



**QUEEN'S
UNIVERSITY
BELFAST**

Measurement of the compressive crack resistance curve of composites using the size effect law

Catalanotti, G., Xavier, J., & Camanho, P. P. (2014). Measurement of the compressive crack resistance curve of composites using the size effect law. *Composites Part A: Applied Science and Manufacturing*, 56, 300-307. <https://doi.org/10.1016/j.compositesa.2013.10.017>

Published in:
Composites Part A: Applied Science and Manufacturing

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
[Link to publication record in Queen's University Belfast Research Portal](#)

Publisher rights

Copyright 2014 Elsevier.

This manuscript is distributed under a Creative Commons Attribution-NonCommercial-NoDerivs License (<https://creativecommons.org/licenses/by-nc-nd/4.0/>), which permits distribution and reproduction for non-commercial purposes, provided the author and source are cited.

General rights

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26

Measurement of the compressive crack resistance curve of composites using the size effect law

15 G. Catalanotti ^{a,*}, J. Xavier ^b, P.P. Camanho ^a

18 ^a*DEMec, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal*

21 ^b*CITAB, University of Trás-os-Montes e Alto Douro, Apartado 1013, 5001-801 Vila Real, Portugal.*

Abstract

27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54

This paper presents a new methodology to measure the compressive crack resistance curve of the longitudinal plies of carbon-epoxy laminates. The methodology is based on three main steps: the first one corresponds to the determination of the energy release rate of cross-ply laminates with two edge cracks using a parametric finite element model. The energy release rate is used in the definition of a relation between the crack resistance curve and the size effect. Finally, experimental tests are performed in scaled double-edge notched specimens to quantify the size effect law, thus proving the last piece of information required to define the crack resistance curve. The full crack resistance curve is obtained for the IM7-8552 carbon epoxy composite material. The methodology proposed in this paper overcomes the inherent limitations of the existing test methods, and it serves as the basis for the identification of cohesive laws used in some analysis models.

Key words:

55
56
57
58
59
60

A. Polymer-matrix composites (PMCs), B. Fracture toughness, C. Analytical modelling, D. Mechanical testing

1
2
3
4 **1 Introduction**
5
6
7

8
9 2 The development of a new generation of tougher composite materials presents
10 3 several challenges to the existing methods for the measurement of the fracture
11 4 toughness associated with cracks that propagate perpendicularly to the fiber
12 5 direction. The intra-laminar fracture toughness is relevant not only for mate-
13 6 rial screening and qualification, but also to define the softening laws used in
14 7 recent analysis models that predict the ultimate strength of composite struc-
15 8 tures [1–5].

9
10 9 The Compact Tension (CT) test specimen [6,7] is normally used to measure
11 10 the fracture toughness and the crack resistance curve (R-curve) of compos-
12 11 ite materials reinforced by unidirectional fibres. While reliable results can be
13 12 obtained for brittle material systems using appropriate data reduction meth-
14 13 ods [8,7], the introduction of tougher resins leads to higher loads for crack
15 14 propagation, which may cause buckling of the unnotched end of the CT test
16 15 specimen [9].

17
18 16 There is also the need to measure the fracture toughness and the correspond-
19 17 ing R-curve associated to the propagation of a kink-band, which shows a
20 18 crack-like behaviour [10–12] with an R-curve that results from the broadening
21 19 of the damage height [11]. It is considered here that the compact compres-
22 20 sion test specimen is inadequate to measure the compressive crack resistance
23 21 curves of polymer composite materials. In fact, the correction factor used in
24 22 the data reduction method of the compact compression test method to calcu-
25 23 late the energy release rate is the same as that used in the compact tension

26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59

* Corresponding author
60 *Email address:* giuseppe.catalanotti@fe.up.pt (G. Catalanotti).
61
62
63
64
65

1
2
3
4 method. However, the contact tractions that occur on the crack faces during a
5
6 compact compression test render the data reduction method inaccurate. This
7
8 was demonstrated in a previous investigation [7] where the \mathcal{J} -integral around
9
10 the crack tip was computed using digital image correlation. It was shown [7]
11
12 that the R-curve of the compact test specimens using the \mathcal{J} -Integral and the
13
14 data reduction method proposed by Pinho et al. [6] are virtually the same for
15
16 tension but not for compression.
17

18
19
20 It should also be noted that the compact compression specimen triggers dif-
21
22 fused damage during the propagation of the kink-band, artificially increasing
23
24 the value of the measured fracture toughness, and that it is not possible to
25
26 identify the location of the tip of the kink band [7]. Therefore, it is considered
27
28 that while the compact compression test method may be used to measure the
29
30 initial value of the fracture toughness it does not provide reliable information
31
32 for the generation of the R-curve.
33

34
35
36 This means that the analysts have no reliable test methods to measure some of
37
38 the required material properties, namely the fracture toughness related with
39
40 the propagation of a kink-band and the corresponding R-curve.
41

42
43 This fact provides the motivation for this paper, whose objective is to propose
44
45 a new methodology to obtain the R-curve of composite materials that fail
46
47 by the propagation of a kink-band. The approach proposed is based on the
48
49 relation between the size effect law and the R-curve, and it will be presented
50
51 in the following order: section 2 describes the analytical model that relates
52
53 the size effect law obtained in cross-ply laminates with two edge notched to
54
55 the R-curve. Section 3 presents the details of the compression tests performed
56
57 in the double-edge notched specimens manufactured using IM7-8552 carbon-
58
59 epoxy. Section 4 combines the analytical model and the test results to fully
60
61
62
63
64
65

1
2
3
4
50 define the R-curve for the material under investigation, and Section 5 presents
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

define the R-curve for the material under investigation, and Section 5 presents the main conclusions of the paper.

2 Analytical Model

Consider the geometry shown in Figure 1. The width is equal to $2w$, the length is $3w$, and a_0 is the initial crack length.

[Fig. 1 about here.]

In a two-dimensional orthotropic body, considering x and y as the preferred axes of the material, the energy release rate (ERR) in mode I for a crack propagating in the x -direction reads [13]:

$$\mathcal{G}_I = \left(s_{11}s_{22} \frac{1+\rho}{2} \right)^{1/2} \lambda^{-1/4} \mathcal{K}_I^2 \quad (1)$$

where s_{lm} are the components of the compliance matrix computed in the x - y coordinate system, \mathcal{K}_I is the stress intensity factor, and λ , ρ are the two dimensionless elastic parameters defined as:

$$\lambda = \frac{s_{11}}{s_{22}}, \quad \rho = \frac{2s_{12} + s_{66}}{2\sqrt{s_{11}s_{22}}} \quad (2)$$

Suppose that the crack is propagating in a $[0/90]_{ns}$ cross-ply laminate. In this case, $s_{11} = s_{22}$, $\lambda = 1$ and equation (1) reads:

$$\mathcal{G}_I = \frac{1}{E} \sqrt{\frac{1+\rho}{2}} \mathcal{K}_I^2 \quad (3)$$

1
2
3
4 where E is the laminate Young's modulus along the x (or y) direction.
5
6

7
8 The stress intensity factor of the double edge notched specimen shown in
9
10 Figure 1 is a function of ρ , of the remote stress σ , and of the shape and the
11
12 size of the specimen:
13
14

$$\mathcal{K}_I = \sigma \sqrt{w} \sqrt{\phi(\alpha, \rho)} \quad (4)$$

15
16
17
18
19
20
21 where $\alpha = a/w$ is the shape-parameter and $\phi(\alpha, \rho)$ is the correction factor
22
23 for the geometry and orthotropy of the material. Substituting (4) in (3) the
24
25 energy release rate reads:
26
27
28
29

$$\mathcal{G}_I = \frac{1}{E} \sqrt{\frac{1+\rho}{2}} \sigma^2 w \phi(\alpha, \rho) = \frac{1}{4wE} \sqrt{\frac{1+\rho}{2}} \left(\frac{P}{t}\right)^2 \phi(\alpha, \rho) \quad (5)$$

30
31
32
33
34
35 where t is the thickness of the specimen.
36
37

38 Following the approach proposed in [7] the dimensionless function $\phi(\alpha, \rho)$
39
40 can be defined for the problem under consideration using the Finite Element
41
42 Method (FEM). For this purpose a parametric Finite Element model was cre-
43
44 ated using Python [14] together with Abaqus 6.8-3 Finite Element code [15].
45
46

47 In this model the characteristic distance w is taken constant and equal to the
48
49 unity, while the variables are: i) the shape parameter or, in other words, the
50
51 crack length $0 < \alpha < 1$; ii) the dimensionless parameter ρ that takes into
52
53 account the effect of the orthotropy of the material ($0 \leq \rho \leq 20$). There is no
54
55 need to take into account λ because for the layup that was chosen (balanced
56
57 cross ply) its value is constant and equal to one; therefore λ is not accounted
58
59 for in the calibration of the model. Figure 2 shows the mesh of the finite
60
61
62
63
64
65

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

83 element model used to define the function ϕ .

84 [Fig. 2 about here.]

85 Taking advantage of the symmetry of the problem, only one quarter of the
86 specimen was modeled. The 4-node quadratic, reduced integration element,
87 CPS4R, is used.

88 The displacement u_y is applied on the top face of the model and represents
89 the loading condition that will be used in the experiments. The nodes on the
90 top faces are free to move in x direction; this means that a frictionless con-
91 tact occurs between the specimen and the loading system. The Virtual Crack
92 Closure Technique (VCCT) [16] is used to compute the energy release rate at
93 the crack tip, \mathcal{G}_I . The applied load P is calculated summing the reactions at
94 the nodes.

Using the results obtained in the FE analysis the correction factor ϕ is ap-
proximated by the following polynomial:

$$\phi(\alpha, \rho) = \frac{\alpha}{1 - \alpha} \sum_{i=1}^M \sum_{j=1}^N \Phi_{ij} \rho^{j-1} \alpha^{i-1} \quad (6)$$

95 where Φ_{ij} is the element of the matrix $\mathbf{\Phi}$ at the row i and at the column j ,
96 and M and N are the number of rows and columns of $\mathbf{\Phi}$ respectively. The
97 matrix $\mathbf{\Phi}$ reads:

$$\Phi = \begin{bmatrix} 4.315050777 & -0.1833177904 & 0.01642021976 & -4.829962430E-4 \\ -5.148136502 & -0.3554678337 & -9.974634025E-4 & 4.975387379E-4 \\ 2.385888075 & 1.339974300 & -0.05966399650 & 7.544565390E-4 \\ -0.2810124370 & -0.8040552990 & 0.04491874691 & -7.869467548E-4 \end{bmatrix} \quad (7)$$

98

99 Figure 3 shows the numerical points obtained in the FE simulations (each point
100 corresponds to one simulation) and the surface fitting function of equation (6).

101

[Fig. 3 about here.]

102 Figure 4 shows the relation between ϕ and α for different values of the di-
103 mensionless elastic parameter ρ obtained by the polynomial approximation
104 and by the FE model. It can be noticed that the fitting obtained is excel-
105 lent. The curve with $\rho = 1$ corresponds to the isotropic case. The close range
106 $\rho \in [0, 20]$ should cover all the practical applications. For example for a IM7-
107 8552 $[0/90]_{ns}$ laminate ($E_1 = 171420\text{MPa}$, $E_2 = 9080\text{MPa}$, $G_{12} = 5290\text{MPa}$,
108 $\nu_{12} = 0.32$) the elastic parameters calculated using the classical lamination
109 theory are $E_x = E_y = 90648\text{MPa}$, $G_{xy} = 5290\text{MPa}$, $\nu_{xy} = \nu_{yx} = 0.032$ and,
110 from (2), $\rho = 8.54$.

111

[Fig. 4 about here.]

112 After defining ϕ , it is observed that the energy release rate $\mathcal{G}(a)$ is an increas-
113 ing function of the crack length, or, in other words, the proposed specimen
114 has a *positive geometry* [17]. Equation (5) can be re-written as:

$$\mathcal{G}_I(a + \Delta a) = \frac{1}{4wE} \sqrt{\frac{1 + \rho}{2}} \left(\frac{P}{t}\right)^2 \phi\left(\alpha_0 + \frac{\Delta a}{w}, \rho\right) \quad (8)$$

where $\alpha_0 = a_0/w$ (see Fig. 1). The energy release rate $\mathcal{G}_I(a + \Delta a)$ and the schematic representation of the R-curve of the material $\mathcal{R}_I(\Delta a)$ are shown in Fig. 5. The dotted line represents the $\mathcal{G}_I(a + \Delta a)$ at a constant load P . The crack cannot propagate at a constant load if $\mathcal{G}_I < \mathcal{R}$ while it will propagate dynamically if $\mathcal{G}_I > \mathcal{R}$. The dashed line in Fig. 1, tangent at the R-curve, represents the crack-driving force at the peak load, P_u (or at the maximum remote stress, σ_u).

[Fig. 5 about here.]

In summary, for different sizes w_k the driving-force curves \mathcal{G}_I corresponding at the peak loads P_{uk} are tangent to R-curve, \mathcal{R} . This fact will be used to measure the R-curve.

2.1 Obtaining the R-curve from size effect

Based on the previous observations, the peak load, P_u , or the ultimate nominal stress, $\sigma_u = P_u/(2wt)$, can be obtained from the following system of equations:

$$\begin{cases} \mathcal{G}_I(a + \Delta a) = \mathcal{R}(\Delta a) \\ \frac{\partial \mathcal{G}_I(a + \Delta a)}{\partial \Delta a} = \frac{\partial \mathcal{R}(\Delta a)}{\partial \Delta a} \end{cases} \quad (9)$$

Assuming that the size effect law, $\sigma_u = \sigma_u(w)$, is known, substituting (5) in the first of equations (9) yields:

$$\frac{1}{E} \sqrt{\frac{1+\rho}{2}} \sigma_u^2 w \phi\left(\alpha_0 + \frac{\Delta a}{w}\right) = \mathcal{R}(\Delta a) \quad (10)$$

This equation holds for every w . Following [17], differentiating (10) with respect to w , under the hypothesis that geometrically similar specimens are tested (α_0 is not a function of w) and remembering that, by definition, the R-curve does not depend on the size of the specimen w ($\partial\mathcal{R}/\partial w = 0$), results in:

$$\frac{1}{E} \sqrt{\frac{1+\rho}{2}} \frac{\partial}{\partial w} \left(\sigma_u^2 w \phi\left(\alpha_0 + \frac{\Delta a}{w}\right) \right) = 0 \quad (11)$$

The R-curve, $\mathcal{R}(\Delta a)$, can be obtained solving (11) for $w = w(\Delta a)$, and by replacing this solution in equation (10).

The proposed method provides the R-curve of the laminate. The R-curve of the 0° plies is obtained neglecting the fracture toughness of the 90° plies, as previously proposed by Pinho [6]. The energy balance for self-similar crack propagation da reads:

$$\mathcal{R} h da = \mathcal{R}_0 h_0 da + \mathcal{R}_{90} h_{90} da \quad (12)$$

where \mathcal{R}_0 and \mathcal{R}_{90} are the R-curves for the 0° and 90° plies respectively while h , h_0 and h_{90} are respectively the thickness of the laminate, of the 0° and of the 90° plies respectively. Taking into account that $\mathcal{R}_{90} \ll \mathcal{R}_0$, and $h_0 = h_{90} = h/2$, equation (12) results in:

$$\mathcal{R}_0 = 2\mathcal{R} \quad (13)$$

1
2
3
4 Therefore the fracture toughness in longitudinal direction \mathcal{R}_0 is simply ob-
5
6 tained as twice the value of the fracture toughness of the laminate \mathcal{R} .
7
8
9

10 11 **3 Experiments** 12 13 14

15 16 *3.1 Material and specimen configurations* 17 18 19

20 The material used is the Hexcel IM7-8552 carbon fiber reinforced epoxy. The
21
22 elastic properties of the material were previously obtained in [18] and are
23
24 shown in Table 1, where E_1 and E_2 are the longitudinal and the transverse
25
26 Young’s modulus respectively, G_{12} is the shear modulus, and ν_{12} is the major
27
28 Poisson’s ratio.
29

30
31
32 [Table 1 about here.]
33
34

35 The $[0/90]_{8s}$ layup with a nominal thickness of 4mm is used. The laminate was
36
37 cured in a hot–press according to the manufacturer’s specifications. The curing
38
39 cycle consists of three different stages: i) keeping the laminate at 110°C for 1
40
41 hour; ii) increasing the temperature up to 180°C and keeping this temperature
42
43 for 2 hours iii) cooling at 3°C/min. The pressure of 7 bar was used during all
44
45 the curing cycle.
46
47

48 After curing, the laminate was cut using a diamond saw disk to the nominal
49
50 specimens size and the notches were machined using a vertical mill equipped
51
52 with 1mm diameter drill bit.
53
54

55 Six sizes were chosen for the specimens, corresponding to the references A to
56
57 F. The nominal dimensions of the specimens are shown in Figure 6. Three
58
59 specimens were tested for each specimen type. It should be noticed that the
60
61
62
63
64
65

1
2
3
4 168 initial crack length a_0 was chosen to be one-half of the characteristic length
5
6 169 for all specimens. Therefore the parameter α_0 is equal to 0.5.
7
8

9
10 [Fig. 6 about here.]
11

12
13 171 The specimens manufactured are scaled from the specimen shown in Figure 1.
14
15 172 It should be noted that in the actual specimen the crack faces are separated
16
17 173 by a finite distance of 1mm. This ensures that there is no contact of the crack
18
19 174 faces; such contact would have invalidated the experimental results. It should
20
21 175 also be noted that the shape of the crack tip (semicircular, 1mm of diameter)
22
23 176 does not have an influence on the correct determination of the R-curve of the
24
25 177 material because: i) the crack tip shape does not influence the value of the
26
27 178 fracture toughness in compression [19], and ii) after the initial propagation
28
29 179 the crack tip can be considered *sharp* and this should not influence the value
30
31 of the fracture toughness at unstable crack propagation.
32
33

34
35 181 After manufacturing, the specimens were painted with a matte white. After
36
37 182 drying the specimens, a speckle was made for all the specimens with the help
38
39 183 of an airbrush (see Fig. 7).
40
41

42
43 184 [Fig. 7 about here.]
44
45
46

47 185 *3.2 Photo-mechanical compression tests*

48
49
50

51
52 186 The compression tests were performed with the support of the digital image
53
54 187 correlation (DIC) system (Fig. 8). The tests were carried out on a Instron
55
56 188 4208 universal testing machine under displacement control, with a cross-head
57
58 189 displacement rate of 0.1 mm/min and at room temperature of 22°C and rel-
59
60 190 ative humidity of 56%. The load was measured using a 100 kN load cell. The
61
62
63
64
65

1
2
3
4 191 Spider data acquisition system along with Catman software was used to record
5
6 192 the strain gauge signal (in the specimens equipped with a strain gauge). 350
7
8 193 Ω strain gauges from Vishay Micro-Measurement, C2A-06-062LW-350, were
9
10 194 used to verify if buckling occurs in the longest specimens (F specimens). To
11
12 195 replicate the same frictionless condition imposed numerically for the calcula-
13
14 196 tion of the correction factor ϕ , a thin layer of molybdenum disulfide (MoS₂)
15
16 197 was used between the specimen and the loading system. The self alignment
17
18 198 system shown in Figure 8(b) was also used. The system consists of two parts
19
20 199 connected by a spherical joint. The upper part of the self alignment system is
21
22 200 in contact with the specimen and it is equipped with a tungsten-carbide insert
23
24 201 to prevent indentation; the bottom part is bolted to the frame of the testing
25
26 202 machine. By allowing small rotations between the two parts, the self alignment
27
28 203 device avoids premature failure of the specimen that may result from possible
29
30 204 parallelism errors between the two faces in contact with the loading system.
31
32

33
34
35 205 [Fig. 8 about here.]
36
37

38
39 206 At the scale of observation used in this work, the textured pattern required for
40
41 207 the digital image correlation was created across the gauge section by means of
42
43 208 aerosol and airbrush painting, as shown in Fig. 9. The Aramis DIC-2D v6.0.2-
44
45 209 6 system was used in this work [20,21]. An 8-bit Baumer Optronic FWX20
46
47 210 camera coupled with a Nikon AF Micro-Nikkor 200 mm f/4D IF-ED lens were
48
49 211 used for image acquisition. The optical devices and measuring parameters are
50
51 212 summarised in Table 2 for each specimen configuration (see also Fig 7).
52
53

54
55 213 [Fig. 9 about here.]
56
57

58
59 214 A subset size and a subset step of 15×15 pixel² and 13×13 pixel² were chosen,
60
61 215 respectively (Table 2). In order to guarantee a suitable speckle pattern for this
62
63
64
65

1
2
3
4 216 virtual gauge an airbrush was used (Iwata Custom 181 Micron CM-B model
5
6 217 with a fluid nozzle of 0.18 mm in diameter and spray 182 range lower than
7
8 218 60 microns). With this marking technique the accurate uniformity, isotropy
9
10 219 and spot size can be achieved. A slightly overlapping of 2 pixels between ad-
11
12 220 jacent subsets was chosen in order to enhance spatial resolution in the strain
13
14 221 reconstruction. A strain base length of 5 subsets was selected for strain com-
15
16 222 putation. The displacement and strain resolution of the measured is expected
17
18 223 in the range of 2×10^{-2} pixel and 0.02-0.04% respectively (Table 2).

22 [Table 2 about here.]

25 3.3 *Experimental results*

26 Representative load vs. displacement curves are shown in Figure 10 for the
27
28 227 different specimens.

29 [Fig. 10 about here.]

30
31
32 229 The F specimens (the longest specimens) were equipped with a strain gauge
33
34 230 on the back side of the specimen (the side without the speckle for the DIC).
35
36 231 The strain measured by the strain gauge was compared to the strain measured
37
38 232 by the DIC (virtual strain gauge) on the front side in the location correspond-
39
40 233 ing to the physical strain gauge. The strain of the virtual strain gauge was
41
42 234 obtained as the average of the longitudinal strain on the rectangular region
43
44 235 symmetric to the strain gauge and with the dimension of the electrical grid of
45
46 236 the physical strain gauge. Figure 11 shows the comparison between the signal
47
48 237 of the physical and of the virtual strain gauge for a representative specimen
49
50 238 and the location of the strain gauge in the specimen. As virtually there are no
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

1
2
3
4 239 differences, it is concluded that no buckling occurred in the test specimens.
5
6
7

8
9 240 [Fig. 11 about here.]
10

11
12
13 241 The digital image correlation can be used to further verify the validity of the
14
15 242 tests performed. Figure 12 shows the contour plot of the shear strain, γ_{xy} . The
16
17 243 perfect antisymmetry of the strain field with respect to the longitudinal axis
18
19 244 of the specimen and the zero value of the shear strain all over the central part
20
21 245 of the specimen (and in particular near the crack tip) shows that the loading
22
23 246 system used is appropriate. Any misalignment would result in a different field
24
25 247 of the shear strain, and mixed mode crack propagation would have occurred.
26
27
28
29

30
31 248 [Fig. 12 about here.]
32
33
34

35 249 The fracture surface for the two types of specimens (A and F) are shown
36
37 250 in Figure 13. It is observed in both specimens that the compressive fracture
38
39 251 occurred along the direction of the initial notches.
40
41
42
43

44 252 [Fig. 13 about here.]
45
46
47
48

49 253 The summary of the results is shown in Table 3. For each type of specimen
50
51 254 the characteristic size w , the peak load P_u , and the standard deviation are re-
52
53 255 ported. The results obtained are used in the following section in the calculation
54
55 256 of the R-curve.
56
57
58
59

60 257 [Table 3 about here.]
61
62
63
64
65

4 Obtaining the R-curve from size effect

As previously explained, the use of the size effect method to obtain the R-curve of the material requires the knowledge of the size-effect law $\sigma_u = \sigma_u(w)$ that relates the nominal remote stress to the characteristic size of the body. According to Bažant and Planas [17] different kind of regression plots can be used to find the size-effect laws: i) bilogarithmic plot ($\ln \sigma_u$ vs. $\ln w$); ii) linear regression I (σ^{-2} vs. w); iii) linear regression II ($(\sigma w)^{-2}$ vs. $1/w$); iv) inverse bilogarithmic plot ($\ln w$ vs. $\ln \sigma_u$). Those methods yield approximatively the same results.

After the inspection of the data obtained experimentally and reported in Table 3, the linear regression I was selected. Figure 14 shows σ^{-2} as a function of the characteristic size w . The experimental points and the linear fitting are also reported.

[Fig. 14 about here.]

The size effect law of the material can be expressed as:

$$\sigma_u = (m w + q)^{-\frac{1}{2}} \quad (14)$$

where m and q are the slope and the intercept of the linear fit respectively. It should be noted that the linear fit was obtained with a coefficient of determination of $R^2 = 0.92$.

Knowing the size effect law, equation (11) can be solved for $w = w(\Delta a)$. All the other parameters are known: $\alpha_0 = 0.5$, $E = 90648\text{MPa}$, and $\rho = 8.54$. Substituting $w = w(\Delta a)$ in equation (10) enables the calculation of the R-

1
2
3
4 278 curve of the laminate, \mathcal{R} . Finally \mathcal{R}_0 is obtained by multiplying by two the
5
6 279 value of \mathcal{R} for every (Δa) .

7
8
9
10 280 Figure 15 shows the \mathcal{R}_0 obtained using this methodology as the envelope of the
11
12 281 crack driving force curves. The value of the steady-state value of the fracture
13
14 282 toughness can be obtained as:

$$15 \quad \mathcal{R}_{ss} = \lim_{w \rightarrow \infty} \mathcal{R}_0 = \frac{\sqrt{2(1+\rho)} \phi_0}{E} \frac{\phi_0}{m} = 61 \text{kJ/m}^2 \quad (15)$$

16
17
18
19
20
21
22
23 283 where $\phi_0 = \phi|_{\alpha=\alpha_0}$. It is interesting to note that the value of the fracture tough-
24
25 284 ness previously measured using the compact compression test specimens [7],
26
27 285 47.5kJ/m^2 , corresponds to just one point in the rising R-curve.

28
29
30
31 The length of fracture process zone reads [17]:

$$32 \quad l_{fpz} = \frac{f_0}{2 f'_0} w_0 \quad (16)$$

33
34
35
36
37
38
39 286 where $f_0 = \sqrt{\phi}|_{\alpha=\alpha_0}$, $f'_0 = \partial \sqrt{\phi} / \partial \alpha|_{\alpha=\alpha_0}$ and w_0 is the constant of Bažant size-
40
41 287 effect law with the dimension of a length. For the linear regression used [17]
42
43
44 288 $w_0 = q/m$; therefore equation (16) reads:

$$45 \quad l_{fpz} = \frac{f_0}{2 f'_0} \frac{q}{m} = 1.43 \text{mm} \quad (17)$$

46
47
48
49
50
51
52
53 289 [Fig. 15 about here.]

54
55
56 290 To simplify the use of the R-curve it is necessary to express it in an analytical
57
58 291 form that fits the points obtained by solving equations (10) and (11). The
59
60 292 following expression can be used [17]:

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

$$\begin{cases} \mathcal{R}_0 = \mathcal{R}_{ss} [1 - (1 - \kappa \Delta a)^n] & \text{if } \Delta a < l_{FPZ} \\ \mathcal{R}_0 = \mathcal{R}_{ss} & \text{if } \Delta a \geq l_{FPZ} \end{cases} \quad (18)$$

293 By optimal fitting the following values are obtained: $\kappa = 0.5126$ and $n = 4.289$.

294 Figure 16 shows the analytical fitting of the R-curve.

295 [Fig. 16 about here.]

296 5 Conclusion

297 This paper demonstrates that it is possible to obtain the compressive R-curve
298 of the longitudinal plies of carbon-epoxy laminates by quantifying the size
299 effect law observed in compressive tests of scaled specimens with two edge
300 cracks. The limiting factors of the previously proposed compact compression
301 test method, such as the difficulty in predicting the exact location of the
302 extremity of the kink band, do not play any role in the model developed in
303 this paper.

304 Using both electrical strain gauges and the digital image correlation system it
305 is demonstrated that the test method and specimens' geometry proposed do
306 not result in premature buckling failure. In addition, the strain field measured
307 by digital image correlation system indicates that the specimens are properly
308 aligned. Therefore, the peak loads measured in the tests can be confidently
309 used for the definition of the size effect law.

310 While the previous tests methods are only able to provide one single value for
311 the compressive fracture toughness, the proposed method enables the identi-
312 fication of the full R-curve. The steady-state value of the fracture toughness

1
2
3
4 313 of the IM7-8552 carbon-epoxy composite material is 61kJ/m^2 , corresponding
5
6 314 to a length of the fracture process zone of 1.43mm.
7
8

9
10 315 The work presented in this paper can be used as the basis for material screen-
11
12 316 ing and selection, to identify cohesive laws used in the recent analysis models,
13
14 317 and to estimate the fracture toughness of multidirectional laminates using the
15
16 318 fracture toughness of the 0° plies.
17
18
19
20

21 319 **Acknowledgements**

22
23
24

25
26 320 This work was funded by AIRBUS under the project 2genComp – Second gen-
27
28 321 eration composite structural mechanics. The authors gratefully acknowledge
29
30 322 the support provided by AIRBUS.
31
32

33 323 The first and second authors would like to acknowledge the support of the
34
35 324 Portuguese Foundation for Science and Technology under the grant FCT-
36
37 325 DFRH-SFRH-BPD-78104-2011 and *Ciência 2008* program, respectively.
38
39
40
41

42 326 **References**

43
44
45

46
47 327 [1] Vogler M, Rolfes R, Camanho PP. Modeling the inelastic deformation and
48
49 328 fracture of polymer composites part I: Plasticity model. *Mechanics of Materials*,
50
51 329 59(0):50 – 64, 2013.
52

53
54 330 [2] Camanho PP, Bessa MA, Catalanotti G, Vogler G, Rolfes R. Modeling the
55
56 331 inelastic deformation and fracture of polymer composites part II: Smearred
57
58 332 crack model. *Mechanics of Materials*, 59(0):36 – 49, 2013.
59

60
61 333 [3] Lapczyk I, Hurtado JA. Progressive damage modeling in fiber-reinforced
62
63
64
65

- 1
2
3
4 334 materials. *Composites Part A: Applied Science and Manufacturing*, 38(11):2333
5
6 335 – 2341, 2007.
7
8
9 336 [4] Donadon MV, Iannucci L, Falzon BG, Hodgkinson JM, de Almeida SFM. A
10 progressive failure model for composite laminates subjected to low velocity
11 337 impact damage. *Computers & Structures*, 86(11–12):1232 – 1252, 2008.
12 338
13
14
15
16 339 [5] Raimondo L, Iannucci L, Robinson P, Curtis PT. A progressive failure model
17 for mesh-size-independent FE analysis of composite laminates subject to low-
18 340 velocity impact damage. *Composites Science and Technology*, 72(5):624 – 632,
19 341 2012.
20 342
21
22
23
24 343 [6] Pinho ST, Robinson P, Iannucci L. Fracture toughness of the tensile and
25 compressive fibre failure modes in laminated composites. *Composites Science*
26 344 and *Technology*, 66:2069–2079, 2006.
27 345
28
29
30
31 346 [7] Catalanotti G, Camanho PP, Xavier J, Dávila CG, Marques AT. Measurement
32 of resistance curves in the longitudinal failure of composites using digital image
33 347 correlation. *Composites Science and Technology*, 70(13):1986 – 1993, 2010.
34 348
35
36
37
38 349 [8] Laffan MJ, Pinho ST, Robinson P, McMillan AJ. Translaminar fracture
39 toughness testing of composites: A review. *Polymer Testing*, 31(3):481 – 489,
40 350 2012.
41 351
42
43
44
45 352 [9] Blanco N, Pinho ST, Robinson P. Design and analysis of a compact tension
46 (CT) specimen for intralaminar fracture toughness characterisation of woven
47 353 composite laminates. In *ECCM 13, 13th European Conference on Composite*
48 354 *Materials*, Stockholm, June 2 - 5, 2008.
49 355
50
51
52
53 356 [10] Bažant ZP, Kim J-JH, Daniel IM, Becq-Giraudon E, ZI G. Size effect on
54 compression strength of fiber composites failing by kink band propagation. *Int.*
55 357 *J. Fracture*, 95:103–141, 1999.
56 358
57
58
59
60 359 [11] Moran PM, Liu XH, Shih CF. Kink band formation and band broadening in
61
62
63
64
65

- 1
2
3
4 360 fiber composites under compressive loading. *Acta Metallurgica et Materialia*,
5
6 361 43(8):2943 – 2958, 1995.
7
8
9 362 [12] Fleck NA, Sutcliffe MPF, Sivanshanker S, Xin XJ. Compressive failure
10 363 of composites due to microbuckle growth. *European Journal of Mechanics*
11 364 *A/Solids*, 16:65–82, 1997.
12
13
14
15 365 [13] Suo Z, Bao G, Fan B, Wang TC. Orthotropy rescaling and implications for
16 366 fracture in composites. *Int. J. Solids and Structures*, 28:235–248, 1990.
17
18
19
20 367 [14] Lutz M. *Learning Python*. O’Reilly Media, Inc., 3rd edition, 2008.
21
22
23 368 [15] Dassault Systèmes. *Abaqus Version 6.8–3 Documentation*, 2008.
24
25
26 369 [16] Krueger R. The virtual crack closure technique: History, approach and
27 370 applications. Technical Report NASA/CR-2002-211628 ICASE Report No.
28 371 2002-10, ICASE, Hampton, Virginia, April 2002.
29
30
31
32 372 [17] Bažant ZP, Planas J. *Fracture and Size Effect in Concrete and Other*
33 373 *Quasibrittle Materials*. CRC Press LLC, 1997.
34
35
36
37 374 [18] Camanho PP, Maimí P, Dávila CG. Prediction of size effects in notched
38 375 laminates using continuum damage mechanics. *Composites Science and*
39 376 *Technology*, 67(13):2715–2727, 2007.
40
41
42
43
44 377 [19] Jackson WC, Ratcliffe JG. Measurement of fracture energy for kink-band
45 378 growth in sandwich specimens. In *Proceedings of the 2nd international*
46 379 *conference on composites testing and model identification, Comptest 2004*,
47 380 *University of Bristol, Bristol, U.K., 2004*.
48
49
50
51
52 381 [20] Xavier J, de Jesus A, Morais J, Pinto J. Stereovision measurements on
53 382 evaluating the modulus of elasticity of wood by compression tests parallel to
54 383 the grain. *Construction and Building Materials*, 26:207–215, 2012.
55
56
57
58
59 384 [21] GOM (www.gom.com). *ARAMIS, User Manual - Software - v6.0.2-6*, 2009.
60
61
62
63
64
65

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

385 **List of Figures**

386	1	Double edge notched compression specimen.	22
387	2	Finite element model of the specimen.	23
388	3	Relation between ϕ and α for different values of ρ .	24
389	4	Numerical points and ϕ for different values of ρ .	25
390	5	Crack driving force and resistance curves.	26
391	6	Specimens' geometry (dimensions in mm).	27
392	7	Specimens' configuration.	28
393	8	Photomechanical set-up.	29
394	9	Speckle pattern typically used in the digital image correlation measurements (region of interest of 1543×1025 pixel ² (22.0×14.6 mm ²)).	30
397	10	Typical load displacement curves for the specimens A, B, C, D and F.	31
399	11	Comparison between the signal of the physical (SG) and virtual (DIC) strain gauge.	32
401	12	Typical contour plot of the shear strain, γ_{xy} .	33
402	13	Tested specimens (not at scale).	34
403	14	σ_u^{-2} vs. w and linear fitting.	35
404	15	Envelope of driving force curves and R-curve.	36
405	16	Analytical fitting of the R-curve.	37

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

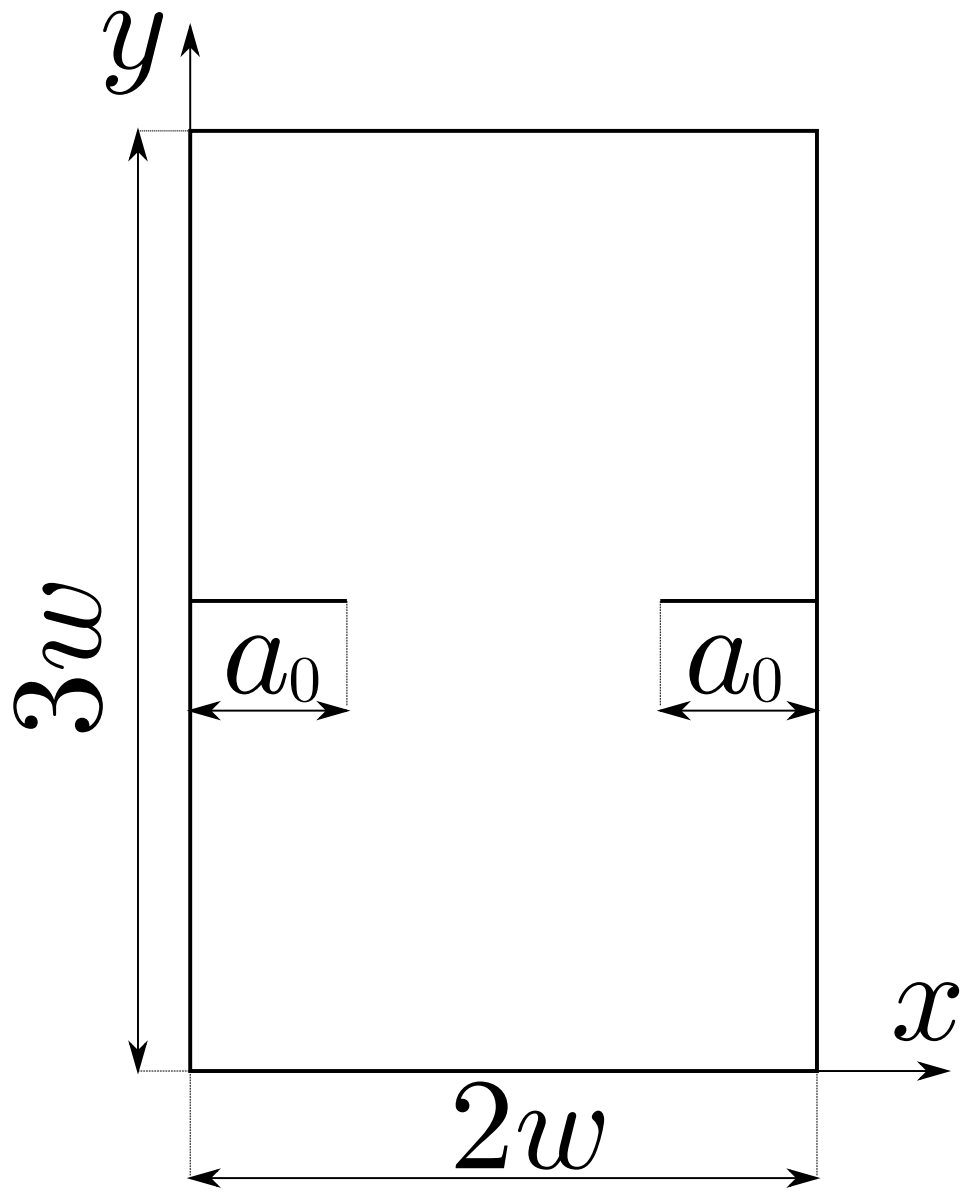


Fig. 1. Double edge notched compression specimen.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

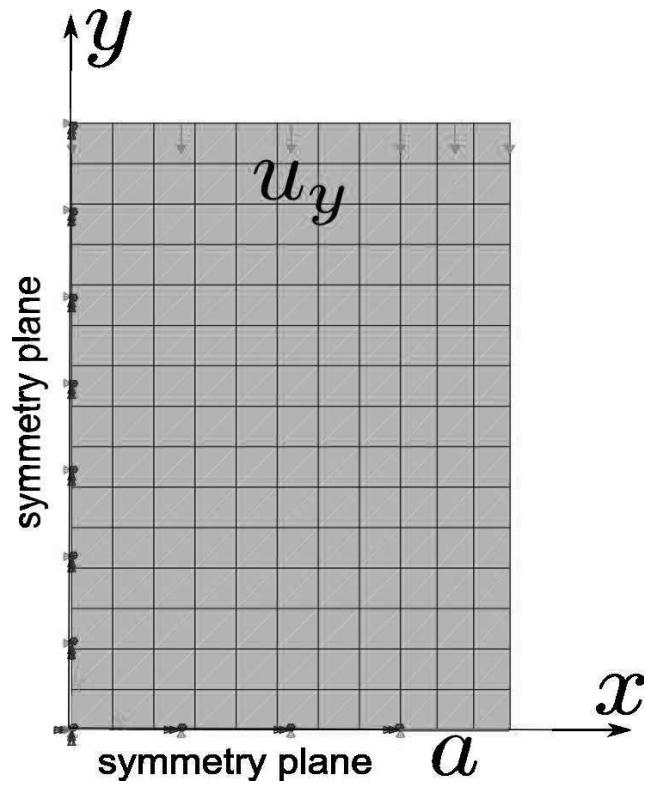


Fig. 2. Finite element model of the specimen.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

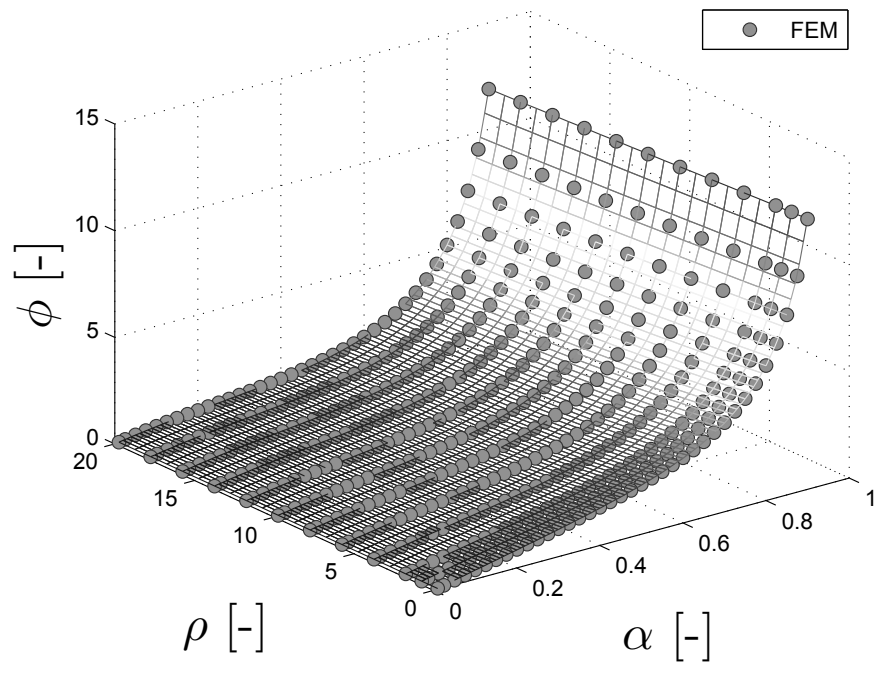


Fig. 3. Relation between ϕ and α for different values of ρ .

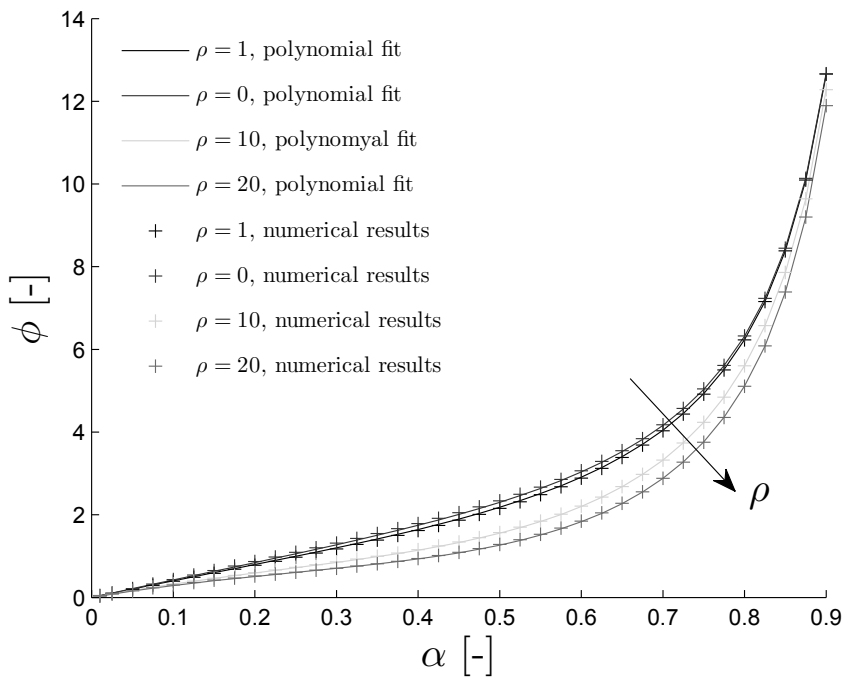


Fig. 4. Numerical points and ϕ for different values of ρ .

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

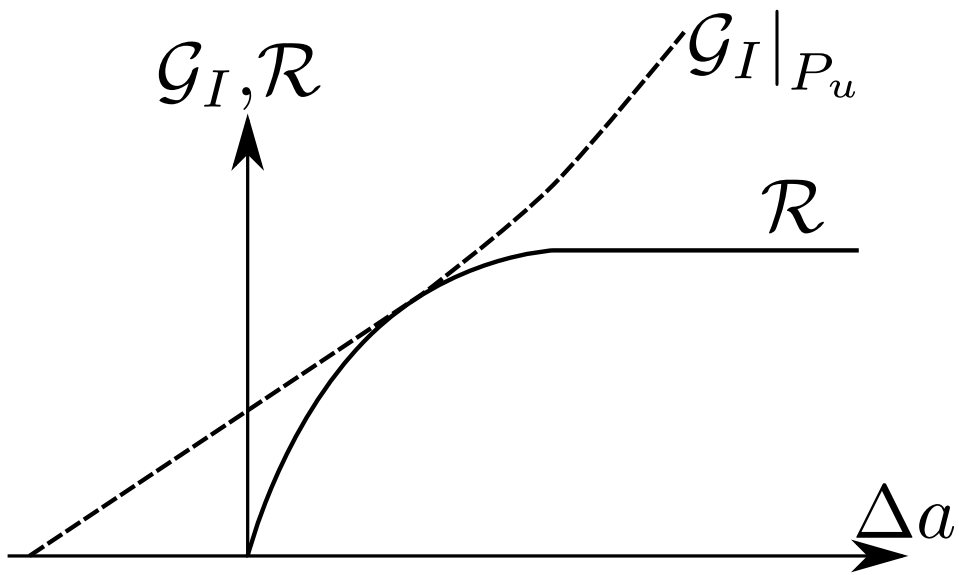


Fig. 5. Crack driving force and resistance curves.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

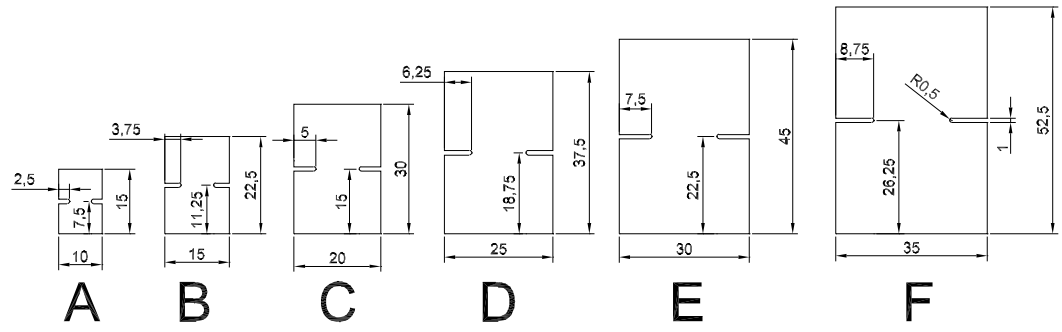


Fig. 6. Specimens' geometry (dimensions in mm).

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

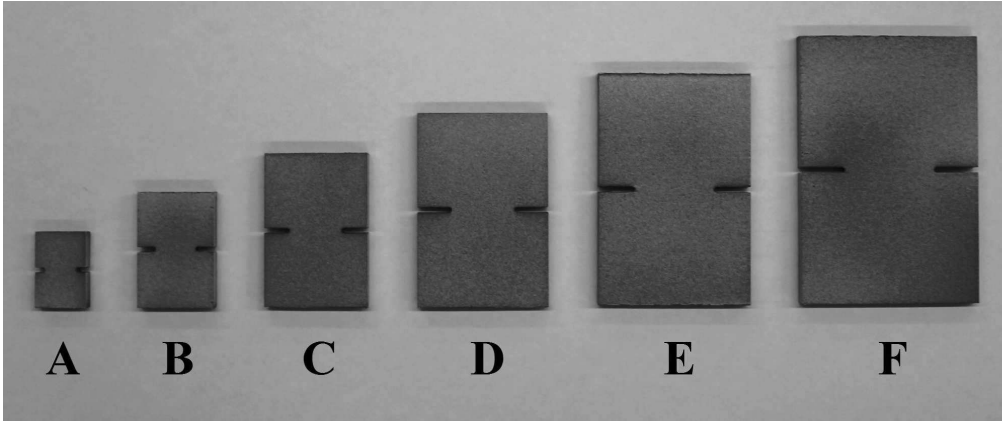


Fig. 7. Specimens' configuration.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

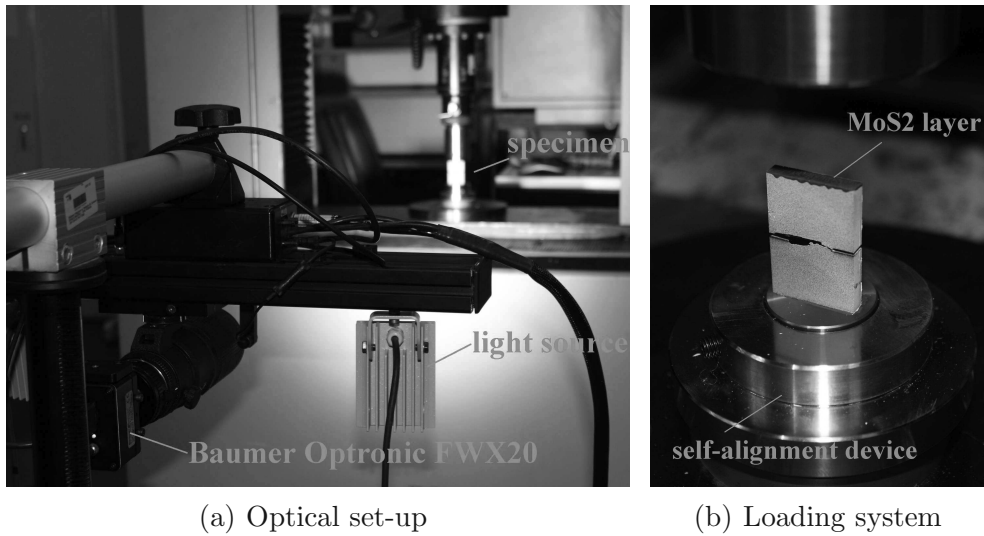


Fig. 8. Photomechanical set-up.

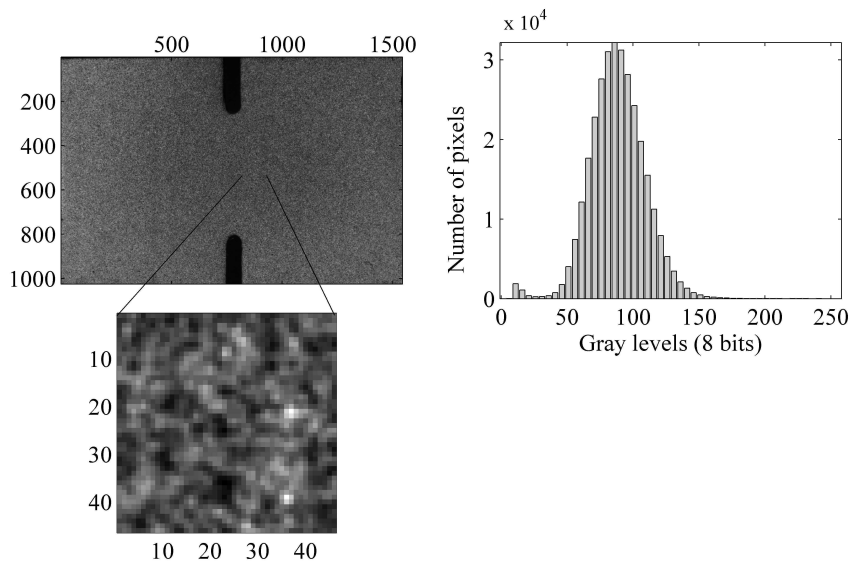


Fig. 9. Speckle pattern typically used in the digital image correlation measurements (region of interest of $1543 \times 1025 \text{ pixel}^2$ ($22.0 \times 14.6 \text{ mm}^2$)).

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

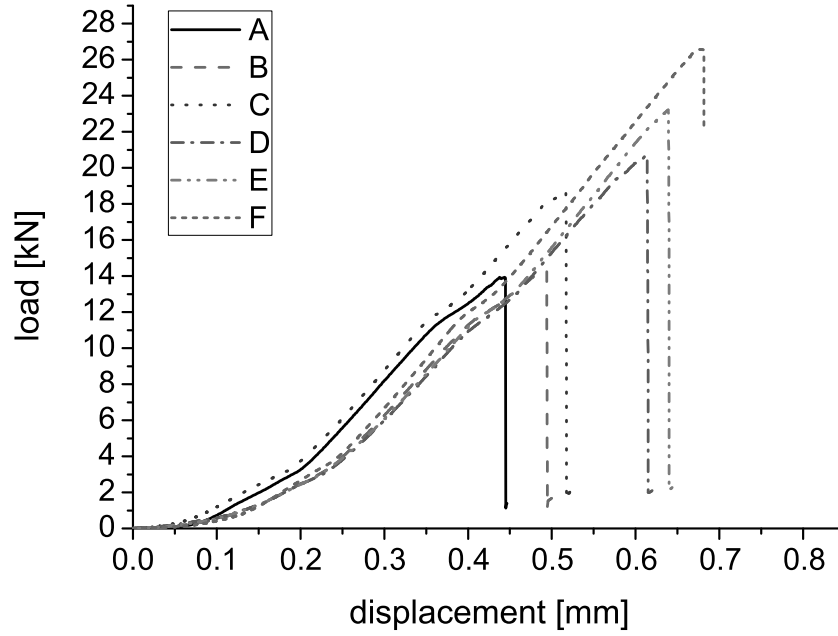


Fig. 10. Typical load displacement curves for the specimens A, B, C, D and F.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

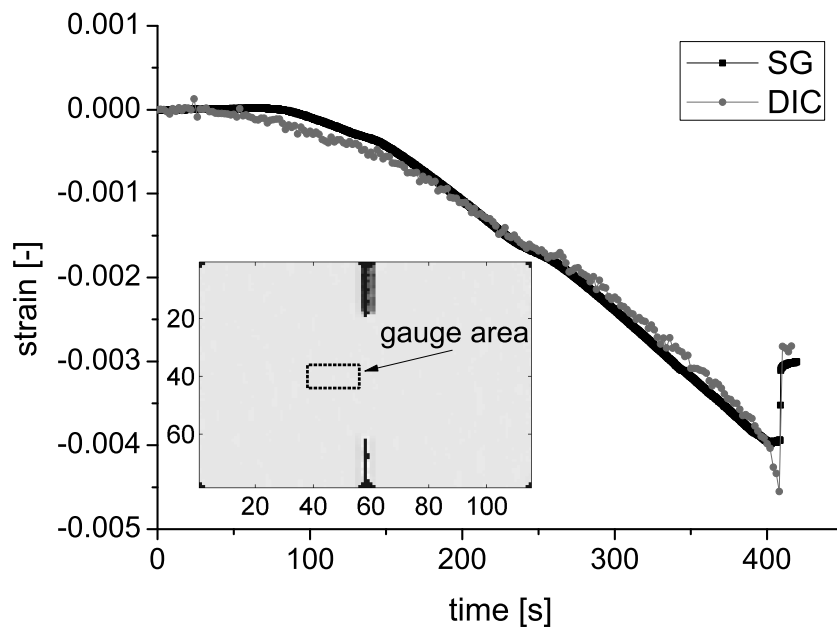


Fig. 11. Comparison between the signal of the physical (SG) and virtual (DIC) strain gauge.

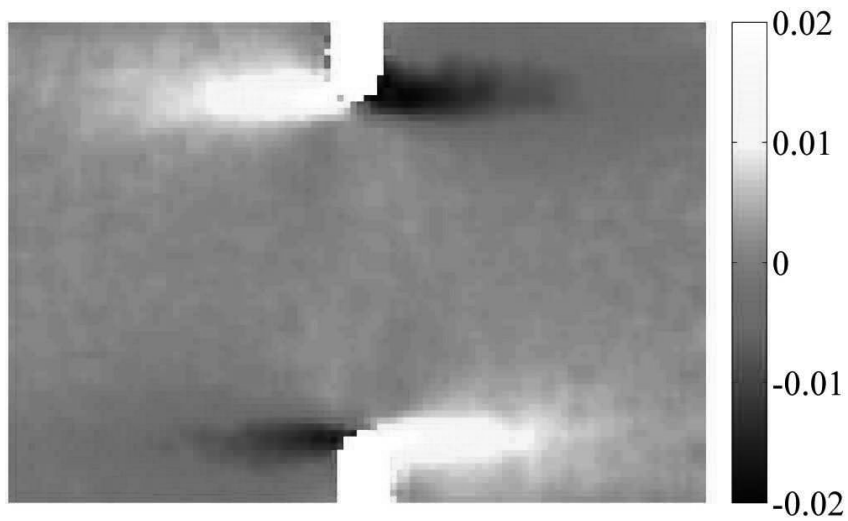


Fig. 12. Typical contour plot of the shear strain, γ_{xy} .

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65



(a) type A (b) type F

Fig. 13. Tested specimens (not at scale).

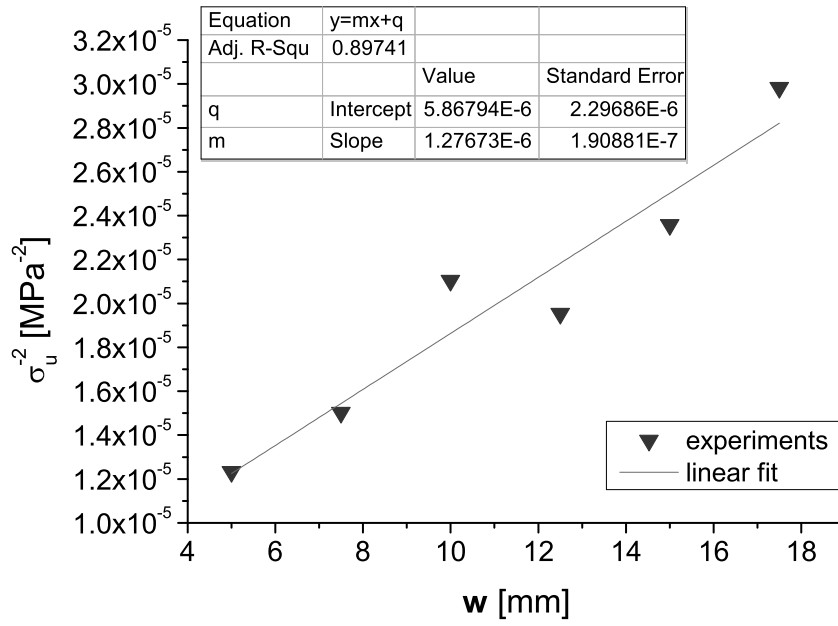


Fig. 14. σ_u^{-2} vs. w and linear fitting.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

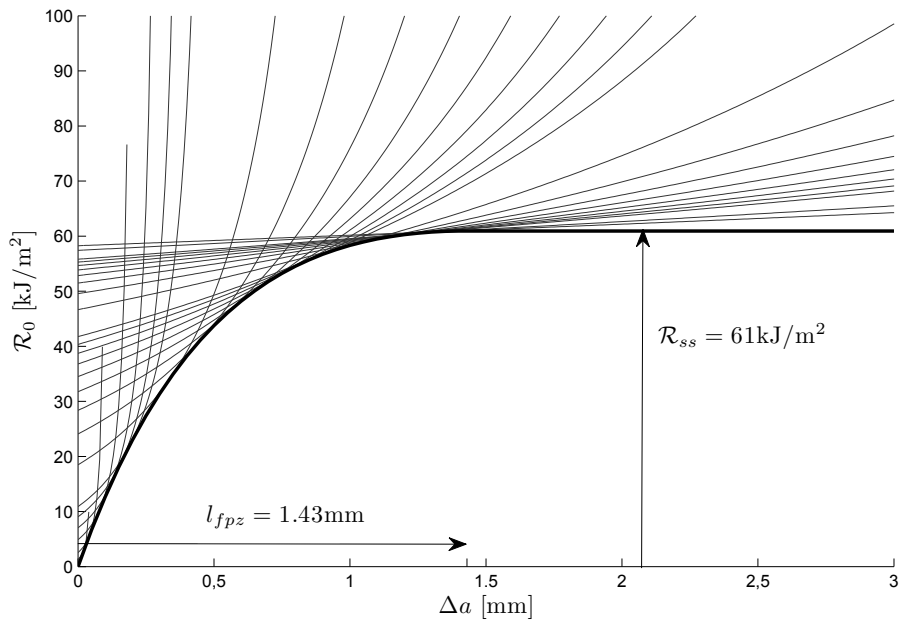


Fig. 15. Envelope of driving force curves and R-curve.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

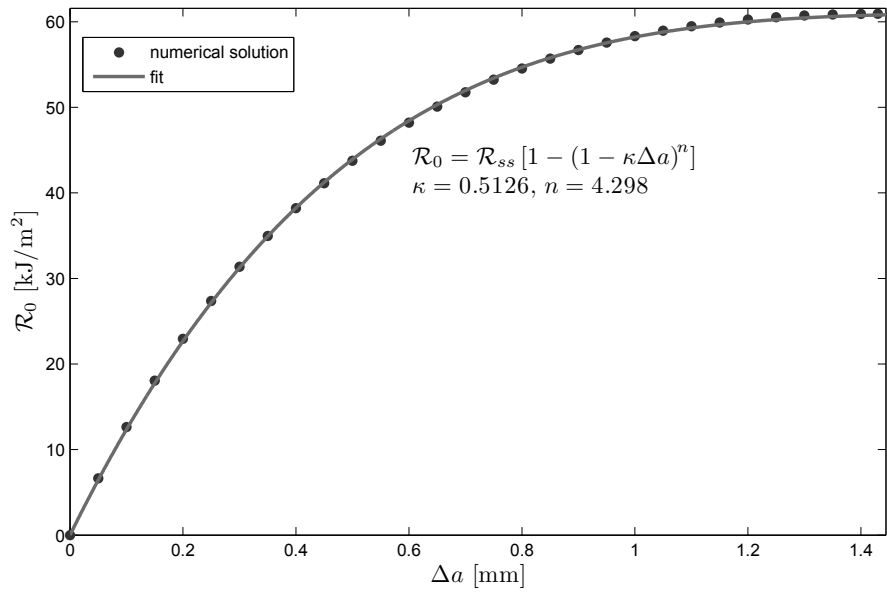


Fig. 16. Analytical fitting of the R-curve.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

406 **List of Tables**

407	1	Elastic properties of IM7-8552.	39
408	2	Optical system components and measurement parameters (see	
409		Fig. 7 for A, B, C, D, E and F specimen configuration).	40
410	3	Summary of the experimental results.	41

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

E_1 [MPa]	E_2 [MPa]	G_{12} [MPa]	ν_{12}
171420	9080	5290	0.32

Table 1
Elastic properties of IM7-8552.

Table 2
Optical system components and measurement parameters (see Fig. 7 for A, B, C, D, E and F specimen configuration).

Camera-lens optical system						
CCD camera	Baumer Optronic FWX20 (8 bit, 1624×1236 pixels)					
Lens	Nikon AF Micro-Nikkor 200 mm f/4D IF-ED (Working F-number: 11)					
Lighting system	LED RAYLUX 25					
Image recording	A	B	C	D	E	F
Field Of View (mm ²)	15.2×11.5	23.2×17.7	31.2×23.7	38.8×29.5	46.2×35.1	55.3×42.1
Working Distance (mm)	560	750	1000	1250	1420	1690
Conversion factor (mm/pixel)	0.0093	0.0143	0.0192	0.0239	0.0284	0.034
Acquisition frequency (Hz)	1	1	1	0.5	0.5	0.5
Exposure time (ms)	4	4	7	7	12	20
DIC (Aramis) parameters						
Subset size	15×15 pixels ²					
Subset step	13×13 pixels ²					
Strain base length	5×5 subsets					
DIC resolution						
Displacement	2×10 ⁻² pixel					
Strain	0.02-0.04%					

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

specimen label	A	B	C	D	E	F
w [mm]	5	7.5	10	12.5	15	17.5
P_u [N]	11390	15477	17436	22622	24706	25628
STDV(P_u) [N]	2262	1421	3268	1793	1827	1028
σ_u [MPa]	285	258	218	226	206	183

Table 3
Summary of the experimental results.