

## Training mathematical skills in school children: Some preliminary results

### *Treino de habilidades matemáticas em crianças em idade escolar:*

### *Alguns resultados preliminares*

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#### Abstract

The present study is a further development of an earlier research on school failure. The aim is to develop sound, simple, effective evaluation and training procedures for children with difficulties to learn mathematics. A review of research on mathematical knowledge reveals some confusion in using terms and a lack of empirical studies on intervention strategies. Nevertheless, some progress has been done in defining the necessary skills for children to solve mathematical problems. In the current study, two cases of primary school children (ISCED 1) with difficulties to learn mathematics are presented. They were trained to acquire a set of pre-mathematical or mathematical skills. The core of the intervention procedures is Behavior Skills Training (BST), a highly effective technique for teaching individuals with different disabilities a wide variety of skills. Evaluation of the training is carried out by comparing the percentage of attained objectives before (pre-test) and after training (post-test). Future developments of this research are explained.

**Keywords:** Mathematical knowledge in children, difficulties to learn mathematics, Behavioral Skills Training.

#### Introduction

The present study must be understood simultaneously as a further development of our research on school failure and as a first step of a new research project. The project on school failure was developed in primary schools, first in slum-like environments in Lisbon (2005-2008), and then in Alentejo, an underdeveloped rural region in Portugal (2013-2017). Educational success in the first school years (ISCED 1) was defined as a result of the joint action of a set of behavioral skills. Three main developmental areas at school were defined, namely, basic skills, social skills and academic skills. The main concern of teachers is usually a set of academic goals to be attained by children at the end of a school semester or school year. These goals were defined as academic behavioral skills, according to teachers expectancies and to the school handbooks for the subjects of Reading, Writing, Mathematics and Environment. Children with school failure problems were trained to attain the defined objectives with Applied Behavior Analysis techniques. The results of these experiences were published in several papers and books. The most important is probably a practical handbook for teachers, aimed to give intervention tools for persons interested in helping children with school problems (Galindo, 2013, 2015; Galindo, & Coradinho, 2017, and Galindo, Candeias, Pires, Grácio, & Stück, 2018).

The current project, whose preliminary results are here presented, is a further development of the above explained intervention. The aim is to develop sound, simple, effective, scientifically founded evaluation and

training procedures for children with difficulties to learn mathematics. The core of the intervention procedures is the so called Behavior Skills Training (BST), a teaching package using together several methods developed by Applied Behavior Analysis. It has been proved to be a highly effective technique for teaching individuals with different disabilities a wide variety of skills (see Gianoumis, Seiverling, & Sturmey, 2012; Homlitas, Rosales, & Candel, 2014; Johnson et al., 2006; Miltenberger, Gross, Knudson, Bosch, Jostad, & Breitwieser, 2009). Ward-Horner & Sturmey define BST as, “an effective training package that consists of instructions, modeling, rehearsal, and feedback” (Ward-Horner, & Sturmey, 2012, p. 75). BST has been the main part of the training procedures applied to children with school failure problems (Galindo et al, 2018), and remains the core in the present development aimed to children with difficulties to learn mathematics. It seems to be a convenient way to approach the problem, in the context of the current state of the art concerning the acquisition of mathematical knowledge.

In the last 25 years, significant advances have been made to understand the processes involved in the development of mathematical knowledge. It is known that these processes begin in the early childhood (Geary, 1994; Haith & Benson, 1998; Wynn, 1992) and develop extensively in the first 5 years of life (Bisanz, Sherman, Rasmussen, & Ho, 2005; Ginsburg, Klein, & Starkey, 1998). Early mathematical knowledge is supposed to involve numerical skills like the so called “subitizing” (Starkey, & Cooper, 1995). Subitizing is considered a general perceptual mechanism. It has been defined as a rapid, accurate and confident judgment of numbers (Kaufman, Lord, Reese, & Volkman, 1949; Plaisier, & Smeets, 2011, and Riggs et al., 2006). It is then supposed to be an immediate knowing of how many things are seen by a person. This is only possible within a determined range of items, from one to four, while an increased number of items decreases speed, accuracy and confidence of judgement. Some more attention has been devoted to the so called “number sense”, a set of elementary numerical skills for the initial learning of mathematics, and its role in mathematical performance (Dehaene, 2002/1997; Gersten, Jordan, & Flojo, 2005). Other mentioned skills are “counting” (Cordes, & Gelman, 2005), “resolution of arithmetical tasks” (Bisanz et al., 2005), and “spatial thinking” (Clements, Battista, Sarama, & Swaminatha, 1997; Newcombe, & Huttenlocher, 2000). The researchers think these elementary numerical skills are developed during the pre-school age and give a basis for the acquisition of formal mathematical knowledge at primary school. (Geary, 1994; Ginsburg, Klein, & Starkey, 1998).

For instance, “number sense” (NS) is defined as a universal inborn capacity to represent and manipulate nonverbal quantities or as a set of acquired skills, which develops through experience and teaching and is responsible for the initial learning of mathematics (see Berch, 2005). Now, NS is understood differently by different authors. In one of the best papers in the field, Berch (2005, p.334) observes “...no two researchers define number sense in exactly the same way. What makes this situation even more problematic, however, is that cognitive scientists and math educators define the concept of number sense in very different ways.” Unfortunately, the discussion on number sense is not an exception, rather it is a good example of the confusion caused by using non operationalized definitions to describe the mathematical knowledge. For instance, Geary (2011) founded that children with math difficulties have deficits in “understanding and representing numerical magnitude”, “retrieving basic arithmetic facts from long-term memory”, “learning mathematical procedures related to working memory”, as well as “number and memory delays”. It is difficult to use these concepts in order to create intervention procedures aimed to solve the mathematical difficulties of individual children.

The same seems to be truth in the case of intervention strategies for children with math difficulties: “In some respects, the field of mathematics instruction has been plagued with too much theory and theorizing and far too little programmatic, empirical research.” (Gersten et al., 2005, p. 293). Consequently, it seems to be a lack of scientifically founded intervention strategies: “Students with learning disabilities who experience difficulties in mathematics are frequently taught (...) using the same procedures and sequences that are used with students without difficulties ...” Wood, Franck and Wacker (1998, p. 323)

That is the bad new. But there is a good one, namely, in spite of all differences, researchers have shown some ways to solve math problems in school children:

“...practice in “counting on,” practice in listening to coins being dropped in a box and counting, practice in counting backwards, practice in linking adding and subtracting to the manipulation of objects (...) to encourage young students in kindergarten and first grade to learn how to talk about numbers and relationships (...) viewing models of proficient performance and being provided with steps that help them solve a problem (...) learning the vocabulary of mathematics (...) simple arithmetic computations are, at some point in their development, extremely difficult problems to solve, requiring a good deal of effort and orchestration of several knowledge bases (counting knowledge, number–symbol correspondence, order irrelevance) (Gersten et al., 2005, pp. 301-302)

It is hard to say if we are speaking about inborn or learned capacities, but we know perfectly, that it is possible to train successfully a wide variety of skills to help children with language deficiencies and/or reading and writing difficulties. It must just be the same in the case of mathematics.

On the other hand, it is known since more than 25 years ago, that a lack of verbal, social and cognitive precursors causes difficulties to learn writing, reading and mathematics (Hallahan, Kauffman, & Lloyd, 1999; and Wallace, Larsen, & Elksnin, 1992). Since then, several authors in different countries and from different approaches have corroborated these findings (Carroll, Snowling, Hulme, & Stevenson, 2003; Connor, Son, Hindman, & Morrison, 2005; DiLalla, Marcus, & Wright-Phillips, 2004; Guevara, 2008; Guevara, & Macotela, 2006; Leppänen, Niemi, Aunola, & Nurmi, 2004).

Consequently, the main purpose of the current enterprise is to search sound, simple, efficient, scientifically founded techniques to train skills for children with mathematical difficulties. In the present study, a first step in this direction will be shown, namely two case studies on the skills to be trained to children with mathematical difficulties.

## 1. Objectives

The general objective of this project is to develop behavioral diagnostic and training procedures to help children with difficulties to learn mathematics. Success in mathematics was defined in terms of a set of skills proposed by teachers. Failure to learn mathematics was then understood as a lack of one or more of these skills. The skills to be mastered by a single child were defined in terms of behavioral objectives, according to the definitions of school teachers. Last but not least, precurrent skills were defined for each proposed mathematical skill, i.e., pre-mathematical skills which are a previous condition for the learning of a given mathematical skill.

Defining all skills has been a hard enterprise, due to the widely different opinions of researchers and to the absence of precise definitions. Nevertheless, an effort has been made to define a sound set of pre-mathematical and mathematical skills taking into account the recommendations of previous authors, but mainly observing the performance of children with difficulties to learn mathematics.

Specific objectives of this study are:

- 1) To develop behavioral diagnostic procedures based on school defined skills for mathematics at elementary school.
- 2) To develop training programs based on skills to learn mathematics.

Children participating in this study were part of a broader contingent with school failure problems. All of them were subject of a thorough evaluation in order to identify their existing and non-existing skills in terms of the teacher's expectations. On the basis of a precise description of the problems of each child, one (or more) intervention program(s) tailored specifically for each one were developed (see Galindo et al., 2018).

As a first step of the current project, two case studies were carried out, in order to define by direct observation and evaluation a set of pre-mathematical and mathematical skills to be taught in the future. These

children were chosen because they showed difficulties to learn mathematics. Two main areas of intervention were defined:

- 1) Pre-mathematical skills.- Precurrent skills, i.e., pre-academic skills that are important as a basis to learn mathematical skills.
- 2) Mathematical skills.- Skills necessary to solve mathematical problems in the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> school years.

Mathematical skills were defined in terms of the expectations of the teachers and the goals established by the corresponding handbooks for each school year.

## **2. Method**

Two case studies were carried out at an elementary school in Evora (Portugal). The purposes of these studies were 1) to observe the behavior problems of children with difficulties to learn mathematics, 2) to define the necessary skills to solve successfully mathematical tasks in the first school year, and 3) to define precurrent skills necessary to learn how to solve successfully mathematical skills in the first school year.

### **2.1. Participants**

Two<sup>2</sup> 6-9 y.o. children. Referred by teachers because of difficulties to learn mathematics. One child ("Mary") received training during one semester, the second child ("Amalia") received training during four semesters.

Five tutors, psychology students.

### **2.2. Instruments**

The central element and only instrument of this intervention is a set of behaviorally designed training programs, especially tailored to solve the difficulties with mathematics of specific children. The programs are the result of a careful analysis of the performance of the children trying to solve mathematical tasks (behavioral diagnosis). Nevertheless, they share a general structure with all programs applied to help children with school failure problems:

"...(1) a general objective, (2) skills defined in terms of specific objectives (a set of correct responses to be given by the child), (3) a definition of the previous skills necessary to learn the new skill (precurrents), (4) a pre-test (% of attained objectives/correct responses), (5) a training package based on instructions, modeling, rehearsal, and feedback, (6) a post-test (% of attained objectives/correct responses). The specific objectives of the program are defined in terms of responses given by a specific child. Every program has a set of previously established correct responses Evaluation of the efficacy of the program was carried out by comparing the percentage of attained objectives before (pre-test) and after training (post-test). "(Galindo et al., 2018, p. 5).

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<sup>2</sup> Several children have been trained during this period to solve very different problems. These two children were selected as case studies because of their specific difficulties with mathematics.

As it was above explained, children were subject of a careful behavioral diagnosis before intervention in mathematics, with a set of check-lists (see Galindo et al., 2018), in order to obtain a whole picture of their skills and school problems.

### **2.3. Procedure**

Written consent was obtained from the parents and children were told that the training was expected to help them. Training was applied by Psychology students (tutors) for 2 hours a week (2 sessions  $\times$  1 h) during one semester (15 weeks) in Case 1 and during 4 semesters in Case 2 (60 weeks). Before training in mathematics, the children were evaluated to identify other academic problems (behavioral diagnosis). A hierarchy of problems was elaborated and an intervention strategy was designed, according to the identified specific difficulties of each child (see Galindo et al., 2018). The percentage of correct responses in a given program attained before and after intervention are the main data. 80% of correct responses is considered good enough for the children performance at school.

The core of the training procedure is the above mentioned BST (see Galindo et al., 2018). During intervention hours, children were collected from the classroom and brought to another room at school. The duration of intervention depended on the children needs and on the support given by the university and the school to carry out the study.

## **3. Some results and implications**

### **3.1. Case study 1**

Mary, a 6 years old girl, attending the 1<sup>st</sup> school year (ISCED 1), was referred by the teachers because of problems to learn mathematics. A complete diagnosis procedure (see Galindo et al., 2018) showed that the girl had no further problems in basic skills nor social behavior, but evident deficiencies in pre-mathematical skills (precursors). A further test showed, she had deficiencies in a pre-defined set of pre-mathematical skills called “quantitative concepts”

The most difficult task in teaching children with problems to learn mathematics is to define precisely pre-mathematical and mathematical skills. In this case, it was observed that Mary had problems with concepts like “more than”, “less than”, “longer than”, “shorter than”, “bigger than”, “smaller than”, “the biggest”, “the first”, “the last”, “the next” and so on. Recognizing more than five concrete objects was difficult. Consequently, a program called “Quantitative Concepts” was tailored for the girl, in order to train the above mentioned basic skills, as well as other skills like verbal counting to ten, telling the number of objects in a given set (until ten), putting a number of objects in a set (until ten), recognizing written numbers (until ten), adding one object to a given set (from one to nine). The child obtained 60% of correct objectives before training and could attain 90% of defined objectives in 9 training sessions (1 hour  $\times$  session) (see Table 1). It seems to be a relatively short time, but it must be taken into account, that the precise definition of the problem during the diagnosis phase takes several sessions.

The main problems of Mary at school seem to have disappeared, but it was not possible to do a controlled follow-up.

### **3.2. Case study 2**

Amalia, a 7 y.-o. girl, attending the 2<sup>nd</sup> school year, was referred by the teachers because of problems in reading, writing and mathematics. She was in danger of repeating the 2<sup>nd</sup> school year. Diagnosis procedures

showed that the girl had no further problems in basic skills nor social behavior, but evident deficiencies in reading, writing and mathematical skills. A hierarchy of problems and an intervention strategy were elaborated: It was decided to direct attention first of all to reading and writing and only later to mathematics. Consequently, the first intervention semester was devoted exclusively to reading & writing, with positive results. Once these problems were solved, training in mathematics followed in the subsequent three semesters (see Table 2)<sup>3</sup>.

In the second intervention semester, an intervention strategy for mathematical skills was developed, according to the specific problems of Amalia. A more precise diagnosis procedure showed that she was not able to solve some tasks defined by teachers for the 1<sup>st</sup> school year and none of the tasks of the second year. A training program was elaborated, based on all the tasks defined by teachers for the 1<sup>st</sup> and a half of the tasks for the 2<sup>nd</sup> school year (Mathematics intermediate for 1<sup>st</sup> - 2<sup>nd</sup> Year). This training program contained tasks like: Verbal counting to one hundred, telling the number of objects in a given set (until twenty), putting a number of objects in a set (until twenty), recognizing written numbers (until one hundred), adding one object to a given set (from one to twenty, then from twenty to twenty and so on), subtracting objects from a give set (from twenty to zero, from nineteen to zero and so on), verbal addition, verbal subtraction, written addition, written subtraction. Table 2 shows that the girl attained 75% of correct objectives before training and could attain 100% of objectives in 5 training sessions.

In the beginning of the next semester, still in the 3<sup>rd</sup> year, Amalia's mathematical skills were evaluated again. Results showed, she was able to solve only 50% of the tasks defined by the teacher for the second year. A training program (Mathematics for 2<sup>nd</sup> year) was tailored devoted mainly to multiplication and division tasks, containing skills like: a) Multiplication until 100 with beans (2x1, 2x2, 2x3 and so on), the same multiplication tasks using images, the same multiplication tasks using only words, the same multiplication tasks writing the operations; b) division until 100 (same procedure); c) tasks solution until 100 (same procedure). A second part of the program was devoted to the "logical reasoning", including tasks like "double", "triple", "quadruple", "a quarter", "a half" the same procedure going from sequentially from the manipulation of beans to solving written tasks was applied. Table 2 shows that the child obtained a 50% of correct responses in the pre-test and 100% in the post-test, i.e., she was able to solve all tasks of the program after 11 hours of training.

In the 4<sup>th</sup> and last training semester, actually the first semester of the 4<sup>th</sup> school year, the girl was able to solve before training 63% of the tasks defined by the teachers. They were multiplication and division tasks similar to those of the 2<sup>nd</sup> school year, but more complicated. Several sessions were devoted to learn the "multiplication table", a very important task in the Portuguese school. The same procedures were applied and Amalia was able to solve a 82% of the defined tasks in 6 training sessions (s. Table 2), a result considered good enough.

It means that, in the middle of the 4<sup>th</sup> school year Amalia was able to master all tasks of the previous years. It is not the best situation for a child, but she was in a much better position than in the beginning of the training. Her performance at school was better, the teacher was satisfied and, last but not least, her so called "self-esteem" was in the best level.

The next phase of this project has as general objective the development of sound, effective, simple, scientific based evaluation and training procedures for children with difficulties to learn mathematics, specific objectives are 1) to evaluate thoroughly children with difficulties to learn mathematics in the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> school years, in order to identify existing and non existing pre-mathematical and mathematical skills, and 2) to create and apply systematically training programs to teach pre-mathematical and mathematical skills to each child, according to their specific needs. A multiple base line design should be applied with a successive introduction of the IV (training program) to different children, much in the way described in Galindo et al. (2018). Participants will be 15 6-12-years-old children referred by the teachers, divided in three intervention

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<sup>3</sup> This part of the training is not explained, because it is not the main concern of this paper.

groups. The training phase will be based in BST. Results will be evaluated in terms of the percentage of objectives attained by the child before and after training.

The final result should be a practical handbook for teachers similar to Galindo (2015).

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## Tables

Table 1. Case study 1. Mary (F). The table shows training programs applied to Mary during one semester, the results obtained before (pre-test) and after training (post-test), as well as the number of sessions. 1 session = 1 hour.

	Age	School	Training	Pre-test	Post-test	Sessions
	year	year	Program	(% correct responses)	(% correct responses)	
1 <sup>st</sup> SEMESTER	6-7	1 <sup>st</sup>	Quantitative concepts	60	90	9

Table 2. Case study 2. Amalia (F). The table shows training programs applied to Amalia during four semesters, the results obtained before (pre-test) and after training (post-test), as well as the number of sessions. 1 session = 1 hour.

	Age	School	Training	Pre-test	Post-test	Sessions
	year	year	Program	(% correct responses)	(% correct responses)	
1 <sup>st</sup> SEMESTER	7-8	2 <sup>nd</sup>	Reading&Writing	80	100	5

2 <sup>nd</sup>	8-9	3 <sup>rd</sup>	Mathematics	75	100	10
SEMESTER			Intermediate 1 <sup>st</sup> & 2 <sup>nd</sup> year			
3 <sup>rd</sup>	8-9	3 <sup>rd</sup>	Mathematics	2 <sup>nd</sup> 50	100	11
SEMESTER			year			
4 <sup>th</sup>	9-10	4 <sup>th</sup>	Mathematics	3 <sup>rd</sup> 63	82	6
SEMESTER			year			