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Dynamics on certain sets of stochastic matrices

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Abstract We study iteration of polynomials on symmetric stochastic matrices. In particular, we focus on a certain one-parameter family of quadratic maps which exhibits chaotic behavior for a wide range of the parameters. The well-known dynamical behavior of the quadratic family on the interval, and its dependence on the parameter, is reproduced on the spectrum of the stochastic matrices. For certain subclasses of stochastic matrices the referred dynamical behavior is also obtained in the matrix entries. Since a stochastic matrix characterizes a Markov chain, we obtain a discrete dynamical system on the space of reversible Markov chains. Therefore, depending on the parameter, there are initial conditions for which the corresponding reversible Markov chains will lead under iteration to a fixed point, to a periodic point, or to an aperiodic point. Moreover, there are sensitivity to initial conditions and the coexistence of infinite repulsive periodic orbits, both features of chaos.

Keywords Matrix dynamics · Stochastic matrices · Iterated interval maps · Reversible Markov chains

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