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Universidade de Évora

Welcome to the International Conference on Nonlinear Differential Equations and Applications

Cari Amici,
Ho optato per un piccolo saluto nelle nostre due lingue, italiano e portoghese,lingue latine di due paesi della comunità europea, che ora si ritrovano in una città, Ebora Cerealis di Plínio, Liberalitas Julia di Cesare, che ha avuto momenti di gloria già ai tempi dell'Impero Romano. Un incontro a memoria dell'origine comune delle nostre due culture, un legame con questa città.
A nome del Comitato Scientifico della Conferenza vorrei porgere a tutti voi il nostro sincero benvenuto, ed anche esprimere un caloroso ringraziamento di aver arricchito con la vostra presenza e con la vostra gradita collaborazione la nostra iniziativa. Un saluto particolare va alle famiglie dei partecipanti che li hanno gentilmente accompagnati.
Spero che questo breve soggiorno a Évora sia sufficiente per darvi l'opportunità di apprezzare questa bellissima città e per decidere di tornarci per meglio conoscere le sue bellezze e pregi. Impossibile evitare la tentazione di nominare l'alto livello della sua variegata cucina, frutto del contributo di tante civiltà.
”Last but not least”, per usare il linguaggio che verrà utilizzato principalmente durante il convegno, l'alto livello scientifico della Conferenza è garantito dall'elenco degli ”speaker”. Ma questo è merito vostro.
Ancora una volta vi auguro un piacevolissimo soggiorno a Évora, città famosa per la sua proverbiale ospitalità.

Il presidente del Comitato Scientifico
Hugo Beirão da Veiga

Caros Amigos,
Optei por uma pequena saudação nas nossas duas línguas, italiano e português, duas línguas latinas de dois países da comunidade europeia, que agora se encontram numa cidade, a Ebora Cerealis de Plínio, Liberalitas Julia de Cesare, que teve momentos de glória já nos tempos do Império Romano. Um encontro em memória da origem comum das nossas duas culturas, um vínculo com esta cidade.
Em nome da Comissão Científica da Conferência, gostaria de transmitir as nossas sinceras boas-vindas a todos vós, e também de expressar os nossos calorosos agradecimentos por terdes enriquecido a nossa iniciativa com a vossa presença e a vossa bem-vinda colaboração. Uma saudação especial é devida às famílias dos participantes que gentilmente os acompanharam.
Espero que esta curta estadia em Évora seja suficiente para vos dar a oportunidade de apreciar esta bela cidade e decidir regressar para melhor conhecer as suas belezas e méritos. Impossível evitar a tentação de nomear o alto nível de sua variada gastronomia, fruto da contribuição de muitas civilizações.
”Last but not least”, para usarmos a linguagem que será utilizada prevalentemente durante os trabalhos, o alto nível científico da Conferência é garantido pela lista de ”speakers”. Mas este é mérito vosso.
Mais uma vez, desejo-lhe uma estadia muito agradável em Évora, cidade famosa pela sua proverbial hospitalidade.

O presidente da Comissão Científica
Hugo Beirão da Veiga

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Programme

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10:15h–10:50h	**José Francisco Rodrigues**: *José Sebastião e Silva, mathematician from Mértola to Lisboa, via Évora and Roma*		
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12:30h–12:55h	**Elisabetta Chiodaroli**, *On wild initial data for the isentropic Euler system of gas dynamics*	**Alessandro Margheri**, *Fixed points for planar maps with multiple twists, with application to nonlinear equations with indefinite weight*	**Hugo Tavares**, *Yamabe systems, optimal partitions and nodal solutions to the Yamabe equation*
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15:00h–18:00h	**Tour through Évora.** Meeting point: CES main cloisters, 15:00h – http://www.picndea22.uevora.pt/social-program/		
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Wednesday 06.07.2022			
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11:30h	**Closing session** – Auditorium Chairman: **Hugo Beirão da Veiga** **Giuseppe Buttazzo**: *Remembering Ennio De Giorgi*		
13:00h	**Lunch** – Room CES.129		

Plenary talks

Physics-based and data-driven models for PDEs

Alfio Quarteroni[1,2]

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Talk Abstract

In this talk I will present a mathematical model that is suitable to simulate the cardiac function, thanks to its capability to describe the interaction between electrical, mechanical, and fluid-dynamical processes occurring in the heart. The model comprises a system of nonlinear differential equations (either ordinary and partial) featuring a multi-physics and multi-scale nature. Efficient numerical strategies are devised to allow for the analysis of both heart function and dysfunction. These strategies rely on both classical physics-based numerical discretization methods and machine-learning (datadriven) algorithms, as well as on their interplay.

Acknowledgements

The work presented in this talk is part of the project iHEART that has received funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (grant agreement No 740132).

José Sebastião e Silva, mathematician from Mértola to Lisboa, via Évora and Roma

José Francisco Rodrigues[1]

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José Sebastião e Silva, mathematician from Mértola to Lisboa, via Évora and Roma

José Sebastião e Silva (1914-1972), the most original and influential Portuguese mathematician of the 20th century, was born in Mértola, in the South of Portugal, in 1914, where he attended primary school. After completing the secondary school in Évora, in 1933, he entered the Faculty of Sciences in Lisboa, with a fellowship, completed the course in mathematical Sciences in 1937 and started to do research in General Topology and Analysis, under the guidance of António Monteiro, at the new founded Centro de Estudos de Matemática de Lisboa, in 1940. From 1943 until 1947 he pursued his research in Roma, where he contacted L. Fantappiè, F. Enriques and M. Picone and prepared his PhD in Functional Analysis, which he presented at the University of Lisbon in 1949. He was Professor at the Technical University of Lisbon, from 1951 until 1960, when he became Professor of Mathematics at the Faculty of Sciences of the University of Lisbon, until his death in 1972. Director of the Centro de Estudos de Matemática de Lisboa since 1952, his research led him to a new class of locally convex spaces, the (LN*) spaces, which were presented at the 1954 International Congress of Mathematicians in Amsterdam, and became subsequently known as ”Silva spaces”. Author of several books, he led, from 1963 onwards, a pioneer project for the modernisation of Mathematics teaching in the 3rd cycle of secondary education, with national and international impact, and he was also scientific advisor of the Laboratório de Física e Engenharias Nucleares, since 1961, and of the Instituto de Física e Matemática, in Lisbon, since 1966 until his death.

Invited speakers

Periodic solutions to the relativistic Kepler problem: a variational approach

Alberto Boscaggin[1], Walter Dambrosio[1] and Duccio Papini[2]

[1]University of Torino, Department of Mathematics “Giuseppe Peano”, Italy
[2]University of Udine, Department of Mathematics, Computer Science and Physics, Italy

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Talk Abstract

The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $m > 0$ is the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Here, we deal with the forced equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $U(t,x)$ is T-periodic in the first variable, and we show how to apply non-smooth critical point theory to prove the existence of multiple T-periodic solutions, with prescribed winding number around the origin. Joint work [1] with Walter Dambrosio and Duccio Papini.

Keywords: relativistic Kepler problem, periodic solutions, non-smooth critical point theory.

Acknowledgements

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Self-adjoint extensions of the curl operator

Alberto Valli[1]

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Talk Abstract

In electromagnetism, for linear isotropic media the relation between the magnetic induction $\boldsymbol{B}$ and the magnetic field $\boldsymbol{H}$ is given by $\boldsymbol{B} = \mu\boldsymbol{H}$, the scalar function μ being the magnetic permeability. If displacement currents are neglected, as in the case of magnetostatic or eddy current problems, the current density $\boldsymbol{J}$ is given by $\boldsymbol{J} = \mathbf{curl}\,\boldsymbol{H}$. In this situation a magnetic field satisfying $\mathbf{curl}\,\boldsymbol{H} = \eta\boldsymbol{H}$, with η a scalar function, produces a vanishing magnetic force $\boldsymbol{J} \times \boldsymbol{B}$, and it is called a *force-free* field.
In fluid dynamics, a divergence-free field $\boldsymbol{u}$ satisfying $\mathbf{curl}\,\boldsymbol{u} = \eta\boldsymbol{u}$, with η a scalar function, is a steady solution of the Euler equations for incompressible inviscid flows (with pressure given by $p = -\frac{|\boldsymbol{u}|^2}{2}$), and it is called a *Beltrami* field.
Eigenfunctions of the curl operator are therefore force-free fields and Beltrami fields, and are of relevant physical interest. In particular, in plasma physics a magnetic field $\boldsymbol{H}$ which minimizes the magnetic energy with fixed helicity has to satisfy the equation $\mathbf{curl}\,\boldsymbol{H} = \lambda\boldsymbol{H}$ for some constant λ, thus it is an eigenfunction of the curl operator.
In this talk we are concerned with the formulation and analysis of the eigenvalue problem for the curl operator. Drawing inspiration from the results in [1] and extending the previous ones in [2], [3], [4], we prove that the curl operator is self-adjoint in L^2, provided that we choose a suitable domain of definition.
This domain is strictly larger than $H_0(\mathbf{curl};\Omega)$, the space of vector fields $\boldsymbol{v}$ belonging to $L^2(\Omega)$ together with $\mathbf{curl}\,\boldsymbol{v}$ and satisfying $\boldsymbol{v} \times \mathbf{n} = \mathbf{0}$ on the boundary $\partial\Omega$, and when Ω is topologically trivial is given by the space of vector fields for which $\mathbf{curl}\,\boldsymbol{v} \cdot \mathbf{n} = 0$ on the boundary. However, additional conditions must be imposed when Ω is not topologically trivial, and we show that a viable choice is imposing that the values of the line integrals of $\boldsymbol{v}$ on suitable homological cycles lying on the boundary are equal to 0.
Following the results in [5] we devise and analyze a saddle-point variational formulation for the spectral problem associated to the curl operator, and also briefly describe the guidelines of its numerical approximation by means of finite elements.

Keywords: curl operator, spectral problem, saddle-point variational formulation, finite element approximation.

Acknowledgements

This work was partially supported by INdAM-GNCS.

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On the minimization of general energies with attractive-repulsive behaviour

Aldo Pratelli[1]

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Talk Abstract

The celebrated “liquid drop model” by Gamow is one of the oldest and most studied energies of attractive-repulsive type, and it has gathered a huge interest among physicists and mathematicians. In the last years several generalisations of the model have been studied (see for instance [1–3]), and now many important properties are known, though still some fundamental questions are open, even in the original model. In addition, people have started to consider the minimization in the class of L^1 positive functions, instead than in the class of sets. In this talk, we will describe the main features of the problem, and we will concentrate ourselves in the even more general case of minimization among positive measures, already considered by some authors but largely open. We will briefly present some properties, proven very recently, and some open questions (see [4,5]). Some of the results have been proved in collaboration with Carazzato, Fusco, Novaga.

Keywords: non-local energies, charged droplets, regularity of minimizers.

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Fixed points for planar maps with multiple twists, with application to nonlinear equations with indefinite weight

Alessandro Margheri[1], Carlota Rebelo[2] and Fabio Zanolin[3]

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Talk Abstract

We investigate the dynamical properties of planar maps which can be represented as a composition of twist maps together with expansive–contractive homeomorphisms. These maps present some common features both with those arising in the context of the Poincaré–Birkhoff theorem and those studied in the theory of topological horseshoes. We show that the multiplicity results of fixed points and periodic points typical of the Poincaré–Birkhoff theorem can be recovered and improved in our setting. Applications are given to periodic solutions for planar systems of non-autonomous ODEs with sign-indefinite weights. The presence of complex dynamics is also discussed.

Keywords: Twist maps, Poincaré-Birkhoff theorem, expansive-contractive directions, topological horseshoes, indefinite weight.

Acknowledgements

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On the existence and stability of two-dimensional compressible current-vortex sheets

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Talk Abstract

We are concerned with the nonlinear characteristic free boundary problem for the existence of current-vortex sheets in ideal compressible Magneto-hydrodynamics in two space dimension. We first identify a sufficient condition ensuring the weak stability of the linearized current-vortex sheet problem. Then the local existence of the original nonlinear problem is proved in anisotropic Sobolev spaces, by using a suitable modification of Nash-Moser iteration scheme, provided that the stability condition above is satisfied at each point of the initial discontinuity front.
The exposed results are a joint work with Paolo Secchi, Paola Trebeschi and Difan Yuan.

Keywords: magneto-hydrodynamics, current-vortex sheets, free boundary problem.

Current status and perspectives of the Virtual Element Method

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Talk Abstract

On January 2013, I and other five co-authors (Lourenço Beirão da Veiga, Franco Brezzi, Andrea Cangiani, Gianmarco Manzini, Donatella Marini) published the paper *Basic principles of Virtual Element Methods* [1]. In the Abstract we wrote: "We present, on the simplest possible case, what we consider as the very basic features of the (brand new) virtual element method. …The idea is quite general, and could be applied to a number of different situations and problems. Here however we want to be as clear as possible, and to present the simplest possible case that still gives the flavor of the whole idea."
At the time of writing, (May 15, 2022) the "volley" paper (the code name comes from the fact that there were six players) has reached the number of 1001 citations on Google Scholar, which is remarkably high for a paper in basic numerical analysis that is less than 10 years old.
In my talk I will try to give an overview of the Virtual Element Method and to explain why it has become so popular in our scientific community. I will end up with a discussion of future perspectives.

Keywords: finite element method, virtual element method.

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A tree-cotree technique for the approximation of the spectrum of the curl operator

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Talk Abstract

We present a new algorithm for the finite element approximation of the eigenvalue problem for the curl operator in a multiply-connected domain Ω. In a simply-connected domain the curl operator is self-adjoint when restricted to the space of vector fields $\mathbf{v}$ that satisfy the boundary condition $\operatorname{curl}\mathbf{v}\cdot\mathbf{n} = 0$. When Ω is multiply-connected additional constraints must be imposed: a viable choice is the vanishing of the line integrals of $\mathbf{v}$ on suitable homological cycles lying on the boundary (see [1]). The new algorithm that we propose is based on the weak formulation and finite element approximation of this problem analyzed in [2]. The algorithm exploits the Hodge decomposition of the finite element space and a tree-cotree decomposition of the graph relating the degrees of freedom of the Lagrangian finite elements and those of the first family of Nédélec finite elements to significantly reduce the dimension of the algebraic eigenvalue problem to be solved.

Keywords: Spectrum of curl operator, Multiply connected domain, Edge elements, Tree-cotree decomposition.

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Generalized solutions of the Navier-Stokes equation

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Talk Abstract

We present a stochastic variational principle for the (deterministic) Navier-Stokes equation. Solutions are understood in the weak sense. The corresponding Lagrangian paths are stochastic and define generalized flows for Navier-Stokes, similar to generalized flows introduced by Y. Brenier for the Euler equation. We discuss both the compressible and the incompressible case. The latter is recent ongoing work while the former corresponds to the paper cited in the bibliography.

Keywords: Navier-Stokes equation, Stochastic Lagrangian flows.

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Some Optimal Design Problems with Perimeter Penalisation

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Talk Abstract

We obtain a measure representation for a functional arising in the context of optimal design problems under linear growth conditions (see Barroso, Matias and Zappale [1]). Starting from an energy $F(\chi, u)$, which has a bulk term depending on the symmetrised gradient of u, as well as a perimeter term, the functional in question is the relaxation of $F(\cdot,\cdot)$ with respect to a pair (χ, u), where χ is the characteristic function of a set of finite perimeter, corresponding to the optimal shape, and u is a function of bounded deformation. The perimeter term, which penalises the interface between the two regions $\{\chi = 1\}$ and $\{\chi = 0\}$, is added to ensure compactness of minimising sequences.
In Barroso and Zappale [2] and [3], a similar investigation was undertaken in the case of non-standard p-q growth conditions on the original bulk energy densities, which now depend on the full gradient of the u variable, and where the energy also includes a perimeter penalisation term. In this setting, we showed in [2] that one of the relaxed functionals under consideration only admits a weak measure representation, whereas for the other a strong measure representation holds. Under some convexity assumptions, we provided a partial characterisation of the corresponding measures, a full representation was obtained in the one-dimensional setting.
In [3] we further identified some conditions under which the relaxation process gives rise to no concentration effects. In this case, we showed that the integral representation in question is composed of a term which is absolutely continuous with respect to the Lebesgue measure, and a perimeter term, but has no additional singular term.

Keywords: measure representation, relaxation, functions of bounded deformation, symmetric quasiconvexity, non-standard growth conditions

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Optimal boundary control for steady Navier-Stokes equations with directional do-nothing boundary condition

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Talk Abstract

Inspired by the modelling of blood flow redirected by a bypass surgery in a tract of an artery (see [1]), we study the Navier-Stokes equations with mixed boundary conditions. As in [1], a non-homogeneous Dirichlet boundary condition is considered in the inlets of the fluid domain. In our work, instead of the classical do-nothing condition, we impose a directional do-nothing (DDN) outflow boundary condition (see [2–5]).
Based on a saddle point approach (see [1]), we begin by establishing the well-posedness of the direct problem. Then, aiming at flow regularization, we analyse the boundary control problem which consists in the minimization of quadratic cost functionals of the velocity field (tracking-type or vorticity) by means of the inflow velocity in one inlet. We prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map and deduce the first order necessary conditions for optimality. The results are obtained under smallness restrictions on the inflow boundary controls.

Keywords: Navier-Stokes equations, mixed boundary conditions, directional do-nothing condition, optimal boundary control.

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Donati type theorems and compensated compactness in the periodic casee

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Abstract

In 1890 Donati gave a classic result : a necessary condition on a strain tensor field that is orthogonal to all divergence free stress tensor fields. The condition is written in terms of second order derivatives of the strain tensor field, and for simply connected domains is equivalent to the so-called Saint Venant compatibility condition. 84 years later, in 1974, T.W. Ting (see [1]) obtained an extension of Donati's result : the necessary condition on a strain tensor field that is orthogonal to all divergence free stress tensor fields, is to be the symmetric gradient of some vector field belonging to a Sobolev space. In the last 20 years P. G. Ciarlet and his collaborators and students (see [2] and [3]) wrote several papers where extensions of the Donati:s theorem were obtained for Dirichlet, Neumann and mixed boundary conditions. However, the periodicity condition was not considered and remained open until now. In the Homogenization Theory applied to Elasticity, a periodicity condition appears naturally when characterizing the homogenized elastic tensor in terms of the so called cellular problems, which are PDEs with periodic boundary conditions. In order to obtain the equivalence between the variational formulations in strain, in stress and in displacement, we are led naturally to a compensated compactness result stated by P. Suquet in 1987 (see [4]). The notion of compensated compactness has been introduced by L. Tartar in 1979 (see [5]) as a condition for the weak convergence of products between strain and stress fields. We will present three extensions of Donati's theorem in the periodic context, where the obtained conditions are necessary and sufficient.

Keywords: homogenization, homogenized elastic tensor, cellular problems.

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On the logistic equation for the fractional p-Laplacian

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Talk Abstract

We study the following Dirichlet type problem for a nonlinear nonlocal elliptic equation:

$$(P_\lambda) \qquad \begin{cases} (-\Delta)_p^s u = \lambda u^{q-1} - u^{r-1} & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p \geq 2$, $s \in (0,1)$ s.t. $ps > N$, $q \in (1, p_s^*)$, $r \in (p, p_s^*)$, $\lambda > 0$ and the leading operator is the degenerate fractional p-Laplacian

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \searrow 0} \int_{\{|x-y|>\varepsilon\}} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}} \, dy.$$

The reaction is of logistic type with powers $q \in (1, p_s^*)$, $r \in (p, p_s^*)$, depending on a parameter $\lambda > 0$. Previous results on logistic equations with nonlocal operators are in [1,4,5]. We distinguish three cases:

(a) in the subdiffusive case $q < p$, for all $\lambda > 0$ problem (P_λ) has a unique solution u_λ with $u_\lambda \to 0$ as $\lambda \to 0^+$;

(b) in the equidiffusive case $q = p$, problem (P_λ) has no solution for $\lambda \leq \lambda_1$, and a unique solution u_λ for all $\lambda > \lambda_1$, with $u_\lambda \to 0$ as $\lambda \to \lambda_1^+$ ($\lambda_1 > 0$ is the principal eigenvalue of $(-\Delta)_p^s$ in Ω);

(c) in the superdiffusive case $q \in (p, r)$, there exists $\lambda_* > 0$ s.t. (P_λ) has no solution for $\lambda < \lambda_*$, (P_{λ_*}) has a solution u_*, and for all $\lambda > \lambda_*$ (P_λ) has two solutions $u_\lambda > v_\lambda$, with $u_\lambda \to u_*$ as $\lambda \to \lambda_*^+$.

In all cases, the map $\lambda \mapsto u_\lambda$ is increasing. In cases (a), (b) we use the direct variational method (minimization) and a Brezis-Oswald approach for uniqueness, while in case (c) we find a second solution via truncations and the mountain pass theorem. Monotonicity results stem from a new strong minimum/comparison principle (see [2] for details). All results make use of the weighted Hölder regularity proved in [3].

Keywords: fractional p-Laplacian, logistic equation, comparison principle.

Acknowledgements

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Virtual Element Methods for Elasticity Problems in mixed form

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Talk Abstract

The Virtual Element Method (VEM) is a methodology for the approximation of partial differential equation problems, whose popularity is growing fast, both in the mathematical and in the engineering community. The initial motivation of VEM is the need to construct an accurate conforming Galerkin scheme with the capability to deal with highly general polygonal/polyhedral meshes, including "hanging vertexes" and non-convex shapes. In the framework of the infinitesimal elasticity problems, we present some 2D and 3D Virtual Element schemes based on the Hellinger-Reissner variational principle, see [1–3]. As it is well-known, imposing both the symmetry of the stress tensor and the continuity of the tractions at the inter-element is typically a great source of troubles in the framework of classical Galerkin schemes, such as the Finite Element Method (FEM), for instance. We exploit the great flexibility of VEM to present an alternative to FEM, which provide symmetric stresses, continuous tractions and is reasonably cheap with respect to the delivered accuracy. A significant feature of our methods is the possibility to employ the so-called hybridization procedure to solve the resulting linear system in an efficient way, see [4]. In addition, the hybridization strategy leads to the construction of a post-processed displacement approximation of higher accuracy.
In this talk, we detail the ideas which led to the design of our VEM schemes and their hybridization, we state the theoretical results, and we present several numerical tests to assess the actual computational performance of our approach.

Keywords: virtual element method, Hellinger-Reissner formulation, hybridization.

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Predator-prey and competitive models with seasonality: an overview of recent results

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Talk Abstract

In this talk we will consider predator-prey and competition models. We will give an overview of recent results in the case when seasonality is considered.
In what concerns predator-prey models we prove persistence results and also coexistence ones both with a logistic type growth for the prey or, in the case of two species, considering an Allee effect (both the cases of weak and strong Allee effects). These results were obtained for a general class of functional responses.
In the case of competition models, we analyse in detail the case of a two-species competition model in which competition can be quite general. We give conditions for the extinction of one or both species and for coexistence.
The results obtained with topological methods are described in the papers [1–3].

Keywords: predator prey, competition, persistence, extinction, coexistence.

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Fluid-structure interaction problems in computational hemodynamics

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Talk Abstract

In this talk we consider fluid-structure interaction (FSI) problems arising in the context of hemodynamics. In such a field, the numerical solution of FSI may be very challenging due to the similar values of fluid and structure densities. In the first part of the talk we discuss the efficiency, stability and accuracy of a family of loosely coupled partitioned algorithms, based on the solution of just one fluid and structure problem at each time step. We report theoretical results about stability and then numerical results in a real data-set of human carotids to study the effect of different plaque typologies on plaque stability. We also propose and apply such algorithms for cardiac FSI problem where also the coupling with electro-physiology is addressed. In the second part of the talk, we provide some preliminary results obtained in the direction of modeling plaque progression. To this aim, we introduce a model composed by the FSI problem coupled with other partial differential equations describing at the macroscopic level the cellular processes leading to plaque progression. We propose a numerical method to solve this highly non-linear system of PDEs characterized by different time scales and we present some numerical results.

Keywords: Fluid-structure interaction, loosely coupled algorithms, plaque progression

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Limiting cases in Choquard type equations

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Talk Abstract

We will present recent results for a class of Choquard type equations of the following form

$$(C) \quad -\Delta_N u + V(x)u = (I_N * F(x,u))f(x,u), \quad x \in \mathbf{R}^N, N \geq 2$$

where I_N is the Riesz logarithmic kernel, V is a bounded Schrödinger potential and the nonlinearity $f(x,u)$, whose primitive in u vanishing at zero is $F(x,u)$, exhibits the highest possible growth – for functions with membership in $W^{1,N}(\mathbf{R}^N)$ – which is of exponential type. The competition between the logarithmic kernel and the exponential nonlinearity demands for new tools. A proper function space setting is provided by a new weighted version of the Pohozaev–Trudinger inequality which enables us to prove the existence of variational, in particular finite energy solutions to (C). Equivalence issues with connected higher order fractional Scrödinger-Poisson systems will be also discussed, as well as related open problems.

Keywords: Higher order fractional Schrödinger-Poisson systems, Schrödinger-Newton equations, Weighted Trudinger-Moser type inequalities in $\mathbf{R}^N$, Variational methods.

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Quadratic PDE and optimal ballistic problems

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Talk Abstract

Optimal ballistic problems are somewhat similar to the shortest path (geodesic) problems. In the finite-dimensional setting, say, on a Lie group with a Riemannian metric, we minimize the kinetic energy along all curves with presribed final point and initial velocity. Remarkably, such problems naturally arise in the infinite-dimensional setting from a certain dual formulation of nonlinear evolutionary PDE, cf. [1,2]. The examples include the incompressible Euler, inviscid Burgers, ideal incompressible MHD, the template matching equation, the multidimensional Camassa-Holm (also known as the H(div) geodesic equation), EPDiff, Euler-alpha, KdV and Zakharov-Kuznetsov equations, the equations of motion for the incompressible isotropic elastic fluid and for the damping-free Maxwell's fluid. This yields the existence of a new type of generalized solutions to the initial-value problems for the above mentioned PDE. We also discuss a sharp upper bound on the optimal value of the dual problem and the weak-strong uniqueness issue.

Keywords: optimal transport, fluid dynamics, hidden convexity, Euler-Arnold equations.

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Power law approximation for supremal functionals

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Talk Abstract

I will discuss the power law approximation of supremal functionals, expecially in connection with optimal design problems.
The results, mostly contained in [3,4], follow in connection with PDEs and norm approximation, and generalize some previous theorems contained in [1,2].

Keywords: supremal functionals, relaxation, Γ-convergence.

Acknowledgements

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A connection between symmetry breaking for Sobolev minimizers and stationary Navier-Stokes flows past a circular obstacle

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Talk Abstract

Fluid flows around a symmetric obstacle generate vortices which may lead to symmetry breaking of the streamlines. We study this phenomenon for planar viscous flows governed by the stationary Navier-Stokes equations with constant inhomogeneous Dirichlet boundary data in a rectangular channel containing a circular obstacle. In such (symmetric) framework, symmetry breaking is strictly related to the appearance of multiple solutions. Symmetry breaking properties of some Sobolev minimizers are studied and explicit bounds on the boundary velocity (in terms of the length and height of the channel) ensuring uniqueness are obtained after estimating some Sobolev embedding constants and constructing a suitable solenoidal extension of the boundary data. We show that, regardless of the solenoidal extension employed, such bounds converge to zero at an optimal rate as the length of the channel tends to infinity.

Keywords: symmetry breaking, fluid-structure interaction.

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Existence of solutions to the Navier-Stokes Cauchy problem in the L^3 setting

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Talk Abstract

We investigate on the existence of solutions to the Navier-Stokes Cauchy problem with initial datum u_0 in L^3 and divergence free. It is known that this kind of result is not new. Indeed, there is a wide literature on it, with a first contribution due to T. Kato in [4]. Our chief goal is to establish the existence interval $(0, T)$ by uniquely considering the size of the initial datum in L^3 and the absolute continuity of $|u_0(x)|^3$.
A similar analysis has been developed in the recent paper [1], where it is employed the dimensionless weighted functional $||U_0||^2_{wt} := \sup_x \int_{\mathbb{R}^3} \frac{U_0^2(y)}{|x-y|} dy$ and, in the set L^2_{wt}, where $|| \cdot ||_{wt} < \infty$, the subset of the so called Kato class K_3 is considered. In this regard, we recall that $|| \cdot ||_{wt}$ is not equivalent to the L^3-norm.
The result, proved in paper [2] for the Cauchy problem, will represent the starting point for the same result, in a forthcoming paper [3], in the case of the initial boundary value problem in $(0, T) \times \Omega$, where $\partial\Omega$ is assumed a sufficiently regular compact set, or is the half-space.

Keywords: Navier-Stokes equations; existence; regular solutions.

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Existence and non-existence of positive solutions for the critical p-Laplace equation in the ball

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Talk Abstract

We focus on positive radial solutions to the Dirichlet problem associated with the generalized scalar curvature equation

$$\begin{cases} \Delta_p u + K(|x|)u^{q-1} = 0, & x \in B_R(0) \\ u(x) = 0 & |x| = R, \end{cases}$$

where $\Delta_p u = \text{div}(\nabla u|\nabla u|^{p-2})$ denotes the p-Laplace operator, $B_R(0)$ is the ball of radius $R > 0$ in $\mathbb{R}^n$, $2n/(2+n) \leq p \leq 2,\ n > p > 1$ and q is the Sobolev critical exponent

$$q = p^* = \frac{np}{n-p}.$$

The function K is assumed to be C^1, bounded, positive and to satisfy the ℓ-flatness condition. In particular, we show that the existence of positive solutions depends on the slope ℓ of K at zero, and on the length of the radius R. Our main purpose is to improve and extend the result in [1] to the p-Laplacian case. Interesting results can also be achieved under an additional monotonicity assumption on K. Our approach, based on Fowler transformation, invariant manifold theory, phase plane analysis, and energy estimates, offers a new geometrical perspective and exploits the construction of suitable barrier sets for the solutions.

Keywords: Scalar curvature equation, Fowler transformation, Invariant manifold, Phase plane analysis.

Acknowledgements

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A mountain pass lemma and multiple solutions to nonlinear differential problems

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Talk Abstract

A mountain pass lemma ([1]) and characterizations of the mountain pass geometry ([2,3]) as well as a local minimum theorem ([4]) are pointed out. As a consequence, the existence of two, three, infinitely many solutions for nonlinear differential problems are established ([5–8]).

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Variations on a theme by Cheeger

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Talk Abstract

We present some generalizations of the well-known Cheeger inequality $4\lambda(\Omega) \geq h^2(\Omega)$, where $\lambda(\Omega)$ denotes the principal eigenvalue of the Dirichlet Laplacian and $h(\Omega)$ is the Cheeger constant. The domain Ω may vary either in the class of *all* open bounded subsets of $\mathbb{R}^d$ or in the subclass of *convex* domains. In particular, the existence of optimal domains for the shape functional $\lambda(\Omega)h^{-2}(\Omega)$ is deeply discussed.

Keywords: Cheeger constant, principal eigenvalue, shape optimization, p-Laplacian.

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Periodic solutions to the relativistic Kepler problem: a dynamical systems approach

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Talk Abstract

The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $m > 0$ is the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic and quasi-periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $U(t,x)$ is T-periodic in the first variable and $\varepsilon \in \mathbb{R}$. The analysis of the action-angle coordinates and an application of an higher dimensional version of the Poincaré–Birkhoff fixed point theorem allow to prove that, for ε small enough, the perturbed problem admits T-periodic solutions with prescribed winding number, bifurcating from invariant tori of the unperturbed problem. The talk is based on the paper [1] written in collaboration with Alberto Boscaggin and Walter Dambrosio.

Keywords: relativistic Kepler problem, periodic solutions, invariant tori, nearly integrable Hamiltonian systems, action-angle coordinates.

Acknowledgements

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Existence results for a turbulence $k-$epsilon model governing flows through permeable media

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Talk Abstract

In this talk, we consider turbulent flows through permeable media governed by the following general one-equation turbulence $k-$epsilon model,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \mathbf{div}\,(\nu_T(k)\mathbf{D}(\mathbf{u})) + \nabla p = \mathbf{g} - \mathbf{f}(\mathbf{u}) \ \text{ in } \ Q_T, \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0 \ \text{ in } \ Q_T, \tag{2}$$

$$\frac{\partial k}{\partial t} + \mathbf{u}\cdot\nabla k - \operatorname{div}(\nu_D(k)\nabla k) = \nu_T(k)|\mathbf{D}(\mathbf{u})|^2 + P(\mathbf{u},k) - \varepsilon(k) \ \text{ in } \ Q_T, \tag{3}$$

$$\mathbf{u} = \mathbf{u}_0 \quad \text{and} \quad k = k_0 \quad \text{in } \ \Omega\times\{0\}, \tag{4}$$

$$\mathbf{u} = \mathbf{0} \quad \text{and} \quad k = 0 \quad \text{on } \ \Gamma_T, \tag{5}$$

set in a cylinder $Q_T := \Omega\times(0,T)$, where $\Gamma_T := \partial\Omega\times(0,T)$, $\Omega\subset\mathbb{R}^d$, $d\geq 2$, is a bounded domain, with its boundary denoted by $\partial\Omega$, and T is a given positive constant. The velocity field $\mathbf{u}$ and the pressure p are, in fact, averages that result by the application of the two averaging concepts. The averaged tensor $\mathbf{D}(\mathbf{u})$ is the symmetric part of the averaged gradient $\nabla\mathbf{u}$, k is the turbulent kinetic energy, ν_T is the turbulent, or eddy, viscosity, ν_D is the turbulent diffusion and ε is the dissipation of turbulent kinetic energy. For the sake of simplicity, the medium porosity is assumed to be constant. The feedback term $\mathbf{f}(\mathbf{u})$ accounts for the resistance made by the rigid matrix of the permeable medium to the flow, whereas $\mathbf{g}$ stands for a body forces field. The function $P(\mathbf{u},k)$ appears as an output of the averaging process, and it is a production term of turbulent kinetic energy that accounts for the solids inside the fluid. In this talk, we study the existence of weak solutions to both the steady-state and the transient version of the problem (1)-(5), under distinct conditions on the growth of $\mathbf{f}(\mathbf{u})$, $\nu_T(k)$, $\nu_D(k)$, $\varepsilon(k)$ and $P(\mathbf{u},k)$. This talk is based in the papers [1,2] and in some recent work [3] by the author.

Keywords: fluid flows through permeable media, turbulence $k-$epsilon model, existence of weak solutions.

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Yamabe systems, optimal partitions and nodal solutions to the Yamabe equation

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Talk Abstract

We give conditions for the existence of regular optimal partitions, with an arbitrary number $\ell \geq 2$ of components, for the Yamabe equation on a closed Riemannian manifold (M, g). To this aim, we study a weakly coupled competitive elliptic system of ℓ equations, related to the Yamabe equation. We show that this system has a least energy solution with nontrivial components if dim $M \geq 10$, (M, g) is not locally conformally flat and satisfies an additional geometric assumption whenever dim $M = 10$. Moreover, we show that the limit profiles of the components of the solution separate spatially as the competition parameter goes to $-\infty$, giving rise to an optimal partition. We show that this partition exhausts the whole manifold, and we prove the regularity of both the interfaces and the limit profiles, together with a free boundary condition.
For $\ell = 2$ the optimal partition obtained yields a least energy sign-changing solution to the Yamabe equation with precisely two nodal domains.

Keywords: Competitive elliptic system, Riemannian manifold, critical nonlinearity, optimal partition, free boundary problem, regularity, Yamabe equation, sign-changing solution.

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Hamilton–Jacobi procedure to Pareto depth analysis for fast colonic polyp detection

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Talk Abstract

Colon cancer is a common type of cancer and a leading cause of cancer death worldwide. Its early detection increases the survival rate, and this correlates with the accurate detection of colonic polyps. These are lesions (fleshy growth) that can be visualized *in vivo*, by medical doctors, during a colonoscopy exam, for instance, either with a conventional colonoscopy or a wireless capsule endoscocope. Here it is described a novel method for fast colonic polyp detection, presented in [1], for *in vivo* images of conventional colonoscopy. Firstly, polyp detection is formulated as a similarity-based anomaly detection method (see [2]), based on multiple objectives, which formally involves Pareto depth analysis (also called as non-dominated sorting). The chosen objectives rely on the main physical and visible differences, observed in colonoscopy images, between regions containing colonic polyps and the surrounding normal mucosa (see also [3]). These differences are defined primarily according to the contrast in shape, texture, and color. Secondly, as Pareto depth analysis is of combinatorial nature and is costly to compute, it is replaced by a fast algorithm that approximates the sorting in the continuum limit. The fast algorithm involves numerical solutions to a particular Hamilton–Jacobi equation (see [4,5]). The proposed similarity-based anomaly detection is thus reformulated into a fast polyp detection method. Several experiments were conducted with a proprietary medical data set, containing 1640 instances of 41 different colonic polyps. The results show that the proposed Hamilton–Jacobi approach to non-dominated sorting speeds up the non-dominated sorting procedure, by more than 500%, and, when compared with other existing methods, it is also faster without lost of accuracy. Moreover, the tests conducted for streaming data, reveal an outstanding performance, in terms of sensitivity and specificity, as well as, a fast auto-adaptability, which demonstrate the power of the proposed

approach towards a real-time and automatic detection, undoubtedly beneficial for clinical practice.

Keywords: image processing, multi-criteria optimization, Hamilton–Jacobi equation, colonic polyp.

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Unbounded spectral minimal partitions?

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Talk Abstract

Spectral minimal partitions (SMPs) of bounded domains in Euclidean space have been studied intensively over the last two decades, ever since the pioneering work of Conti, Terracini and Verzini (e.g., [1]; see also [2] for a relatively recent survey). In general, one seeks to minimise, among all suitable partitions of the domain, a functional built on Laplacian eigenvalues of the partition pieces; the solution can also typically be attained as a limit solution to competing species-type elliptic systems as the competition term becomes stronger. Beyond their links to competing systems, SMPs are studied both due to their links to spectral properties of the Laplacian on the whole domain, and the problem of dividing a domain optimally into a given number of pieces which are "equal" in some analytic sense. Corresponding problems on graphs are closely related to the search for clusters in the graphs. We will explore the problem of partitioning *unbounded* domains $\Omega \subseteq \mathbb{R}^n$ of infinite volume in Euclidean space, equipped with a nonnegative potential $q : \Omega \to \mathbb{R}$ acting as a "landscape" on the domain. We will formulate a number of conjectures relating the existence, or non-existence, of SMPs of Ω to the infimum of the essential spectrum of the Schrödinger operator $-\Delta + q$ on Ω. This is in large part based on ongoing joint work with Matthias Hofmann (Texas A&M University) and Andrea Serio (Faculdade de Ciências da Universidade de Lisboa), where we prove analogous results in the setting of quantum graphs.

Keywords: Schrödinger operator, eigenvalues, quantum graph, unbounded domain.

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Compactness of radial solutions to the Lin-Ni-Takagi equation in the asymptotically critical regime

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Talk Abstract

I will consider the Lin-Ni-Takagi equation in the ball $B_R = B(0,R) \subset \mathbb{R}^N$, $N \geq 3$, for some $R > 0$ under a Neumann boundary condition:

$$\begin{cases} -\Delta u + u = |u|^{p-2}u & \text{in } B_R \\ \partial_\nu u = 0 & \text{in } \partial B_R. \end{cases}$$

More precisely, I will focus on radial solutions in the asymptotically critical case i.e. when $p = \frac{2N}{N-2} + \varepsilon$, for some $0 \neq \varepsilon \to 0$. I obtain a complete picture of the behavior of radial solutions with finite energy. In particular, I prove that if $\varepsilon > 0$ and $N \geq 7$ then solutions with finite energy are precompact. I will interprete this result in term of a bifurcation analysis done for radial solutions with respect to the parameter p. This is based on a joint work with Denis Bonheure and Bruno Premoselli.

Keywords: Asymptotically critical equation, Neumann boundary condition, blowing-up solutions.

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Variational approaches in blood flow modeling

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Talk Abstract

The mathematical investigation of the cardiovascular system and, in particular, of blood flow in major arteries, has seen clear progress in the last three decades. Such progress allows, for the first time, to obtain 3D computational solutions which can capture important features for the understanding of the mechanical physiology of the vascular system, as well as some of its pathologies. However, the use of such simulations in clinical practice, either for diagnosis or prognosis purposes, depends on their value as patient-specific simulations. Surrogate models based on geometric multiscale couplings have been addressed by several authors. An alternative approach relies on using the so-called variational or control techniques. In this talk, we will give an overview of some of these later techniques, present numerical results, as well as some ongoing research directions.

Keywords: Navier-Stokes equations, artificial boundaries, optimal control.

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Schrödinger meets De Giorgi: entropic interpolation in metric spaces

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Talk Abstract

The classical Schrödinger problem (~ 1931) consists in determining the most likely evolution of a system of independent Brownian particles, given the observation of their statistical distributions at two initial and terminal times. Recently, this interpolation problem was also identified as a noisy perturbation of the geodesic optimal transport problem (in the Wasserstein space of probability measures). The noise is driven by the standard Boltzmann-Gibbs-Shannon entropy at small temperature $\varepsilon > 0$. In the small-noise limit it is known that the blurred problem Gamma-converges towards the deterministic one, which is actually remarkably useful for numerics. In this talk I will discuss a natural extension to dynamical Schrödinger problems driven by general entropy functionals on arbitrary metric spaces, for which the same Gamma-convergence as well as convexity properties can be established.
Based on joint works with A. Baradat, H. Lavenant, L. Tamanini, and D. Vorotnikov.

Keywords: optimal transport, entropy, interpolation, metric spaces.

Acknowledgements

References

Equilibrium analysis of an immersed rigid leaflet with virtual elements

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Talk Abstract

In this talk we present a study [1], both theoretical and numerical, on the equilibrium of a hinged rigid leaflet with an attached rotational spring, immersed in a stationary incompressible fluid within a rigid channel.
Through a careful investigation of the properties of the domain functional describing the angular momentum exerted by the fluid on the leaflet (which depends on both the leaflet angular position and its thickness), we identify sufficient conditions on the spring stiffness function for the existence (and uniqueness) of equilibrium positions.
We propose a numerical technique that exploits the mesh flexibility of the Virtual Element Method [2] (VEM). A (polygonal) computational mesh is generated by cutting a fixed background grid with the leaflet geometry, and the problem is then solved with stable divergence-free VEM Stokes elements [3] of degrees 1 and 2 combined with a bisection algorithm. We prove quasi-optimal error estimates and present an array of numerical experiments to document the accuracy and robustness with respect to degenerate geometry of the proposed methodology.

Keywords: Virtual Elements, fluid-structure interaction, divergence-free elements.

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Pseudo monotone operators and the unsteady rotational Smagorinsky model

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Talk Abstract

We show that the rotational Smagorinsky model for turbulent flows can be put in the setting of Bochner pseudo-monotone evolution equations. This allows to prove existence of weak solutions identifying a proper weighted spaces and checking some easily verifiable assumptions, at fixed time. We also will briefly discuss the critical role of the exponents present in the model (power of the distance function and power of the curl).

Keywords: turbulence, degenerate parabolic equations.

Acknowledgements Partially supported by a grant of the group GNAMPA of INdAM and by the University of Pisa within the grant PRA_2018_52 UNIPI: "*Energy and regularity: New techniques for classical PDE problems.*"

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Artificial Stress Diffusion in Numerical Simulations of Viscoelastic Fluids Flows

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Talk Abstract

Numerical simulations of viscoelastic fluid flows continue to be a very challenging problem for high values of Weissenberg ($\mathcal{W}e$) especially due the High Weissenberg Number Problem (HWNP) [2]. The HWNP is characterized the instability of the numerical solution for higher values than some critical value of the parameter $\mathcal{W}e$ [3]. The most widely used stabilization methods consists on the introduction of an extra (artificial) numerical diffusion term into the transport equations for viscoelastic stress tensor, at the discretization stage, leading to more stable simulations [1]. However, we should have special care to keep the modified model consistent with the original problem because the additional term affects the solution of the problem [4].
In this talk, several variants of tensor artificial diffusion are presented, focusing on practical aspects of its implementation and use.

Keywords: Finite Element Method, Oldroyd-B Fluid, Numerical Stabilization, Stress Diffusion.

Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UIDB/04674/2020 (Centro de Matemática e Aplicações).

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How to recognize constant functions through double integrals of their difference quotients

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Talk Abstract

We present some results contained in [1] that address a classical open question by H. Brezis [2] and R. Ignat [3] concerning the characterization of constant functions through double integrals that involve their difference quotients.
On the negative side, we present a counterexample that shows that the natural necessary conditions introduced in [3] are not sufficient without some kind of summability assumption. On the positive side, we show that the answer to the question is positive if we restrict either to functions that are bounded and approximately differentiable almost everywhere, or to functions with bounded variation.
We also discuss the cases that remain open, and how they motivate further questions in measure theory.

Keywords: difference quotients, nonlocal functionals, bounded variation functions, approximate differentiability.

Acknowledgements

We would like to thank H. Brezis and R. Ignat for personally introducing us to this problem, and L. Ambrosio for some interesting comments.

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An isoperimetric problem with capacitary repulsion

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Talk Abstract

I will discuss a classical variational model of a charged droplet, given by the sum of a surface energy and a capacitary term, showing that it is mathematically ill-posed irrespectively of the degree to which the liquid is electrified. More specifically, an isolated spherical droplet is never a local minimizer, no matter how small is the total charge on the droplet, since the energy can always be lowered by an arbitrarily small distortion of its surface. This is in contrast with the experimental observations that a critical amount of charge is needed in order to destabilize a spherical droplet.
On the other hand, if the repulsion is stronger than the usual Coulombic one, the surface energy dominates the capacitary term at small scales. In this case, one can prove existence and regularity of minimizers for small charges. Combining this result with the stability of the ball under small regular perturbations, this leads to the minimality of the ball for small charges.

Keywords: charged droplets, nonlocal energies, isoperimetric problems.

Asymptotic behavior of wave fronts in parameter-dependent Born-Infeld models

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Talk Abstract

We present some recent results for parameter-dependent Born-Infeld reaction-diffusion models. In particular, after providing some estimates of the critical speed, we discuss the behavior of the critical traveling front as the parameters vary. We mainly focus on two results: on the one hand, we show the convergence to the critical front for the linear diffusion equation when the maximal field strength goes to infinity. On the other hand, we highlight a new phenomenon of front sharpening in the limit for vanishing diffusion, which distinguishes the considered diffusion from linear and saturating ones.

Keywords: Born-Infeld models, critical wave fronts, critical speed, vanishing diffusion limit.

Acknowledgements

This work was partially supported by the PRIN 2017 project “Direct and inverse problems for partial differential equations: theoretical aspects and applications” (Italy) and by GNAMPA (Gruppo Nazionale per l’Analisi Matematica, la Probabilità e le loro Applicazioni, Rome, Italy).

Shock waves in 2D compressible elastodynamics

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Talk Abstract

We study the two-dimensional structural stability of shock waves in a compressible isentropic inviscid elastic fluid in the sense of the local-in-time existence and uniqueness of discontinuous shock front solutions of the equations of compressible elastodynamics in two space dimensions. By the energy method based on a symmetrization of the wave equation and giving an a priori estimate without loss of derivatives for solutions of the constant coefficients linearized problem we find a condition sufficient for the uniform stability of rectilinear shock waves. Comparing this condition with that for the uniform stability of shock waves in isentropic gas dynamics, we make the conclusion that the elastic force plays stabilizing role. In particular, we show that, as in isentropic gas dynamics, all compressive shock waves are uniformly stable for convex equations of state. Moreover, for some particular deformations (and general equations of state), by the direct test of the uniform Kreiss–Lopatinski condition we show that the stability condition found by the energy method is not only sufficient but also necessary for uniform stability. As is known, uniform stability implies structural stability of corresponding curved shock waves.
The result obtained is a joint work with Alessandro Morando and Yuri Trakhinin.

Keywords: elastodynamics, shock waves, free boundary problem.

A topological degree theory for rotating solutions of planar systems

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Talk Abstract

We present a generalized notion of degree for rotating solutions of planar systems. We prove a formula for the relation of such degree with the classical use of Brouwer's degree and obtain a twist fixed-point theorem providing information of the rotation number of the associated periodic solutions [1]. We then apply the result in a short proof of the sharp lower bound on the number of periodic solutions of planar Hamiltonian systems asymptotically linear at zero and infinity [2], illustrating the complementarity of our theorem with the Poincaré–Birkhoff Theorem.

Keywords: periodic solutions, fixed-point theorem, Maslov's index, asymptotically linear Hamiltonian systems.

Acknowledgements

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Geometric optics for hyperbolic free boundary problems

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Talk Abstract

In this talk we discuss the geometric optics approach for the construction of weakly nonlinear, highly oscillating solutions to some free boundary problems in Fluid Dynamics and MHD. The analysis depends on the nature of the roots of the Lopatinskii determinant associated to the problem. As evidenced in earlier works, for problems where the Lopatinskii determinant vanishes at an *elliptic* frequency, high frequency oscillations may give rise to surface waves on either side of the free boundary that decay exponentially in the normal direction. Such a case occurs for current-vortex sheets and the plasma-vacuum interface problem in MHD. In other problems where the Lopatinskii determinant vanishes at an *hyperbolic* frequency, such as detonation waves or compressible vortex sheets, there occur non-decaying radiative surface waves that generate bulk waves propagating away from the boundary into the interior of the space domain.

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Three solutions for Dirichlet boundary value problems driven by an a-Laplacian operator

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Talk Abstract

An overview on some results concerning the existence of at least three solutions for nonlinear elliptic Dirichlet boundary value problems driven an a-Laplacian operator and with sub-linear reaction terms at zero, is given. No asymptotic conditions on the nonlinearity at infinity are required. In particular, one looks on those results obtained combining variational and truncation techniques.

Keywords: positive solutions, nodal solutions, nonlinear nonhomogeneous elliptic equations.

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Fractional elliptic systems with critical nonlinearities

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Talk Abstract

Elliptic systems arise in biological applications (e.g. population dynamics) or physical applications (e.g. models of a nuclear reactor) and have been drawn a lot of attention. In the nonlocal case, there are not so many papers on weakly coupled systems in $\mathbb{R}^N$. In this talk we present some recent existence, uniqueness and multiplicity results for positive solutions of a class of weakly coupled nonlocal systems of equations in $\mathbb{R}^N$, which are new also in the local case. Moreover, we also provide a global compactness result, which gives a complete description of the Palais-Smale sequences of the treated systems. To the best of our knowledge, this decomposition has been studied only for systems of equations in bounded domains.

Keywords: Nonlocal systems, uniqueness, ground state solutions, Palais-Smale decompositions, energy estimates, positive solutions, min–max methods.

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Existence and Stability of almost finite energy weak solutions to the Quantum Euler-Maxwell system

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Talk Abstract

We prove the existence of global in time, finite energy, weak solutions to a quantum magnetohydrodynamic system [7] (QMHD) with large data, modeling a charged quantum fluid interacting with a self-generated electromagnetic field. The analysis of QMHD relies upon the use of Madelung transformations. The rigorous derivation requires non-trivial smoothing estimates, which are obtained by assuming slightly higher regularity for the electromagnetic potential. These assumptions are motivated by the nonlinear dependence of the hydrodynamic system in terms of the underlying wave function dynamics, which is supercritical with respect to the bare energy bounds. [2,**?**,4]
Due to quantum effects on the dispersive properties of QMHD, our approach requires neither smallness nor high regularity, unlike a large amount of existing literature for Euler-Maxwell's classical system [9,8]. In fact, the difficulty posed by the presence of the nonlinear electromagnetic force field (Lorentz) severely restricts the possibility to get existence and stability results in the general framework of finite energy solutions. In the classical case the dispersion is not able to deal with the transport of a non-trivial vorticity, therefore almost GWP holds in a life span, reciprocal of the amplitude of the vorticity. GWP can be proved in the irrotational case, where in any case smallness and high regularity assumptions are needed.
For quantum MHD system the irrotationality and the presence of a highly nonlinear quantum stress tensor induce much stronger dispersive properties, as a byproduct of a close relationship with the classical Maxwell-Schrödinger system. Therefore the core argument is shifted to the analysis of the nonlinearities related to the formulation of the hydrodynamic variables through the Madelung transformations. The analysis carried out in section 4 shows that it is necessary to go through non-trivial smoothing estimates and these require us to assume regularity conditions, just above the energy norms, for the initial data of the Maxwellian electromagnetic potential. In the same regime of regularity, with the help of suitable local smoothing estimates, we also prove stability of both the hydrodynamic variables and the Lorentz force associated with the electromagnetic field. These results can be found in [6]

Keywords: Quantum hydrodynamics, Quantum MHD, Maxwell-Schrödinger, Madelung transformation, Finite-energy weak solutions, Large data.

Acknowledgements

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Sharp regularity results for the one-dimensional curvature equation via the maximum principle

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Talk Abstract

We present a novel proof of some sharp regularity results that we have recently obtained in [1,**?**,**?**,**?**] for the bounded variation solutions of non-autonomous quasilinear equations driven by the one-dimensional curvature operator. The alternative approach adopted provides a new interpretation of the considered assumptions clarifying their meaning and making their connection with the strong maximum principle transparent.

Keywords: non-autonomous quasilinear equation, bounded variation solution, strong solution, strong maximum principle.

Acknowledgements

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A new approach to singularities in solid mechanics via a notion of generalized Jacobian determinant

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Talk Abstract

We introduce a notion of Jacobian determinant for some R^2-valued maps of bounded variation on a bounded planar domain. This notion extends the Distributional Determinant and allows for some applications in solid mechanics. We will focus on classical singularities appearing in Ginzburg-Landau model and dislocations mechanics. This is a joint work with Nicolas Van Goethem and Lucia De Luca.

Keywords: Singularities in solid mechanics, Jacobian determinant, dislocations, Ginzburg-Landau model.

Acknowledgements We thank the organizers for the invitation.

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Stationary solutions to some Ginzburg-Landau equations

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Talk Abstract

We consider the Ginzburg-Landau equation either on a bounded domain or over the whole space. For this equation, stationary solutions solve an elliptic PDE with complex coefficients. Consequently, there is a lack of structure that permits the use of variational arguments. We present some existence and nonexistence results, either through explicit computations or performing a bifurcation analysis. As a byproduct, we study some bifurcation problems starting from multiple eigenvalues, thus generalizing the results of [3].

Keywords: Ginzburg-Landeu, stationary solutions, bifurcation.

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Periodic solutions for systems of differential equations with delays and impulses

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Talk Abstract

For a family of periodic systems of differential equations with (possibly infinite) delay and nonlinear impulses, sufficient conditions for the existence of at least one positive periodic solution are established. The main technique used here is the Krasnoselskii fixed point theorem on cones. Although fixed points methods have been extensively employed to show the existence of positive periodic solutions to scalar delay differential equations (DDEs), the literature on n-dimensional impulsive DDEs is very scarce. Our criteria are applied to some classes of mathematical biology models, such as Nicholson-type systems with patch structure. See [1].

Keywords: Delay differential equations, impulses, positive periodic solutions, Krasnoselskii's fixed point theorem, Nicholson systems.

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Unique continuation from the boundary for fractional problems

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Talk Abstract

I will present the recent results of paper [1], concerning the problem of unique continuation from boundary points for some fractional elliptic equations under outer homogeneous Dirichlet boundary conditions. I will describe a blow-up procedure which involves an Almgren type monotonicity formula and provides a classification of all possible homogeneity degrees of limiting entire profiles. The Caffarelli-Silvestre extension provides an equivalent formulation of the fractional equation as a local degenerate or singular problem in one dimension more, with mixed Dirichlet and Neumann boundary conditions. In the development of a monotonicity argument, the mixed boundary condition raises delicate regularity issues, which turn out to be quite difficult in dimension $N \geq 2$ due to the positive dimension of the junction set and some role played by the geometry of the domain. Such difficulties are overcome by a double approximation procedure: by approximating the potential with functions vanishing near the boundary and the Dirichlet N-dimensional region with smooth $(N+1)$-dimensional sets with straight vertical boundary, it is possible to construct a sequence of approximating solutions which enjoy enough regularity to derive Pohozaev type identities, needed to obtain Almgren type monotonicity formulas and consequently to perform blow-up analysis.

Keywords: fractional elliptic equations, unique continuation, monotonicity formula, boundary behavior of solutions.

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Contributed talks

Quasiconvexity and the norms of the Beurling–Ahlfors transform

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Talk Abstract

The Beurling–Ahlfors transform $\mathcal{S}\colon L^p(\mathbb{C}) \to L^p(\mathbb{C})$ is a fundamental singular integral operator in the complex plane. A well-known conjecture due to Iwaniec asserts that

$$\|\mathcal{S}\|_{L^p(\mathbb{C})\to L^p(\mathbb{C})} = \max\{p-1, (p-1)^{-1}\}.$$

This conjecture has a deep connection with Morrey's problem, which relates quasiconvexity and rank-one convexity in the vectorial Calculus of Variations. We will discuss recent progress in both problems by establishing quasiconvexity for a large family of geometric integrands. As applications, we derive sharp L^p estimates for the derivatives of quasiconformal mappings and sharp $L \log L$ estimates for the Jacobian determinant of an orientation-preserving mapping. Based on joint work with Kristensen [2] and Astala, Faraco, Kristensen and Koski [1].

Keywords: quasiconvexity, rank-one convexity, quasiconformal mappings.

Acknowledgements

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Gradient estimates for an orthotropic nonlinear diffusion equation

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Talk Abstract

We consider a quasilinear degenerate parabolic equation driven by the orthotropic p-Laplacian. We prove that local weak solutions are locally Lipschitz continuous in the spatial variable, uniformly in time.

Keywords: Degenerate parabolic equations, Lipschitz continuity, anisotropic operators.

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The motion of the director field of a nematic liquid crystal

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Talk Abstract

In this talk, based in a joint paper with Paulo Amorim and Assis F. Martins, cf. [1], we study the motion of the director field of a nematic liquid crystal submitted to a magnetic field and to a laser beam. For planar deformations depending on a single space variable and in a particular case we prove the existence of a weak solution for the corresponding initial value problem.

Keywords: nematic, magnetic field, laser beam, Cauchy problem.

Acknowledgements

J.P.Dias was partially supported by the Fundação para a Ciência e Tecnologia (FCT) through grant UIDB/04561/2020.

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Fractional Sobolev regularity for fully nonlinear elliptic equations

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Talk Abstract

We study high-order fractional Sobolev regularity for fully nonlinear, uniformly elliptic equations, in the presence of unbounded source terms. More precisely, we show the existence of a universal number $0 < \varepsilon < 1$, depending only on ellipticity constants and dimension, such that if u is a viscosity solution of $F(D^2u) = f(x) \in L^p$, then $u \in W^{1+\varepsilon,p}$, with appropriate estimates. Our techniques are based on touching the solution with $C^{1,\alpha}$ cone-like functions to produce a decay rate of the measure of certain sets.

Keywords: regularity theory, viscosity solutions, $C^{1,\alpha}$-aperture functions.

Acknowledgements

This work was supported by PUC-Rio Archimedes Fund.

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A mass optimization problem with convex cost

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Talk Abstract

We consider a mass optimization problem where instead of imposing a constraint on the total mass of the competitors, we penalize the classical compliance by a convex functional defined on the space of measures. We obtain a characterization of optimal solutions to the problem through a suitable PDE. This generalizes the case considered in the literature of a linear cost (see [1]) and applies to the optimization of a conductor where very low and very high conductivities have both a high cost, and then the study of nonlinear models becomes relevant. The results are obtained in collaboration with G. Buttazzo and D. Lučić
.

Keywords: Mass optimization problems, convex functionals on measures, Sobolev spaces with respect to measures, Fenchel duality.

Acknowledgements

This work was partially supported by the two projects PRIN 2017 -*Gradient flows, Optimal Transport and Metric Measure Structures*, and PRIN 2017-*Variational Methods for Stationary and Evolution Problems with Singularities and Interfaces*, both funded by the Italian Ministry of Research and University.

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Pathwise stochastic control and a class of stochastic partial differential equations

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Talk Abstract

We consider a class of control problems where we study the minimization of cost in the pathwise sense. We derive the associated Bellman's dynamical programming principle in the pathwise sense and show that the corresponding Hamilton-Jacobi-Bellman equation is well-posed in the class of stochastic viscosity solutions. Moreover, we give a characterization of the drift of the control problem. This is an ongoing work.

Keywords: stochastic control, pathwise, viscosity solutions.

Regularity theory for the parabolic normalized p-Laplace equation

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Talk Abstract

In this talk, I will discuss regularity estimates for viscosity solutions to the parabolic normalized p-Laplacian. We prove that the gradient of bounded viscosity solutions is locally asymptotically Lipschitz continuous when p is sufficiently close to 2. We also establish regularity estimates in Sobolev spaces. This is joint work with Makson Santos.

High-order interpolatory/quasi-interpolatory serendipity virtual element method for semilinear parabolic problems

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Talk Abstract

An efficient method for the numerical approximation of a general class of two dimensional semilinear parabolic problems on polygonal meshes is presented. The proposed approach takes advantage of the properties of the serendipity version of the virtual element method (VEM), which not only reduces the number of degrees of freedom compared to the classical VEM, but also allows for the introduction of an interpolatory or quasi-interpolatory approximation of the nonlinear term that is computable from the degrees of freedom of the discrete solution with a low computational cost, thus significantly improving the efficiency of the method. An error analysis for the semi-discrete formulation is carried out, and an optimal estimate for the error in the L_2-norm is obtained. The accuracy and efficiency of the proposed method when combined with a second order Strang operator splitting time discretization is illustrated with numerical experiments, with approximations up to order 6.

Keywords: serendipity virtual element method, interpolant operator, operator splitting method, semilinear parabolic equations.

Stability of a periodically perturbed point-vortex

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Talk Abstract

We present a result about the stability of a periodic Hamiltonian system in the plane with a singularity: a periodically perturbed point-vortex, [1]. In a perfect fluid, a point-vortex is essentially a singularity of the vorticity, and can be modeled by the Hamiltonian

$$H_0(x,y) = \frac{1}{2} ln(x^2 + y^2),$$

being x and y the usual rectangular coordinates in the plane. The associated system is integrable, with the particles rotating around the vortex in circular paths and the origin is trivially stable. We have studied this system after introducing an external periodic perturbation $p(t,x,y)$. The perturbed system models ideally the passive transport of particles in a perfect fluid under the action of a steady vortex placed at the origin and an external time-dependent background flow. We will see which hypothesis must be imposed on the perturbation $p(t,x,y)$ to preserve the stability of the origin. In this context, we apply the *Invariant Curve Theorem* in the analytical version presented in [2]. This allows to find a family of invariant curves by the Poincaré map of our system. These curves surround the vortex and due to the low dimensionality, act as barriers to the solutions; therefore, the stability of the origin can be guaranteed. Recently, in [3] the authors proved a similar stability result under the action of a periodic background flow induced by a general polynomial field

$$\sum_{1 \le i+j \le N} a_{ij}(t) x^i y^j,$$

where a_{ij} are 2π-periodic continuous differentiable functions. The proof is obtained from a finite differentiable version of the *Invariant Curve Theorem* [4].

Joint work with Rafael Ortega and Pedro J. Torres.

Keywords: Vortex dynamics, KAM theory, particle advection, Mosers invariant curve theorem

Acknowledgements

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