NUMERICAL STUDY OF NEWTONIAN FLUID FLOWS IN TREE-SHAPED STRUCTURES ¶

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From the Constructal Theory standpoint, living systems are those where there is a flow with a purpose. Concerning their shapes, a great variety may be encountered, and, often, the artificial seeks its inspiration in the natural. Analogous patterns and shapes in animate systems are numerous, from the honeycomb configuration in tissues and cells to the tree-shaped configuration of lightning, neurons, roots, and branches of plants, blood circulatory systems, and watersheds [1]. The internal flow of fluids through tree-shaped systems has been an important object of investigation due to its importance in understanding natural systems' behavior and designing artificial systems [1-4]. For a fluid transport system, the best configuration, which connects a point-to-volume or volume-to-point, occurs in the shape of a tree, and an optimal ratio between the large and the small duct is the unknown to be specified [1-6]. For the vascular system, assuming a Hagen-Poiseuille flow through the vessels, Hess [7] and Murray [8] determined the optimal branching diameter. For symmetrical vessels, the ratio between the daughter and parent diameters (a_{D_i}) is $a_{D_i} = 2^{-1/3}$ (Hess-Murray's law). Although first derived from the principle of minimum work, the Hess-Murray law can be obtained in the light of the constructal law of design in Nature [2,5,6]. In accord with this law. For a finite-size open system to persist in time (to live), it must evolve such that it provides easier access to the imposed (global) currents that flow through it [1]. The shape (design) is the constructal way to transport fluid, heat, mass, or information to achieve its purpose under global restrictions. This work is about fluid

¶Poster available on page 262

networks to provide easy access to fluid flow. We address the effect of the network size-limiting constraints on the optimal design. The optimal design of a symmetrical dichotomous tree structure with several branching levels for laminar flow of a Newtonian fluid is studied numerically. For optimal flow design to emerge, it is necessary to include size constraints in the study. In several studies, only the volume occupied by the network is preferred [5,7-11]. Here, the volume of each branching level is fixed. Among other results, we showed that the network designed according to the Hess-Murray law does not represent the design with minimum resistance, but the network built on this law is the one with the most uniform resistances at the different levels of bifurcation. Another outcome of our study is that freedom inside a fixed size flow system is needed for preventing non-optimal designs from appearing, which corroborates the constructal thinking of ”freedom is good for design.”

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1. INTRODUCTION

From the Constructal Theory standpoint, living systems are those where there is a flow with a purpose. Concerning their shapes, a great variety may be encountered, and, often, the artificial seeks its inspiration in the natural. Analogous patterns and shapes on animate systems are numerous, from the honeycomb configuration in tissues and cells to the tree-shaped configuration of lightning, neurons, roots, and branches of plants, blood circulatory systems, and watersheds [1]. The internal flow of fluids through tree-shaped systems has been an important investigation object due to its importance in understanding natural systems' behavior and designing artificial systems [1-4]. For a fluid transport system, the best configuration, which connects a point-to-volume or volume-to-point, occurs in the shape of a tree, and an optimal ratio between the large and the small duct is the unknown to be specified [1-6]. For the vascular system, assuming a Hagen-Poiseuille flow through the vessels, Hess [7] and Murray [8] determined the optimal branching diameter. For symmetrical vessels, the ratio between the daughter and parent diameters (a_{Di}) is a_{Di} = $2^{-1/3}$ (Hess-Murray's law). Although first derived from the principle of minimum work, the Hess-Murray law can be obtained in the light of the constructal law of design in Nature [2,5,6].

2.METHODS

The system has the global geometric constant, which is defined by the ducts' volume at each branch level (V_i), which is kept fixed for the different cases studied. The degree of freedom of the system is defined by the ratio between the diameter of the daughter and parent ducts (a_{Di}). Thus, fluidic structures with Tree-shaped bifurcations are determined by Eqs. (1) to (4):

$$V_i = V_{i+1} = V_{i+2} = V_{i+3} \quad (1)$$

$$V_i = \frac{\pi}{4}\, 2^i\left({D_i}^2 L_i\right) = const \quad (2)$$

$$V_T = \frac{\pi}{4}\left({D_0}^2 L_0 + 2{D_1}^2 L_1 + 4{D_2}^2 L_2\right) \quad (3)$$

$$a_{Di} = D_{i+1} \,/\, D_i \quad (4)$$

where D is the diameter, L is the length and a_{Di} is the ratio between the diameters and the indices i and $i+1$ signify the parent and child ducts respectively. The index i ranges from 0 to 2, thus informing the branch's level, where a larger index means that the network is more branched. Thus, the balances of mass and momentum applied to the system under study are defined according to Eq. (5), respectively.

$$\nabla \boldsymbol{u} = 0 \quad \text{and} \quad -\nabla p + \mu \nabla^2 \boldsymbol{u} = 0 \quad (5)$$

where $\boldsymbol{u}$ is the velocity vector, p is the pressure and μ is the dynamic viscosity.

The objective of the work is to evaluate different geometric configurations in the search for a system that minimizes losses and facilitates the access of fluid flow. In view of this, the parameter used to quantify this objective is the overall resistance (R_T) of the flow determined by Eq. (6).

$$R_i = \Delta p_i \,/\, \dot{m}_i \quad \text{and} \quad R_T = \sum R_i \quad (6)$$

where, Ri is the level flow resistance, Δp_i is the partial drop pressure, $\dot{m}$ is the mass flow, and R_T is the overall resistance. HMR is the ratio of Hess-Murray resistances, R_T the total flow resistance defined as the sum of the resistances at each branch level, and $R_{Hess\text{-}Murray}$ the tree-shaped structure resistance numerically simulated and projected according to the geometric $a_{Di} = 2^{-1/3}$.

$$HMR = R_T \,/\, R_{Hess-Murray} \quad (7)$$

Fig. 1 shows the dichotomous fluid structure in the form of a tree with two levels of branching and a circular section.

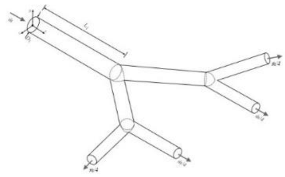
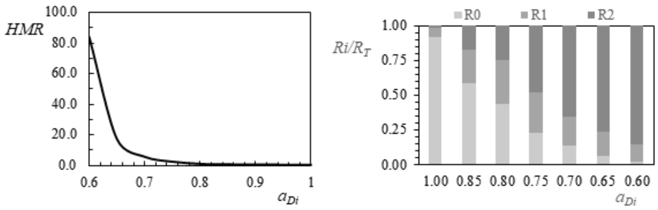


Figure 1: Schematic representation of a branched structure in two shape

The governing Eq. (5) were solved using a finite volume method and employing the segregated method with implicit formulation. A constant mass flow rate and an outflow boundary condition are used at the inlet and at the outlet, respectively. No-slip boundary conditions were applied at walls. The residual values of the governing Eqs. (5) were all set to 10^{-6}. Details can be found in [10].

3. RESULTS E REMARKS

Fig. 2 shows the ratio of Hess-Murray resistances (HMR) and the resistance ratio at each branch level (R_i/R_T) as a function of the diameter ratio (a_{Di}).



(a) (b)

Figure 2: (a) HMR x a_{Di} and (b) R_i/R_T x a_{Di} for Newtonian fluid flow with $Re_D = 10^2$

It can be seen that the curve in Fig. 2(a) presents a behavior where the minimum point where the ratio of diameters a_{Di} = 1, which does not correspond to the optimal diameter ratio proposed according to the Hess-Murray Law. In Fig. 2(b), it is possible to observe that the structure with relation to a_{Di} = 0.80 presents the most homogeneous distribution of the flow resistance between the branching levels.

Here, the volume of each branching level is fixed, Eq. (1) . Among other results, we showed that the network designed according to the Hess-Murray law does not represent the design with minimum resistance, but the network built on this law is the one with the most uniform resistances at the different levels of bifurcation (Figure 2). Another outcome of our study is that freedom inside a fixed size flow system is needed for preventing non-optimal designs from appearing, which corroborates the constructal thinking of "freedom is good for design."

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