and (40) are:

$$\lambda_2 = \beta_2 + e^{(\rho - R_N)t}\left(\lambda_{2,i} - \beta_2\right); \qquad (41)$$

$$\lambda_3 = \frac{\beta_2}{\rho}\left(e^{\rho t} - 1\right) + \frac{\lambda_{2,i} - \beta_2}{\rho}\left(e^{(\rho - R_N)t} - 1\right) + \lambda_{3,i} e^{\rho t} . \qquad (42)$$

Where $\lambda_{2,i}$ and $\lambda_{3,i}$ are the initial conditions and β_2 is given by:

$$\beta_2 = \frac{1}{\rho - R_N} \frac{\phi}{N^*} \frac{1}{1-u^*}. \tag{43}$$

For the transversality conditions (eq. 18) to hold and the shadow prices to remain nonnegative along the growth path, from eq. 41 we obtain that the productivity of natural capital, R_N, must lie between zero and ρ and also that $\lambda_{2,i} \geq \beta_2$. The productivity of natural capital is $R_N = r(CC- 2N)$, so the transversality condition poses a constraint on (N, CC). In the general case, the shadow price of the carrying capacity does not cope with the transversality condition. However, if the initial conditions are set so that $\lambda_{2,i} = \beta_2$ and $\lambda_{3,i} = \beta_2 R_{CC}/\rho$ we have $\dot{\lambda}_{2,i} = 0$ and $\dot{\lambda}_{3,i} = 0$ and the transversality conditions hold.

4.3 Summary and discussion of the results

We will now summarise the results from the previous section. Suppose that the technological parameters a, n and α obey $a/n = 1/(1-\alpha)$ and the natural system is in an original state (N_0, CC_0) such that $0 < R_N < \rho$. There is a sustainable value of output per stock of built capital, y^*, determined by the initial stocks of built and intellectual capital, K_0 and A_0, and the initial regeneration of natural capital, $R(N_0, CC_0)$, according to eq. (33). Suppose that the discount rate ρ obeys $\rho > \rho_{min}$ where ρ_{min} is implicitly defined by $\beta_1 = 0$ (eq. 36) and y^* is larger than $\delta + zg$. If $\rho \leq \rho_{min}$, suppose instead that y^* belongs to the set defined by eq. (37). Finally, assume that the initial value of the shadow prices of natural capital and its carrying capacity, $\lambda_{2,i}$ and $\lambda_{3,i}$, are, respectively, β_2 and $\beta_2 R_{CC}/\rho$, where β_2 is defined by eq. (43). If society allocates a fraction of natural capital, u_0, to production, such that eq. (34) holds and decides to consume a flow C_0 such that eq. (31) holds, then society will experience steady economic growth and the natural system will remain in stationary state (eq. 32).

From a technical point of view, the dynamical system described by eqs. (23-30) possesses a 3-dimensional hyper-surface (on y, N and CC) of fixed points if $a/n = 1/(1-\alpha)$. Let X = (y, c, N, CC, P, u, λ_1, λ_2), X^* be one of the fixed points and $J(X^*)$ represent the Jacobian of the system on X^*. With a first order expansion:

$$X - X^* \approx J(X^*) \cdot (X - X^*) \tag{44}$$

The behaviour of the linearised system is determined by the eigenvalues of $J(X^*)$. If $J(X^*)$ possesses at leat one eigenvalue with negative real part, then there is at least one trajectory leading to that fixed point, even though that fixed point may not be a stable equilibrium. We performed numerical calculations for a set of plausible parameters and initial conditions and found out that such a trajectory always exists[13].

The system is structurally unstable, i.e., given a small perturbation of the parameters, the behaviour of the system will change drastically. If $a/n \neq 1/(1-\alpha)$, not only will there be no solution that satisfies unbounded economic growth and biophysical steady state but it will be extremely hard to tell what kind of solution exists at all. Structural instability is an undesirable property for biological and economic models (Clark,

[13] The parameters we found most plausible were: a share of capital, α, of 0.7 (calculations ranging from 0.5 to 0.9); discount, depreciation and knowledge generation rates, ρ, δ and g of 0.05, 0.03 and 0.02 on a year basis (ranging from 0.1 to to 0.15); n and ϕ no larger than 1; $a = n/(1-\alpha)$; regarding the biophysical parameters, r and l, we used a range of 0.1 to 100. The initial conditions were generated by N ranging from 1 to 1000, CC between $2N$ and $2N + \rho/r$ and $(A/K^{1-\alpha})^a$ spawning 3 orders of magnitude over the critical value that verifies $y^* > \delta + zg$.

1976). Structural instability typically appears in conservative systems, while dissipative systems are (typically) structurally stable. An example is the model of the harmonic oscilator (structurally unstable), where the introduction of dampening makes the model structurally stable. In our model "dampening" may be introduced by removing the unrealistic assumptions of exogenous dematerialisation (determined by exogenous accumulation of intellectual capital and constant parameters) and that the allocation of natural capital is reversible, costless and instantaneous. But doing so would add extra degrees of complexity to an already complicated model.

5 Conclusions

We presented an extension of the neoclassical growth model with natural capital and exogenous technological change with two main novelties: allocation of natural capital and dematerialisation. The first idea was taken from conservation biology and it acknowledges that natural capital used for productive processes does not provide the same positive externalities as free natural capital. Therefore, there is a trade-off between the extension of human domination of the biosphere, increasing production, and the maintenance of ecosystem services, necessary both for ecological integrity and to provide direct welfare to humans. The second idea comes from industrial ecology and draws on the asssumption that the environmental impact of the economic process depends on the material throughput of the economy and that throughput per unit of wealth has steadily decreased over time. Therefore, long run sustainability is achievable if dematerialisation, which is caused by the change of the composition of the production sector and by innovation, outweights the environmental impact of economic growth.

We found that, for some set of technological parameters and initial conditions, it is possible to experience unbounded economic growth and to keep the natural system in steady state. For that to happen the ratio of dematerialisation elasticities must exactly offset the inverse of the fraction of natural capital in production, $a/n = 1/(1 - \alpha)$. Natural regeneration and the ratio of the initial stocks of built and intellectual capital determine the sustainable output per stock of built capital, y^*, which must be high enough to allow for both consumption and built capital accumulation. The relation between the optimal sustainable fraction of natural capital allocated to production, u^*, and y^* is U-shaped. If the discount rate is low, there will be a region of y^* for which no sustainable steady growth path is possible. Finally, the productivity of natural capital is bounded by zero and the discount rate.

If some policy implication can be gained from this theoretical model, it is that in order to achieve sustainability the main focus should not be to halt economic growth but rather to change its quality. There is room for policy action in promoting the growth of less environmentally harmful production sectors, in exploring the environmental benefit of innovation in general and in fostering natural capital saving innovation. None of these ideas is new, but they have been set in a formal framework.

Elasticiy of materialisation and the degree of substitutability between man-made and natural capital are key factors determining whether long term economic growth is sustainable. Two promising lines of research are to make these parameters endogenous and to explore a generalised optimal solution of this model. Even though this work has been purely theoretical, we note that the main assumptions of our model (the dynamics of natural capital and dematerialisation) may be subject to empirical testing. In fact, the empirical work currently done in ecosystem service accounting and material flow analysis may in time provide such testing.

In this model we assume that dematerialisation is exogenous and that allocation of natural capital is reversible, costless and instantaneous. These are unrealistic assumptions but they were necessary, to build a simple and manageable descriptive model. Yet, for growth theory to become a general theory of sustainable development, the mechanics of natural capital allocation and dematerialisation must be unveiled, so that we can move from descriptive to mechanistic models.

6 References

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory, MIT Press, Cambridge.

Andreoni, J., Levinson, A., 2001. The simple analytics of the environmental Kuznets curve. Journal of Public Economics, 80:269-286.

Azar, C. and Holmberg, J., 1995. Defining the generational environmental debt. Ecological Economics, 14: 7-19.

Ayres, R. U., van den Bergh, J., Gowdy, J. M., 1998. Viewpoint: weak vs. strong sustainability. Tinbergen Institute, discussion paper 98-103.

Belbute, J., 1998a. Algumas reflexões sobre as interacções entre a economia e o ambiente. Economia e Sociologia, 67: 31-56.

Belbute, J., 1998b. Acumulação de capital num contexto de interacções entre economia e ambiente. Estudos de Economia, XVIII (4).

Belbute, J., 1999. Preferências, crescimento endógeno e sustentabilidade. Estudos de Economia, XIX (3).

Blaug, M., 1991. Methodology in Economics, 4th Ed. Cambridge U. Press, Cambridge.

Bruce, H., 1994. Sense of place: geographic discounting by people, animals and plants, Ecological Economics (10)2: 157-174.

Canas, A., 2001. Análise da intensidade de utilização de materiais na economia. (MSc. thesis) Instituto Superior Técnico, Lisboa.

Cabeza Gutés, M., 1996. The concept of weak sustainability. Ecological Economics, 17: 147-156.

Clark, Colin W., 1976. Mathematical Bioeconomics. John Wiley & Sons, New York.

Clark, Colin W., 1997. Renewable resources and economic growth. Ecological Economics, 22: 275-276.

Cohen, J. E., 1995. Population Growth and Earth's Human Carrying Capacity. Science, 269: 341-348.

Constanza, R., Cumberland, J., Daly, H., Goodland, R., Norgaard, R., 1997a. An Introduction to Ecological Economics. St. Lucie Press, Boca Raton.

Constanza, R., d' Arge, R., de Groot, R., Farber, S., Grasso, M., Hannon, B., Naeem, S., Limburg, K., Paruelo, J., O'Neill, R.V., Raskin, R., Sutton, P., van den Belt, M., 1997b. The value of the world's ecosystem services and natural capital. Nature, 387: 253-260.

Constanza, R., d' Arge, R., de Groot, R., Farber, S., Grasso, M., Hannon, B., Naeem, S., Limburg, K., Paruelo, J., O'Neill, R.V., Raskin, R., Sutton, P., van den Belt, M., 1998. The value of ecosystem services: putting the issues in perspective. Ecological Economics, 25: 67-72.

Daily, G. C. (ed.), 1997. Nature's Services: Societal Dependence on Natural Ecosystems. Island Press, Washington DC.

Daly, H. E., 1977. Steady State Economics: The Political Economy of Biophysical Equilibrium and Moral Growth. W.H. Freeman & Co., San Francisco.

Daly, H. E., 1997. Georgescu-Roegen versus Solow-Stiglitz. Ecological Economics, 22: 261-266.

Dunlap, R. E., 1993. From environmental to ecological problems. In: Calhoun et al (eds.). Social Problems. McGraw-Hill, New York.

Endres, A., Radke, V., 1998. Land Use, Biodiversity, and the Theoretical Structure of a Sustainability Indicator. Global Network of Environmental Economists - Working paper (http:\\www.feem.it/gnee/pap-abs/endre.pdf).

England, R. W., 1998a. Measurement of social well-being: alternatives to gross domestic product. Ecological Economics, 25: 89-103.

England, R. W., 1998b. Should We Pursue Measurement of the Natural Capital Stock? Ecological Economics, 27: 257-266.
England, R. W., 2000. Natural capital and the theory of economic growth. Ecological Economics, 34 (3): 425-431.
Faber, M., Manstetten, R., Proops, J., 1994. On the conceptual foundations of ecological economics: a teleological approach. Ecological Economics, 12: 41-54.
Georgescu-Roegen, N., 1971. The Entropy Law and the Economic Process. Harvard University Press, Cambridge.
Grossman, G. and Krueger, A., 1995. Economic growth and the environment. Q. J. Econ. 110, 353-377.
Gurney, W. S. C., Nisbet R. M., 1998. Ecological Dynamics. Oxford University Press, New York.
Hall, C. A. S., 2000. Quantifying Sustainable Development. Academic Press, New York.
Harte, M. J., 1995. Ecology, sustainability, and environment as capital. Ecological Economics, 15: 157-164.
Hinterberger, F., Luks, F., Schmidt-Bleek, F., 1997. Material flows vs. 'natural capital'. What makes an economy sustainable? Ecological Economics, 23: 1-14.
Kaufmann, R. K., 1995. The economic multiplier of environmental life support: can capital substitute for a degraded environment? Ecological Economics, 12: 67-79.
Keil, T., 1998. Substitution or technical progress: a production theoretic perspective on the sustainability debate. Global Network of Environmental Economists - Working paper (http:\\www.feem.it/gnee/pap-abs/keil.pdf).
Kolstad, C. D., Toman, M., 2001. The Economics of Climate Policy. Resources For the Future. Discussion paper 00-40REV.
Levin, S., 1999. Fragile Dominion. Perseus Books, Reading, Massachusetts.
Lieb, C. M., 2001. The Environmental Kuznets Curve and Flow Versus Stock Pollution - When Abatement Incresases Pollution. 11th Annual Conference of the European Association of Environmental and Resource Economists, Southampton.
Lucas, R., 1988. On the Mechanics of Economic Development. Journal of Monetary Economics, 22(1): 3-42.
Martínez-Alier, J., 1995. The environment as a luxury or "too poor to be green"? Ecological Economics, 13: 1-10.
Nelson, R. H., 1997. In memoriam: On the death of the 'market mechanism'. Ecological Economics, 20: 187-197.
North, D. C., 1990. Institutions, Institutional Change and Economic Performance. Cambridge University Press, New York.
Odum, E. P., 1969. The Strategy of Ecosystem Development. Science, 164: 262-270.
Parker, Philip, 2000. Physioeconomics - the basis for long-run economic growth. MIT Press, Cambridge.
Peters, R. H., 1991. A critique for ecology. Cambridge University Press, Cambridge.
Rabl, A., 1996. Discounting of long-term costs: What would future generations prefer us to do? Ecological Economics, 17: 137-145.
Rebelo, S., 1991. Long-Run Policy Analysis and Long-Run Growth. Journal of Political Economics, 99: 500-521.
Rodrigues, J., Domingos, T., Conceição, P., 2002. Human-Nature interactions: the problem of structural interference. Submitted to Ecological Economics.

Scheffer, M., Carpenter, S., Foley, J., Folke, C., Walker, B., 2001. Catastrophic shifts in ecosystems. Nature, 413: 591-596.

Schneider, E. D., Kay, J. J., 1994. Life as a Manifestation of the Second Law of Thermodynamics. Mathematical and Computer Modelling, 19: 25-48.

Schulze, E.-D., Mooney, H. A. (eds.), 1994. Biodiversity and Ecosystem Function. Springer-Verlag, Berlin.

Serageldin, I., Steer, A., 1994. Epilogue: Expanding the Capital Stock. In Serageldin, I., Steer, A. (eds.). Making Development Sustainable: From Concepts to Action, The World Bank, Washington, D.C.

Solow, R., 1974. Intergenerational equity and exhaustible resources. Rev. Econ. Stu., Symposium: 29-45.

Solow, R. M., 1997. Georgescu-Roegen versus Solow-Stiglitz - Reply. Ecological Economics, 22: 267-268.

Stern, D. I., 1997. Limits to substitution and irreversibility in production and consumption: A neoclassical interpretation of ecological economics. Ecological Economics, 21: 197-215.

Stiglitz, J. E., 1997. Georgescu-Roegen versus Solow-Stiglitz - Reply. Ecological Economics, 22: 269-270.

Ulanowicz, R. E., 1986. Growth and Development. Springer-Verlag, New York

van den Bergh, 1998. Materials, capital, direct/indirect substitution and mass balance production function. Tinbergen Institute, discussion paper 98-065.

van den Bergh, J., de Mooij, R. A., 1997. An assessment of the growth debate: a comparison of perspectives. Tinbergen Institute, discussion paper 97-096.

van den Bergh, J., Hofkes, M. W., 1997. A survey of economic modelling of sustainable development. Tinbergen Institute, discussion paper 97-107.

van den Bergh, J., Verbruggen, H., 1998. Spatial sustainability, trade and indicators: an evaluation of the “ecological footprint”. Tinbergen Institute, discussion paper 98-105.

Vitousek, P. M., Mooney, H. A., Lubchenco, J., Melillo, J. M., 1997. Human Domination of Earth’s Ecosystems. Science, 277: 494-499.

Wackernagel, M., Rees, W. E., 1997. Perceptual and structural barriers to investing in natural capital: Economics from an ecological footprint perspective. Ecological Economics, 20: 3-24.