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Estimation of component redundancy in optimal age maintenance

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ABSTRACT

The classical Optimal Age-Replacement defines the maintenance strategy based on the equipment failure consequences. For severe consequences an early equipment replacement is recommended. For minor consequences the repair after failure is proposed. One way of reducing the failure consequences is the use of redundancies, especially if the equipment failure rate is decreasing over time, since in this case the preventive replacement does not reduce the risk of failure.

The estimation of an active component redundancy degree is very important in order to minimize the life-cycle cost. If it is possible to make these estimations in the early phase of system design, the implementation is easier and the amortization faster.

This work proposes an adaptation of the Optimal Age-Replacement method in order to simultaneously optimize the equipment redundancy allocation and the maintenance plan. The main goal is to provide a simple methodology, requiring the fewer data possible.

A set of examples are presented illustrating that this methodology covers a wide variety of operating conditions. The optimization of the number of repairs between each replacement, in the cases of imperfect repairs, is another feature of this methodology.

KEYWORDS: redundancy; age-replacement; maintenance strategy; life-cycle cost.

1 INTRODUCTION

The classical Optimal Age-Replacement defines the maintenance strategy based on the equipment failure consequences, as function of the failure rate variation over the operating time. The consequences of a failure are measured by the difference between the cost of repair, before the occurrence of failure and the cost of repair after a failure in service. For severe consequences an early equipment replacement is recommended. For minor consequences the repair after failure is proposed.

One method of reducing the failure consequences is by increasing the system reliability. This can be achieved by using more reliable components, by using the components just when their failure rates are minimal or by the use of redundancies. Unlike the first two strategies, which lead to small system reliability increases, the use of redundancies provides an exponential growth. Therefore, when the consequences of failure are relevant, it is necessary to examine the pertinence of using redundancy.

The estimation of an active component redundancy degree is very important in order to minimize the life-cycle cost. If it is possible to make these estimations in the early phase of system design, the implementation is easier and the amortization faster.

This work proposes an adaptation of the Optimal Age-Replacement method, in order to simultaneously optimize the equipment redundancy allocation and the maintenance plan. The main goal is to provide a simple methodology, requiring the fewer data possible.

Because reliability gains are difficult to access when using standby redundancy, since a large quantity of data about equipment performance over the time is necessary, which is very hard to get in the early phase of system design, the application of standby redundancy is not covered by this approach.

2 MATHEMATICAL FORMULATION

For a redundant system of n equal equipment operating simultaneously and in parallel, the system probability of failure function $F_S(t)$, the system reliability function $R_S(t)$, and the system failure probability density function $f_S(t)$, are obtained from the correspondent equipment reliability functions by:

$$F_S(t) = F(t)^n \tag{1}$$

$$R_S(t) = 1 - F(t)^n \tag{2}$$

$$f_S(t) = \frac{dF_S(t)}{dt} = n \times F(t)^{n-1} \times f(t) \tag{3}$$

Where $f(t)$ and $F(t)$ are the failure probability density and probability functions for each equipment.

The basis for the development of this work is the well-known Optimal Age-Replacement formula of Jardine /7./ (also considered in/1./ - /9./), which for only one equipment is given by:

$$C(t_p) = \frac{C_p \times R(t_p) + C_f \times \left[1 - R(t_p)\right]}{t_p \times R(t_p) + \int_{-\infty}^{t_p} t \times f(t)dt} \tag{4}$$

where: $C(t_p)$ – Total expected replacement cost per unit of time.
t_p- Preventive replacement age.
C_p – Cost of a preventive replacement.
C_f – Cost of a failure replacement.

The operation time, can be exchanged by other counting unit: cycles, energy consumption, production units, etc.

Let C_A denote the equipment acquisition cost including all the fixed costs that result from the possession of an equipment: purchase price, operating permits, space, etc. For a system the total acquisition cost is:

$$C_{A_S} = n \times C_A \tag{5}$$

Similarly, for n equipment the system expected preventive replacement cost per replacement cycle is:

$$EC_{P_S}(t_p) = C_{p_S} \times R_S(t_p) = n \times C_P \times \left[1 - F(t_p)^n\right] \tag{6}$$

The preventive replacement cost is the average cost value due to repairing a defect prior to failure occurrence, including all materials and labour costs. At the preventive replacement age, all of the n equipments are replaced.

The system expected failure replacement cost per replacement cycle is:

$$EC_{f_S}\left(t_p\right)=C_{f_S}\times F_S\left(t_p\right)=\left[C_F+C_P\times\left(n-1\right)\right]\times F\left(t_p\right)^n \tag{7}$$

The system failure replacement cost is the sum a component failure replacement cost, with (*n-1*) components preventive replacement costs. It is assumed that the equipment is visited only when the time for preventive repair arrives, therefore it is time to make repairs in all working equipment and the possible failures are also detected only at this time. The failure repair costs are the average costs of in service failure occurrence, includes all costs for materials, labour loss of production, loss of image, etc.

The mean time of good operation, denoted $MTGO_S$, is the expected length of a good operation cycle. It is a function given by summing the equipment operating time, with the already failed equipment mean time of operation:

$$MTGO_S\left(t_p\right)=t\times R_S\left(t_p\right)+\int_0^{t_p} t\times f_S(t)dt \tag{8}$$

The $MTGO(+\infty)$ corresponds to the *MTBF* - Mean Time Between Failures.

Similar to Eq.(4) above, the system total expected replacement cost per unit of time is:

$$C_S(t_p)=\frac{C_{A_S}+EC_{p_S}\left(t_p\right)+EC_{f_S}\left(t_p\right)}{MTGO_S\left(t_p\right)}=\frac{C_{A_S}+C_{p_S}\times R_S\left(t_p\right)+C_{f_S}\times F_S\left(t_p\right)}{t\times R_S\left(t_p\right)+\int_0^{t_p} t\times f_S(t)dt} \tag{9}$$

The preventive replacement time t_p and the number of equipment n that minimize Eq. (9) are the optimal replacement age and redundancy.

Please note that the cost values considered must be chosen with caution, because all the costs that are proportional to the operation time can be neglected, since their induced costs per unit time are constants, which are process or equipment dependent only, and do not influence the t_p or the redundancy.

In the Eq.(9) it is assumed that any repair is perfect, in the sense that it is assumed that the component returns to an as good as new condition. Therefore the failure probability function is always the same. However, Eq.(9) can be generalized to the case of non-prefect repairs or different costs per repair, by considering different functions of reliability before and after each repair, and different costs. In this case, between each replacement, there will be imperfect repairs, where each can have different costs and failure rates. Therefore, the system total expected repair cost per unit of time, is now the ratio between the sum of the costs and the sum of the operation times of each one of these operations:

$$C_S(\vec{t}_p)=\frac{C_{A_S}+\sum_{i=1}^{k}\left[C_{p_{Si}}\times R_{Si}\left(t_{pi}\right)+C_{f_{Si}}\times F_{Si}\left(t_{pi}\right)\right]}{\sum_{i=1}^{k} MTGO_{Si}\left(t_{pi}\right)} \tag{10}$$

where: $\vec{t}_p$ – Vector of preventive repair ages

k – Number of preventive repairs (*k-1* imperfect repairs andone perfect repair).

Note that at the optimal k^{th} preventive repair, the equipment will be replaced instead of repaired, since the k+1 intervention will have a higher cost, because it is not the optimal. Therefore, between the optimal k preventive repairs there will be effectively only k-1 imperfect repairs.

The local minimums of Eq.(10) give, for each redundancy n, the optimal preventive repair ages for each operation t_{pi} (note that after repair i time is reset to 0). The global minimum corresponds to the optimal redundancy n and the corresponding t_{pi}, with lower cost.

3 NUMERICAL IMPLEMENTATION

The presented methodology allows the use of any parametric or nonparametric probability distribution. However, in the examples shown below the Weibull probability function is used, because of its simplicity and the possibility to have increasing and decreasing failure rates. In practice, the probability function that best fits the equipment failure times should be selected. In the presence of experimental data, a first good option for a probabilistic model is to consider the adjustment to a Weibull distribution by parametric estimation, because of its flexibility.

$$R(t) = e^{-\left(\frac{t}{\lambda}\right)^{\beta}} \tag{11}$$

$$F(t) = 1 - R(t) \tag{12}$$

$$f(t) = \frac{\beta}{\lambda} \times \left(\frac{t}{\lambda}\right)^{\beta-1} \times e^{-\left(\frac{t}{\lambda}\right)^{\beta}} \tag{13}$$

Where:

λ - is a scale parameter (sometimes termed characteristic life)

β - is a shape parameter (associated with the variation of the failure rates).

The computation process is easily implemented in spreadsheet software, providing a quick, cheap and standard way of using it and integrating it with other maintenance software. Only the numerical integration for calculating the mean time of good operation *MTGO*(t) can involve some programming depending on the accuracy required. The following examples were all generated using a spreadsheet implementation, however any other mathematical software can be used.

4 EXAMPLES AND DISCUSSION

For simplification, in the next examples all the costs and times were nondimensionalised, by taking the preventive repair cost to be $C_p=1$ monetary unit and the characteristic life of the Weibull model $\lambda=1$ time unit.

The results of the optimal values for operation are presented in tables. Each table is built for one type of equipment and each type of equipment is characterized by the acquisition cost $C_{A.}$ and the Weibull shape parameter β. Depending on the equipment operating conditions there are different failure repair costs. The best degree of redundancy for each situation (column presented in bold) is the one which presents the minimum value of the system total expected repair cost per unit of time $C(t_p)$(represented by the underlined value).

For each equipment operating condition, the values of optimal preventive repair age t_p, the correspondent percentage of failure repair $F(t_p)$, mean time of good operation $MTGO_S(t_p)$ and the system total expected cost of a run-to-failure decision$C(+\infty)$, are also presented.

4.1 Example 1: Low-priced equipment (C_A=C_p=1) with β=2 (increasing failure rate)

Consider a system with low-priced components where the acquisition cost is equal to the preventive replacement cost and the failure rates increase. For the failure repair costs $100C_p$, $18C_p$, $6C_p$, $3C_p$, the best redundancy configuration is analysed using Eq. (9) and the results presented in Table 1.

Table 1: Optimal values of operation for $C_A=C_p=1$ and $\beta=2$.

	C_f							
	$100C_p$			$18C_p$		$6C_p$		$3C_p$
	n=4	**n=3**	n=2	**n=2**	n=1	n=2	**n=1**	**n=1**
t_p **-** Optimal preventive repair time [λ]	0,655	**0,528**	0,358	**0,599**	0,346	0,917	**0,654**	**1,091**
$C(t_p)$ [C_p/λ]	14,48	**14,10**	15,22	**9,45**	11,78	6,67	**6,54**	**4,36**
Run-to-failure cost ***$C(+\infty)$*** [C_p/λ]	77,06	**81,37**	89,89	**18,33**	21,44	7,85	**7,90**	**4,51**
$F(t_p)$ [%Failure repair]	34,9%	**24,3%**	12,0%	**30,2%**	11,3%	56,9%	**34,8%**	**69,6%**
$MTGO_S(t_p)$ [λ]	0,654	**0,527**	0,357	**0,587**	0,333	0,843	**0,572**	**0,777**

It can be seen from Table 1 that for very small failure replacement costs ($C_f=3C_p$) the expected run-to-failure cost is only slightly higher than the optimal cost with preventive replacement. The optimal preventive repair time (1.091λ) and the optimum percentage of failures $F(t)=69,6\%$ (equipment that fail before being preventively replaced), are relatively high.

Without the use of redundancy, n=1, as the costs of failure increase, the percentage of failures and optimal time of preventive replacements decrease quickly.

The use of a$n=2$ redundancy is only useful for failure repair costs $C_f>7C_p$ and a $n=3$ redundancy only compensates for $C_f>36C_p$.

Therefore the redundancy can increase the optimal preventive replacement time and the optimal percentage of failures, without increasing the costs.

4.2 Example 2: Low-priced equipment ($C_A=C_p=1$) with $\beta=0,9$ (decreasing failure rate)

Consider now the system from Example 1 but with decreasing failure rates. The results from Eq.(9) are presented in Table 2.

Table 2: Optimal values of operation for $C_A=C_p=1$ and $\beta=0,9$.

	C_f							
	$100C_p$			$18C_p$		$6C_p$		$3C_p$
	n=9	**n=8**	n=7	**n=5**	n=4	**n=3**	n=2	**n=2**
t_p **-** Optimal preventive repair time [λ]	1,343	**1,219**	1,081	**2,072**	1,771	$+\infty$	$+\infty$	$+\infty$
$C(t_p)$ [C_p/λ]	17,86	**17,86**	18,03	**10,12**	10,31	**5,48**	5,56	**3,71**
Run-to-failure cost ***$C(+\infty)$*** [C_p/λ]	36,44	**37,42**	38,70	**10,62**	10,86	**5,48**	5,56	**3,71**
$F(t_p)$ [%Failure repair]	72,9%	**69,7%**	65,8%	**85,4%**	81,2%	**100%**	100%	**100%**
$MTGO_S(t_p)$ [λ]	1,329	**1,205**	1,070	**1,753**	1,494	**2,006**	1,617	**1,617**

For the case of decreasing failure rate, the equipment becomes more reliable with the age. Therefore, preventive replacements are not recommended since they increase the equipment failure probability. The "as good as new" state is the worst situation in terms of reliability. In this case, the optimal time of preventive repair only makes sense when we have redundancies, and this maintenance operation is just an inspection to substitute the failed components. For this situation the run-to-failure strategy becomes convenient, whenever no redundancy is used and for small failure repair costs $C_f<12$.

Comparing Table 1 and Table 2 it is observed that the optimum degree of redundancy, the optimal preventive replacement time and the minimum costs are higher for decreasing failure rates.

4.3 Example 3: Medium-priced equipment (C_A=5C_p=5) with β=2 (increasing failure rate)

Let's assume a 5 times increase in the acquisition cost and a duplication on the failure costs for the system from Example 1. Table 3 is obtained in this situation.

Table 3: Optimal values of operation for $C_A=5C_p=5$ and β=2.

	C_f							
	200C_p			36C_p		12C_p		6C_p
	n=4	**n=3**	n=2	**n=2**	n=1	n=2	**n=1**	**n=1**
t_p **-** Optimal preventive repair time [λ]	0,701	**0,573**	0,400	**0,675**	0,420	1,042	**0,774**	**1,219**
$C(t_p)$ [C_p /λ]	40,81	**<u>39,16</u>**	41,06	**<u>25,50</u>**	29,40	18,29	**<u>17,02</u>**	**<u>12,17</u>**
Run-to-failure cost ***C***(+∞) [C_p /λ]	160,60	**168,17**	184,15	**41,02**	46,26	20,07	**19,18**	**12,41**
$F(t_p)$ [%Failure repair]	38,8%	**28,0%**	14,8%	**36,6%**	16,2%	66,2%	**45,1%**	**77,4%**
$MTGO_S(t_p)$[λ]	0,699	**0,571**	0,398	**0,655**	0,397	0,920	**0,644**	**0,811**

Comparing Table 3 with Table 1, it is verified that the duplication on the failure costs compensates the 5 times increase in the acquisition cost in the optimal degree of redundancy, suggesting only a small increase in the optimal preventive repair time. As expected, more expensive equipment imply less redundancy and require a longer run between preventive replacements.

4.4 Example 4: High-priced equipment (C_A=20C_p=20) with increasing preventive repairs costs

This example assumes perfect repairs $f_i(t)=f(t)$, but considers the increase of the preventive repair cost with the number of repairs $C_{PSi}=C_P\times\alpha^{i-1}$. In this case, the system total expected repair cost per unit of time must be calculated by Eq. (10). Consider an equipment for which the acquisition cost is 20 times the cost of preventive repair ($C_A=20C_p=20$) and whose failure rate follows the Weibull model of increasing risk ($\beta=2$). In table 4 are presented the results for each one of the optimal preventive repair times, of a redundant system with 3 equipment operating simultaneously in parallel, with a ($C_f=100C_p=100$) failure repair cost and a successive 50% increase in the preventive repairs costs.

Table 4: Values of optimal operation for $C_A=20\times C_p=20$, β=2, C_f=100, n=3 and α=1,5.

Repair	i=1	i=2	i=3	i=4	**i=5**	i=6	i=7	i=8
t_p^i – Optimal preventive repair time [λ]	0,911	0,778	0,728	0,706	**0,696**	0,706	0,728	0,767
$\sum t_p^i$ [λ]	0,911	1,689	2,417	3,123	**3,819**	4,525	5,253	6,020
$\boldsymbol{C}(\vec{t}_p)$ [C_p/λ]	91,71	57,40	45,67	40,47	**<u>38,51</u>**	38,84	41,24	45,86
Run-to-failure cost $\boldsymbol{C}(\vec{t}_p \mid t_p^i{=+\infty})$ [C_p/λ]	125,54	84,64	67,76	58,75	**53,76**	51,50	51,49	53,76
$\boldsymbol{F(t_p^i)}$ [%Failure repair]	56,4%	45,4%	41,1%	39,3%	**38,4%**	39,3%	41,1%	44,5%
$\sum\boldsymbol{MTGO_{Si}(\vec{t}_p)}$ [λ]	0,880	1,646	2,365	3,064	**3,754**	4,453	5,172	5,927

Table 4 indicates that for archiving the minimum system total expected repair cost per unit of time of *38,51 Cp/λ*, the *k=5* preventive repairs have to be performed at the ages of *0,911λ*, *1,689λ*, *2,417λ* and *3,123λ*, and *0,696λ* after the fourth preventive repair (*k-1* imperfect repairs) it is better to replace the system for a new one, instead of executing de fifth repair which would cost $5{,}0625C_p$ ($1{,}5^4 \times C_P$) instead of just Cp. Note that the optimal k* in Eq.(10) corresponds to having (*k-1*) imperfect repairs and a substitution at *k*, since *k+1* interventions will be suboptimal.

The values in Table 4 are not the global minimums, since the number of redundancies *n* was not optimized, as Figure 1 below will also indicate. In Table 5 the number of redundancies is also optimized and it is found that the global minimum cost for *100Cp* corresponds to *n=2* redundancies and *k*=5* preventive repairs (*k-1=4* imperfect repairs and *k=5* is a replacement). Additional optimal values are presented for other *Cp*.

Table 5: Values of optimal operation for $C_A=20\times C_p=20$, β=2 and α=1,5.

	C_f							
	$200C_p$			$100C_p$			$36C_p$	$18C_p$
	n=3	n=5	n=4	n=3	**n=2**	n=1	**n=2**	**n=1**
	k*=5	k*=5	k*=5	k*=5	**k*=5**	k*=6	**k*=5**	**k*=+∞**
t_p^N - Optimal preventive repair time [λ]	**0,595**	0,931	0,826	0,696	**0,524**	0,292	**0,755**	**>5**
$C(\vec{t}_p)$ [C_p/λ]	**<u>44,29</u>**	44,47	41,15	38,51	**<u>38,35</u>**	53,92	**<u>28,40</u>**	**<21**
Run-to-failure cost $C(\vec{t}_p \mid t_p^N=+\infty)$ [C_p/λ]	**84,95**	56,33	54,38	53,76	**57,04**	74,75	**31,40**	**<u><21</u>**
$F(t_p^N)$ [%Failure repair]	**29,8%**	58,0%	49,5%	38,4%	**24,0%**	8,2%	**43,5%**	**100%**
$\sum MTGO_S(\vec{t}_{op})$[λ]	**3,209**	4,919	4,406	3,754	**2,869**	1,912	**3,935**	+∞

The system total expected repair cost per unit of time for a range of redundancy from *n=1* to *n=5* and for a set of more than 10 successive repairs with increasing costs, is presented in Figure 1.

4.5 Example 5: High-priced equipment ($C_A=20C_p=20$) with increasing failure rates.

In this example it is analyzed the introduction of imperfect repairs $f_i(t)\neq f(t)$. Comparing to Example 4, now the Weibull scale parameter is successively decreasing with each repair according to $\lambda_i= \lambda/\varepsilon^{i-1}$. Using Eq.(10), Table 6 presents results for each one of the optimal preventive repair times, of a redundant system of 3 equipment operating simultaneously and in parallel, with a ($C_f=100C_p=100$) failure repair cost and a successive decreasing of the Weibull scale parameter.

Table 6: Values of optimal operation for Ac=20×Pc=20, β=2, Fc=100, K=3 and ε=1,3.

Repair	i=2	i=3	i=4	i=5	i=6	i=7	**i=8**	i=9
t_p^i – Optimal preventive repair time [λ]	0,595	0,425	0,311	0,230	0,173	0,131	**0,099**	0,075
$\sum t_p^i$ [λ]	1,505	1,930	2,241	2,471	2,645	2,775	**2,874**	2,949
$C(\vec{t}_p)$ [C_p/λ]	62,23	52,30	47,62	45,17	43,87	43,26	**<u>43,09</u>**	43,21
Run-to-failure cost $C(\vec{t}_p \mid t_p^i=+\infty)$ [C_p/λ]	95,54	84,30	78,75	75,80	74,25	73,53	**73,34**	73,51
$F(t_p^i)$ [%Failure repair]	42,47%	35,66%	30,77%	27,10%	24,41%	21,95%	**19,97%**	18,15%

$\sum MTGO_{si}(\vec{t}_p)$ [λ]	1,467	1,889	2,198	2,428	2,601	2,731	**2,830**	2,905

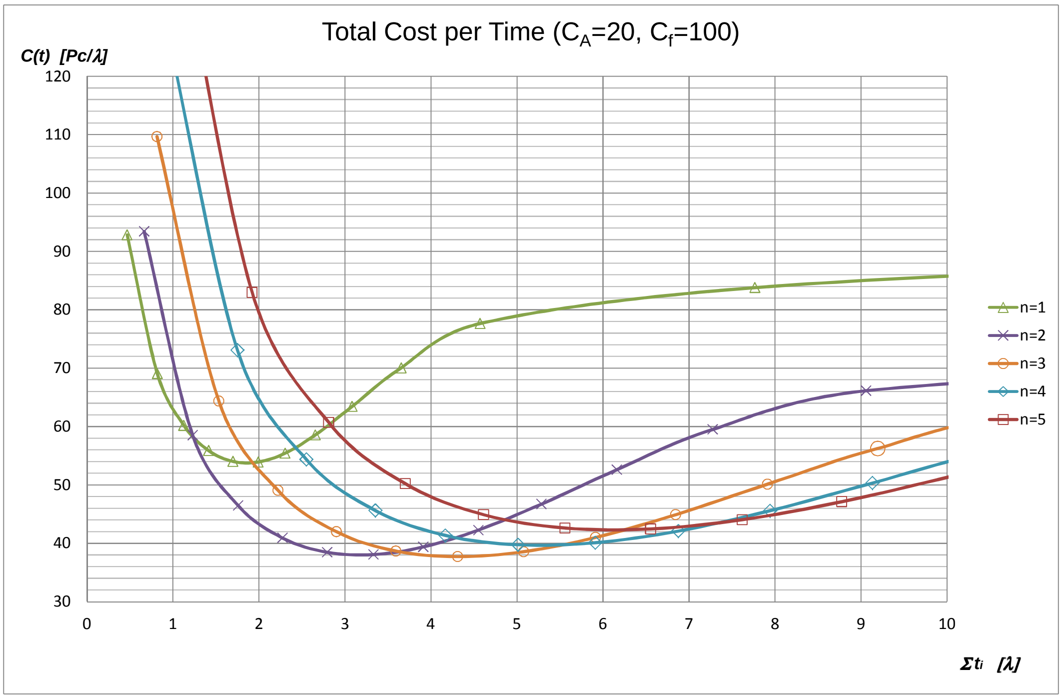

Figure 1: Graph with C_A=20×C_p=20, β=2, C_f=100C_p and α=1,5.

In Figure 2 are presented the system total expected repair cost per unit of time for a range of redundancy from *n=1* to *n=5* and for a set of more than 10 successive repairs with successive decreasing of the Weibull scale parameter.

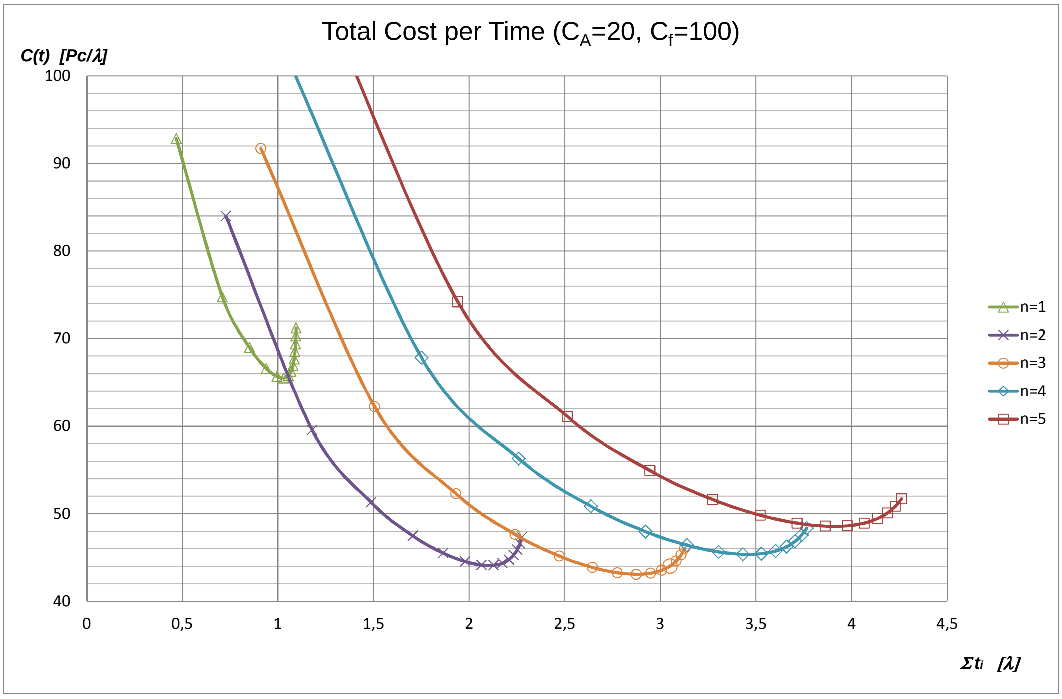

Figure 2: Graph with C_A=20×C_p=20, β=2, C_f=100C_p and ε=1,3.

It is observed form Table 4 and Table 6 (and from Figure 1 and Figure 2) that for the present case (Example 5, decreasing of the Weibull scale parameter) the successive decrease of optimal preventive replacement is more severe, the optimal number of imperfect repairs between replacements is also much higher and the system total expected repair costs per unit of time are slightly higher.

From the examples above some general appreciations can be drawn.

- More expensive equipment require the least use of redundancy.
- The redundancy can increase the optimal preventive repair time and the optimal percentage of failures, without increasing costs.
- In most cases involving small failure repair costs Cf<5Cp, the Run-to-Failure (F(t)=100%) is the most appropriate strategy.
- The degree of growth of the failure rate is a crucial parameter. If there are random failures or decreasing failure rates, the increase in redundancy is inevitable, and so is the applicability of a run-to-failure strategy.
- As the failure repair costs grow (increase of the failure consequences) the optimal percentage of failures and the optimal time to preventive repair decreases, whereas the optimal number of redundancies increases.

5 CONCLUSIONS

The proposed methodology requires a small amount of data and the required optimization process can be easily implemented in a spreadsheet as was done for the examples. The set of examples presented intent to demonstrate the applicability to a wide variety of systems operating conditions, providing quantitative estimations useful in assisting some maintenance decisions and optimization situations.

One of the most important decisions in maintenance is how many imperfect (or increasing costs) repairs should be made between each replacement. The methodology presented allows this optimization according to the degree of imperfection (or the increasing costs rate) of each repair.

For some applications it is convenient to reformulate the acquisition cost definition, incorporating time-dependent values or fixed values independents of the redundancy degree. This can also be included in using the proposed methodology.

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