STRATEGIC INTERACTIONS OF URBAN LAND DEVELOPERS IN THE HOUSING MARKET

Pedro Miguel Moreira Lopes Garcês

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ORIENTADOR : *Cesaltina Maria Pacheco Pires*

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Abstract

This thesis studies the supply side of the housing market taking into account the strategic interactions that occur between urban land developers. The thesis starts by reviewing the literature on new housing supply, concluding that there are very few studies where strategic interactions are taken into account. Next, we develop a model with two urban land developers, who first decide the quality of housing and then compete in prices, considering that the marginal production costs depend on the housing quality. First, we analyze the price competition game and characterize the Nash equilibrium of the price game. Finally, we examine the first stage of the game and determine numerically the subgame perfect Nash equilibrium (SPNE) of the quality-price game.

In the price competition game, our results show that the equilibrium price of an urban land developer is an increasing function of its own quality, while it is a non-monotonic function of the rival's quality. The behavior of the equilibrium profits reveals that, in general, urban land developers gain by differentiating their quality. However, the urban land developer located at the Central Business District (CBD), may prefer to have the same quality than the rival when transportation costs are high by exploiting its locational advantage.

The analysis of the first stage of the game also reveals that, in general, the firms best response is to differentiate their quality and that, in most cases, there are two subgame perfect Nash equilibria that involve quality differentiation. However, the results depend on transportation costs and the quality valuation parameter. For small quality valuations, in equilibrium, the market is not fully covered and, if the unit transportation costs are high, only the urban land developers located at the CBD operates. For higher quality valuations, all the consumers are served. Furthermore, the equilibrium qualities and profits are increasing with quality valuation parameter.

Keywords: land urban developers, strategic interaction, vertical differentiation

Interação estratégica dos produtores no mercado de habitação

Resumo

Esta tese estuda a oferta no mercado da habitação, tendo em conta as interações estratégicas que ocorrem entre os produtores de habitação. A tese revê a literatura sobre a oferta de habitação, concluindo que existem poucos estudos que tenham tido em conta as interações estratégicas. De seguida, desenvolvemos um modelo com dois produtores de habitação, que primeiro decidem a qualidade da habitação e depois competem em preços, considerando que os custos marginais de produção dependem da qualidade. Primeiro analisamos o jogo em preços e caracterizamos o equilíbrio de Nash. Posteriormente, examinamos o primeiro estágio do jogo e determinamos numericamente o equilíbrio perfeito em todos os subjogos (SPNE) do jogo.

No jogo de competição em preços, os resultados mostram que, o preço de equilíbrio, é uma função crescente da qualidade da habitação, sendo uma função não monótona da qualidade do rival. O lucro de equilíbrio revela que, geralmente, os produtores de habitação têm ganhos em diferenciar a qualidade. No entanto, o produtor localizado no Centro (CBD), pode preferir oferecer a mesma qualidade que o rival, caso os custos unitários de transporte sejam elevados, através da sua vantagem de localização. A análise do primeiro estágio do jogo, revela que, geralmente, a melhor resposta de um produtor é a de diferenciar a qualidade. Na maior parte dos casos existem dois SPNE que envolvem essa diferenciação. No entanto, os resultados dependem dos custos unitários de transporte e da valorização da qualidade por parte do consumidor. Para uma reduzida valorização da qualidade, em equilíbrio, o mercado não é totalmente coberto e, se o custo unitário de transporte é elevado, apenas o produtor localizado no CBD opera no mercado. Para uma valorização elevada da qualidade, todos os consumidores são servidos. Além disso, as qualidades e os lucros de equilíbrio são crescentes com a valorização da qualidade.

Palavras-chave: produtores no mercado da habitação, interação estratégica, diferenciação vertical

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Chapter 1

Introduction

The object of this study is the housing supply. We start by a literature review on the supply of new housing. The objective in this chapter is to elaborate a systematic literature review that shows the various theoretical and empirical studies about housing supply and to identify the theoretical bases of these studies, so as to discover how can we make a contribution to a better understanding of this theme.

In the housing market, the demand side has been widely studied. On the contrary, many authors like DiPasquale (1999) state that the supply side is still understudied. Sometimes it is difficult to identify the theoretical underpinnings of certain empirical and theoretical studies. However one can identify three major theoretical foundations: the investment literature; the urban economic theory and more recently the industrial organization literature. In the urban economic theory there is evidence, supported by studies like Arnott and Igarashi (2000), of imperfect competition in the housing market. That enables and justifies the growing literature that applies game theory / industrial organization to the housing market. Our work is in the intersection between industrial organization and urban economic theory. Our review on the articles that use game theory/ industrial organization models of housing supply, shows that the strategic interaction between urban land developers is still understudied. Thus we believe there is a need to develop theoretical models of the urban land developers.

Strategic interaction models of housing supply may enable us to understand how land developers make their decisions regarding the house location, house quality and prices. It may permit us to explore the market structure of the housing market and test if the market is competitive or if the urban land developers have some oligopolistic power. There is a extensive literature in industrial economics about vertical product differentiation models, including models with endogenous quality choice. But most of the literature analyzes general theoretical models, that do not take into account the specificities of the housing market. In particular, one feature that is important in the housing market is the house location. Furthermore, differences in quality are likely to affect both the marginal cost of construction as well as the fixed costs.

Our specific goal is to study the supply side of the housing market taking into account the type of strategic interactions that occur between urban land developers. So in the third chapter we develop a model of the behavior of urban land developers, incorporating the specificities of the housing market, such as the location of the producers and considering variable and fixed costs of quality improvements. We discuss a model where there are two urban land developers, who take quality and price decisions independently and simultaneously. In this model, producers first decide the housing quality and then compete in prices. We assume that one of the producers stays at the CBD while the other has a more decentralized location. Our model also considers, in the utility function a transport cost per unit of distance.

We start by examining the price competition stage game, considering the quality levels as given. We derive the demand functions and we impose some conditions about the quality levels of the two urban land developers, so that the demand is positive for at least one of the urban land developers, we also define some cut-off valuations that enabled us to simplify the exposition. We obtained analytically the Nash equilibrium of the price game considering all possible cases. Next we perform a numerical analysis, using a GAUSS program, to characterize the Nash equilibrium of the price game. For each vector of quality levels, we compute the equilibrium prices, profit and type of equilibrium. Then, we examine how the equilibrium prices and profits vary with the quality levels.

In the fourth chapter we solve the first stage of the quality-price game, obtaining the equilibrium housing qualities. Since it is impossible to get an analytical solution for the equilibrium qualities, we determine the Subgame Perfect Nash Equilibrium (SPNE) numerically, using a GAUSS program. In this chapter we present the results of this numerical analysis. We start by analyzing the best response functions of the two urban land developers for various parameters values. Next we study the impact of changing the unit transportation cost and the lowest quality valuation parameter. In particular, we analyze the type of equilibrium for different combinations of these two parameter values and we study how the equilibrium values of qualities and profits change with the unit transportation cost and with the lowest quality valuation parameter.

Chapter 2

New housing supply: what do we know and how can we learn more?

2.1 Introduction

This study reviews the literature on housing supply. The focus of our review is the literature on the supply of new housing, so we do not consider the renovation and repair of the existing stock.

While the literature on housing supply has grown in the last years, housing supply still remains understudied relatively to demand. In fact, many authors in their reviews about housing supply conclude that it has been understudied (see, among others, Quigley, 1979; Olsen, 1987; Smith, 1988; and DiPasquale, 1999). The reason certainly is not the lack of interest but perhaps, as argues Rosenthal (1999), the inexistence of adequate data for empirical studies. Another reason may be the difficulty of modelling the housing supply as referred by Quigley (1979). The first difficulty is that housing services are difficult to measure. The second is that in the housing market we observe price times quantity, unlike other markets where we see the price for a standard unit. The third difficulty is that housing supply is the result of the decision making by land developers and by the actual owners of housing. To understand the micro foundations of housing supply, we would need data such that the unit of observation is the individual supplier. However it is very difficult to obtain data on the behavior of land developers. This explains why the great majority of the articles of new housing supply analyze aggregate data. Like DiPasquale (1999) says, there are few articles that use micro data (where the decision maker, the developer, is the unit of analysis).

Most studies in the literature of housing supply involve the estimation of an empirical model, with the objective of identifying the determinants of new housing supply and estimating the price elasticity of supply. As a consequence, a great part of this survey is dedicated to the empirical studies on housing supply and summarizes the findings regarding these two issues.

Although, in some cases, it is difficult to identify the theoretical underpinnings of the empirical studies, one can identify two major theoretical foundations: the investment literature and the urban economic theory. The main difference in these two approaches is the treatment of land. Studies based on the investment theory treat land as an input in the production of new housing and tend to ignore the special characteristics of land as a factor of production while those based on urban economic theory incorporate the land market into the theoretical structure. Moreover, the models based on the investment theory assume that the home-building industry is composed of competitive firms and that they face rising factor cost schedules for labor and for building materials. However, according to the urban economic theory, land is different from other factors of production. Land prices depend on the stock of housing, not on the flow or level of building activity, as a result a rise in house prices initially generates excess returns, but the flow of construction increases only temporarily above the normal level. As the stock of housing grows, land prices rise and eventually absorb the excess returns and construction declines to its normal level.

The investment theory framework is well illustrated in the work of Poterba (1984) and Topel and Rosen (1988). Poterba (1984) uses an asset market form to model the housing market and defines supply as net investment in structures. Topel and Rosen (1988) consider housing production decisions as housing investment decisions. On the other hand, Dipasquale and Wheaton (1994) and Mayer and Somerville (2000) are reference papers based on the urban economic theory. Besides the influence of the investment theory and urban economic theory, there is a growing literature that applies game theory / industrial organization to the housing market. We believe that this new branch of the literature can provide an important theoretical contribution to the housing supply and suggest some clues to future empirical work on this theme. We dedicate section 2.5 to the review on strategic interaction models.

Besides the differences in the theoretical foundations, the studies also differ in the type of data and estimation techniques used, thus in our literature review we provide information on these two aspects. In the literature there are two approaches that have been used to estimate housing supply: the reduced-form estimation and the structural form estimation. In the reduced-form estimation the equilibrium price is a function of supply and demand factors. On the other hand, in the structural approach the aggregate supply is estimated directly with construction as a function of price and cost shifters.

As mentioned before it is not always easy to classify the empirical papers according to their theoretical foundations, thus we did not attempt to do so in this work. However, we decided to organize the survey of the empirical articles in two distinct sections. First, we revise the earlier empirical studies, from Maisel (1953) to Topel and Rosen (1988). These studies are influenced by the investment theory. Next we revise the more recent studies, starting with Dipasquale and Wheaton (1992).

The remaining of the chapter is organized as follows. In section 2 we start by revising the earlier empirical studies whereas in section 3 we present the more recent empirical studies. In section 4 we analyze the determinants of housing supply. In section 5 we summarize the game theoretical models that have been used to model the housing supply. Finally, the last section, summarizes the main conclusions of the chapter and presents some ideas for future research.

2.2 Earlier empirical studies

Table 2.1 summarizes the earlier empirical studies, indicating the country, sample period, estimation method, whether the regression is done in levels or differences and, finally the estimates of the price elasticity of supply (PES).

Although Maisel (1953) provides a description of builders of single-family housing in USA, namely in the San Francisco area, and the factors that influence their construction decisions, in the literature on housing supply the study by Muth (1960) is considered the first empirical study. Muth (1960) assumes a neoclassical efficient markets view of the housing market, where supply responsiveness is infinitely elastic in the long run. He develops a stock adjustment model and tests the relation between price and quantity of new housing construction. He was unable to reject the null hypotheses of a perfectly elastic supply. However, there are several problems with the Muth (1960) study. One of the problems is the small sample: annual data from 1915 to 1934 and with the war years omitted. Another critique is the fact that his estimation does not adjust for serial correlation or for the possibility of simultaneity bias between the price and quantity of new housing construction. Olsen (1987) also points out significant methodological problems, particularly on the issue of including both input prices and quantity in the reduced form model.

Leeuw and Ekanem (1971) use a reduced form model. In their paper they use information on rent differences among metropolitan areas in the USA to estimate the elasticity of supply of rental housing. Using cross sectional data, they estimated two equations and combined the results of the reduced form estimation with information from other studies on the parameters of the demand equation to draw conclusions about the behavior of the supply of housing services. Leeuw and Ekanem (1971) estimate an elasticity of supply from 0.3 to 0.7, suggesting that the supply of housing is inelastic. In addition, they suggest that one of the sources of the inelasticity are the diseconomies of scale.

Follain (1979) follows the formulation of Muth (1960). He uses annual aggregated data from 1947 to 1975 and employs two measures of the quantity of new housing stock.

Follain (1979) tests the null hypotheses of a perfectly elastic supply. Like Muth (1960) he finds no significant positive relationship between quantity and price, and concludes that the hypothesis of a perfectly elastic long run supply of new construction cannot be rejected.

Whitehead (1974) used quarterly data from 1955 to 1972 for the UK. With this time series he develops and estimates a series of related stock adjustment models. The results for the price elasticity of supply range from 0.5 to 2.

Rydell (1982) has a very complete study of the price elasticity of housing supply. He examines the components of supply response to demand shifts. Rydell (1982) argues that the supply of housing services available to consumers can increase in three ways: (i) existing housing can be upgraded by repair; (ii) the housing inventory can be expanded either by using existing residential land more intensely or by increasing the amount of residential land; (iii) the proportion of existing housing that is occupied can be increased. So the overall supply elasticity is a composite of these three components. His study supports the conclusion that the repair elasticity is very low, that the inventory elasticity is very large, and that the occupancy elasticity is greater than zero. Rydell (1982) used cross sectional data from 59 metropolitan areas in the USA, in the years of 1974 and 1976. Using a reduced form estimation he estimates a long run price elasticity of supply of 11.3. He finds that the short-run price elasticity of supply (PES) is lower, 0.24 or 0.83, depending on the market occupancy rate.

Table 2.1: Summary of earlier empirical studies.

Author	Country	Sample Period	Data level of aggregation	Estimation method	Levels/ Differences	PES long-run	PES short-run
Muth (1960)	USA	1915-1934b)	National	OLS	levels	perfectly elastic	_
Leeuw and Ekanem (1971)	USA	1967	39 metropolitan areas	OLS	levels	0.3 to 0.7	_
Whitehead (1974)	UK	1955-1972a)	National	OLS	levels	0.5 to 2	_
Follain (1979)	USA	1947-1975b)	National	2SLS	levels	perfectly elastic	_
Rydell (1982)	USA	1974 and 1976	59 metropolitan areas	OLS	levels	11.3	0.24 to 0.83 c)
Poterba (1984)	USA	1964-1982a)	National	IV	levels	0.5 to 2.3	_
Topel and Rosen (1988)	USA	1963-1984a)	National	IV	levels	3	1

a) Quarterly data; b) Annual data; c) The short-run PES is 0.24 (with a 96% occupancy rate) and 0.83 (with a 90% occupancy rate).

The attempts to directly model housing supply, in the 80's, comes from the theoretical background of the investment literature. These models assume that the home-building industry is composed of competitive firms. Two reference studies are Poterba (1984) and Topel and Rosen (1988).

Poterba (1984) models the housing market using an asset approach, he defines supply as net investment in structures. Poterba assumes that investment supply depends on real house price, the real price of alternative investment projects, and the construction wage rate. To explain the impact of credit rationing he includes alternative indicators of credit availability. Knowing that houses take time to build, he uses one-quarter ahead forecasts of real house price and the real price of alternative investment projects. Since new houses take time to sell, he adjusts real house price to reflect interest costs incurred during the period from completion to sale. He estimates various linear models using quarterly data from 1964 to 1982. Investment supply is measured as the value of one-family structures put in place or as a rate of new housing investment defined relative to aggregate real output. In the best-fitting models, the elasticity of the rate of new construction with respect to real house prices varies from 0.5 to 2.3. He detects a significant relationship between credit availability and the rate of housing investment, supporting the "supply effect" hypothesis that credit availability affects the flow of new construction. The measures of construction costs, such as the construction wage, produced unexpected signs and no statistical significance.

Topel and Rosen (1988) study new housing supply by considering whether current asset prices are sufficient for housing investment decisions. If they are, then the short-run and long-run investment supplies are identical; if they are not, because of costs associated with moving resources between industries, then short-run supply is less elastic than long-run supply. As a result, builders and developers must anticipate future asset prices in making current construction decisions.

They incorporate these supply dynamics by specifying the industry's cost function in terms of both the level and the rate of change in construction, along with cost variables. They estimate a myopic model and then a model with expectations and internal adjustment costs. In their myopic model, production costs are unaffected by the rate of change in construction activity so current construction decisions are based solely on current asset price and marginal cost. If production costs are affected by the rate of change in construction activity, then internal adjustment costs are present and short-run supply is less elastic than long-run supply. These internal adjustment costs introduce expectations of future asset prices as determinants of new housing supply since current prices by themselves fail to reflect all relevant information. Using quarterly data for 1963 through 1983, they estimate alternative versions of their myopic and internal adjustment cost models. They measure new housing investment as the number of single-family housing starts. The expected real interest rate, the expected inflation rate, lags of these rates, and alternative measures of construction input prices are included as cost shifters. The number of months from start to sale for single-family homes is included as an indicator of market conditions.

In both the myopic and adjustment cost frameworks, nominal interest rates influence construction activity, but construction costs have insignificant effects on housing investment. The myopic model generates new housing supply elasticities ranging from 1.2 to 1.4. They find that the short-run PES is lower, about 1. Their empirical results reject the myopic model in favor of the adjustment cost model. Supply elasticities are calculated to reveal the investment impact of both transitory and permanent housing price shocks. The presence of the time to sale variable considerably reduces the magnitude of the supply responses. For their preferred model, a permanent 1% rise in housing price increases housing investment by about 1.7% in the short run and 2.8% in the long run. However, nearly all of the change in construction activity occurs within one year.

As in Poterba (1984) their measures of construction costs do not have a significative impact on housing starts, the cost of capital to the builders are explained by real interest rates. Topel and Rosen (1988) conclude that real interest rates and expected inflation have a significative impact on starts. They argue that the impact of inflation is difficult to explain and that the magnitude of the coefficient on real interest rates is too big to just reflect the cost of capital. They also argue that the impact of inflation may reflect changes in the velocity at which houses are sold at market prices, to test this explanation they put the median months on the market for new houses, their results show a significant and negative impact of that variable on house starts. But again they argue that the effect is too big to reflect the holding costs related to sales delay.

2.3 Recent empirical studies

The contributions from the investment literature, such as Poterba (1984) and Topel and Rosen (1988), do not take into account the importance of land as an input. However, as we know, from the literature on urban economic theory, land is different from other factors of production. Urban economic theory incorporates the land market on its theory and gives us equilibrium models in which the stock of houses always equals the urban population.

Table 2.2 summarizes the more recent empirical studies on housing supply.

DiPasquale and Wheaton (1994) approach reflects the dynamic nature of housing supply by incorporating a stock adjustment process and a long run equilibrium framework based on urban spatial theory. The latter theory implies that urban spatial growth generates higher land prices in order to attract the land necessary for new housing. By definition, the net change in the housing stock equals new construction less replacement investment. New construction in turn reflects how quickly the housing stock adjusts to its long run equilibrium level. The long run equilibrium housing stock depends on housing price and input prices.

This housing supply framework has two important implications for understanding new housing supply. First, it implies that construction activity reflects the adjustment process as the current stock moves to its long run equilibrium level. Second, it indicates that the housing price level affects new construction only to the extent that the current housing stock differs from its long run equilibrium level for this price level. As such, changes in housing price rather than its level attract the land necessary for long run urban spatial growth.

Table 2.2: Summary of the more recent empirical studies.

Author	Country	Sample Period	Data level of aggregation	Estimation method	Levels/ Differences	PES long-run	PES short-run
Dipasquale & Wheaton (1992)	USA	1960 to 1989b)	National	OLS	levels	6.8	_
Follain, Leavens and Velz (1993)	USA	1977-1990a)	4 metrop. areas	OLS/2SLS	levels	3 to 5	1 to 2
Dipasquale and Wheaton (1994)	USA	1963-1990b)	National	OLS	levels	1 to 1.2	_
Blackley (1999)	USA	1950-1994b)	National	2SLS	levels	1.6 to 3.7d)	_
Pryce (1999)	UK	1988 and 1992	Local aut. level	2SLS	levels	0.58 in 1988g) 1.03 in 1992	_
Somerville (1999)	USA	1979-1991	3 metrop. areas	IV	differences	5.61 to 14.76	_
Mayer and Somerville (2000)	USA	1975-1994a)	National	IV	differences	6	_
Malpezzi and Maclennan (2001)	USA and UK	USA:1889-1994c) UK: 1850-1995c)	National	OLS	differences	UK Prewar: 1-4 UK Postwar: 0-1 USA Prewar: 4-10 USA Postwar: 6-13	_
Kenny (2003)	Ireland	1975-1998a)	National	OLS/IV ARDLf)	levels	1	_
Meen (2005)	UK	1973-2002a)	English regions	OLS	differences	0 to 0.84	_
Green, Malpezzi and Mayo (2005)	USA	1979-1996b)	45 metrop. areas	IV	differences	1.43 to 21.6e)	_
Hwang and Quigley (2006)	USA	1987-1999b)	75 metrop. areas	2SLS	differences	0.01 to 0.09	_

a) Quaterly data; b) Annual data; c) Droping the war period; d) The model with variables expressed in differences yelds PES of 0.8
e) PES statistically greater than zero in 23 of 43 metropolitan areas; f) ARDL(autoregressive distributed-lag)
g) The Boom period was 1988 and the slump period 1992, in this article the PES corresponds to the weighted average of short-run and long-run

DiPasquale and Wheaton (1994) specify new construction (housing starts) as a linear function of new housing price, the short-term real interest rate (the real cost of short term construction financing), the price of agricultural land, construction costs (indices for construction), and lagged housing stock. The change in aggregate employment and the number of months from completion to sale for new homes are also introduced as indicators of housing market conditions. They estimate alternative linear versions of their supply framework using aggregate annual data from 1963 through 1990. They restrict their analysis to single-family housing and measure new construction as the number of single-family housing starts. In all specifications, the coefficient on housing price is significantly positive.

Their estimates of the long-run PES range from 1.0 to 1.2. They conclude that the stock adjusts to its long run equilibrium through new construction very slowly, the rate of adjustment is about 2% per year. On the other hand, real short-term interest rates have a significant negative impact on construction and land costs do not have a significant impact on construction. Just like Topel and Rosen (1988) and Poterba (1984) they did not find a significant relationship between construction costs and the level of construction. Like Topel and Rosen they add months on the market for new homes to the supply equation and they also find that sales time has a large impact on construction, they argue that the magnitude of the coefficients of sales delays and interest costs is too large and that the importance of those variables indicates that price is not enough to explain housing starts. They also argue that the magnitude of the coefficient appears to be too large to simply reflect holding costs associated with sales delays. They include, as a market indicator, the change in employment, a variable that has a positive impact on construction. Adding this variable and the sales time to the model improved the fit of the model. DiPasquale and Wheaton (1994) presents strong evidence of a gradual price adjustment process in the market for single family housing in contrast to previous studies that made assumptions of instantaneous market clearing. Their results confirms the idea that the housing market functioning is very different from other financial asset markets.

In their model, Mayer and Somerville (2000), incorporate the time taken in the development process. In addition, they use more recent time series econometrics methods. One of the differences of this model relatively to DiPasquale and Wheaton (1994) is that Mayer and Somerville use price and cost changes and not their levels. They argue that housing starts is a flow variable so it should be a function of flow variables. Consequently they use lagged price changes and lagged cost changes in their model. The results of this model is a price elasticity of housing starts of about 6.0 and a low price elasticity of the stock of about 0.08. They justify that difference saying that the low price elasticity of the stock is due to the fact that housing starts are a small percentage of the stock. They also find that changes in construction costs are not statistically significant, and that time to sales is statistically significant and the coefficient is large, which means that time to sale has a significant impact on construction.

A great majority of the studies that try to estimate the supply concentrates on the problem of single-family housing starts but there are two articles that study the problem of the supply of multifamily housing. DiPasquale and Wheaton (1992) estimated a construction equation for multifamily rental housing where the level of multifamily construction, measured by the units in structures with more than one unit, depends on how the asset price of rental housing compares with the construction costs. Asset prices are a function of rents, vacancies, and the capitalization rate. The estimated model explains variation in construction with rents, vacancies, the capitalization rate, construction costs, lagged construction, and construction by the federal government of the USA. With this model they estimate a long-run rent elasticity of supply of 6.8, in this model the construction costs obtained in a firm of construction, is statistically significant and has the expected negative sign.

Malpezzi and Mayo (1997) indicates that there are significant differences in supply elasticities between countries. They argue that those differences seem to be correlated with the stringency of the regulatory framework in place for land and housing developers. Goodman (1998) says that supply conditions vary also within a country. Pryce (1999) used data from England at a local district level and constructed a simultaneous equation model of housing construction. The model compares elasticities of supply between two cross-sectional periods, a boom in 1988 and a slump in 1992. The article discussed the rationality and tested the existence of, a backward-bending supply relationship. Pryce (1999) concludes that supply was concave in both periods and that it bends backwards during the boom period. He finds that there is a structural break between the boom and the bust period, the elasticity of supply is higher in the slump period (1.03) and smaller in the boom (0.58), but he concludes that there are considerable variations across districts.

Blackley (1999) used annual data from USA for the period 1950–1994. The basic model expresses residential construction as a linear function of new housing price, the prices of construction materials and labor, the real interest rate and the expected inflation rates. He also considered the effects of land price, lagged housing stock and the price of nonresidential construction. The variables are expressed in levels. The first conclusion is that the new housing supply is relatively price elastic in the long run. Estimates of the long run price elasticity of new housing supply range from 1.6 to 3.7. However in the models with variables expressed in differences, the long-run elasticity is lower, about 0.8. The second conclusion, is that nominal interest rates influence new housing supply directly. And the third conclusion, is that the temporal properties of each data series should be considered when specifying and estimating time-series models of new housing supply, for example, with variables expressed in levels, supply is elastic, but with explanatory variables expressed in differences, supply is inelastic.

Malpezzi and Maclennan (2001) estimate the PES of housing for USA and for UK. Using a long time series both countries, as we can see from Table 2.2, they divided the sample between prewar and postwar. The results for the PES reveal greater values for USA comparing with UK, concluding that the USA market is more elastic. Moreover, the values of PES are higher in the prewar period both in USA and UK.

Green, Malpezzi, and Mayo (2005) estimated supply elasticities for 45 metropolitan areas in the USA following the model of Mayer and Sommerville (2000). They conclude that the estimates of the price elasticity of supply varied significantly according to the metropolitan area. Metropolitan areas that were strongly regulated have low elasticities while metropolitan areas that are less regulated have a wide range of behavior. In particular, metropolitan areas with low regulation and with fast growth tend to have high price elasticities whereas those with slow growth have low price elasticities. They also conclude that population density is an important variable in explaining supply elasticity and that metropolitan areas with high population density have lower elasticities.

Meen (2005) states that, in comparison with the USA, the price elasticities of supply in England are low, and that the England's price elasticities of supply have been falling since 1970. He concludes that the price elasticities of supply is low in all the regions of England (price elasticities are approximately 0 since 1990 in all the English regions). Meen argues that it is difficult to incorporate information about planning controls into the time-series models, although that may partially explain the results as Malpezzi and Mayo (1997) defend. By introducing dummy variables, Meen (2005) concludes that there are additional factors that explain the low price elasticity of supply.

In their paper, Levin and Pryce (2009) works out the UK market. This paper gives a great contribution to the problem of the price elasticity of supply, first by demonstrating that it varies over time due to changes in real interest rates. They conclude that increases in the long run real interest rates cause house price rises and a low elasticity of supply, this in the absence of restrictive regulation and market imperfections. The article considered also how some market imperfections can interact with planning constrains and building regulations to form the response of supply to price changes. They argue that these may conduce to cyclical asymmetry in price elasticity of supply - the tendency for the quantity supplied to respond very slowly to outward shifts of demand, but very rapidly to inward shifts.

As we can see from Tables 2.1 and 2.2 the estimate of the long-run PES of housing varies considerably across studies. However, excluding some earlier studies like Muth (1960) and Follain (1979), we can reject a perfectly elastic supply of housing, and we can conclude that at least in the long run supply is elastic with respect to price. We can conclude also by the recent studies that the PES of housing is higher in the USA comparing with UK, so the values should be different across countries. Another conclusion that can be made observing the results across studies is that the PES of housing varies at regional and local level, there are several studies that conducted this analysis and came to the same conclusion. We can state that the short-run PES is lower than the long-run PES. We can also state that the results vary with econometric models used and with the specification. For example, the use of variables in differences seem to lead to lower values in the long-run PES of housing.

2.4 Determinants of housing supply

In the last two sections we revised the empirical studies but did not mention, in a systematic manner, the regressors of the housing supply models. However it is worthwhile to summarize the various categories of explanatory variables that have been used as well as the results that have been obtained. This will give us an overall picture of the results obtained in the existing empirical evidence.

The set of explanatory variables and the result regarding their impact on housing supply has varied across studies. Classifying the regressors in 8 categories, Table 2.3 shows selected references that include in their study that category of regressors.

As we can see by the number of references in Table 2.3, the most utilized regressors are those related with financing costs and with construction costs.

To have a more clear view of the sign and significancy of the regressors classified in the same categories of Table 2.3, we show in Table 2.4 the number of papers where that regressor has a positive and statistically significant impact, the number of papers where that regressor has a negative and statistically significant impact, and the number of papers where the regressor are not statistically significant.

In the category of financing costs, which includes the interest rate in various forms, almost all the empirical studies conclude that the cost of financing determines negatively the housing starts. This result is consistent with the theory. Levin and Pryce (2009) concludes that changes in the long-run real interest rate cause a low PES.

Theoretically, construction costs should be an important determinant of housing supply, and should have a negative sign, reflecting the negative relation between housing

Table 2.3: Selected references for each category of regressors.

Category of regressors	Selected References
Financing costs	Follain (1979);Topel and Rosen (1988);Dipasquale and Wheaton (1994); Blackley (1999); Mayer and Somerville (2000); Kenny (2003); Meen (2005); Hwang and Quigley (2006)
Construction costs	Follain (1979); Poterba (1984);Dipasquale and Wheaton (1992); Blackley (1999); Somerville (1999); Mayer and Somerville (2000); Kenny (2003); Meen (2005)
Vacancy rate	Leeuw and Ekanem (1971); Dipasquale and Wheaton (1992)
Sales delay	Topel and Rosen (1988); Mayer and Somerville (2000); Dipasquale and Wheaton (1994)
Inflation rate	Topel and Rosen (1988); Blackley (1999)
Stock of housing	Dipasquale and Wheaton (1994); Blackley (1999); Mayer and Somerville (2000)
Price of agricultural land	Dipasquale and Wheaton (1994); Blackley (1999)
Regulation	Pryce (1999); Hwang and Quigley (2006)

starts and construction costs. However, Table 2.4 shows that the results for the category of construction costs (which include material costs, wage costs or an index of both of them) are inconclusive. Although the expected negative impact is obtained if 5 articles, an equal number of papers shows a positive impact and in 2 other studies the construction costs are not statistically significant. As DiPasquale (1999) refers, most of the empirical literature on housing supply has the problem of the measurement of construction costs. Thus one possible explanation for the inconclusive results is the quality of the data used to measure the variable. It is interesting to note that studies that use more disaggregated data, such as Somerville (1999), conclude that the variable has significant and negative impact on housing supply.

The evidence on the impact of the vacancy rate is scarce, since only 3 studies include this variable as a regressor. Two of this studies find a negative impact, which is accor-

Table 2.4: Results of the empirical studies by category of regressors.

Category of regressors	Positive	Negative	Not significant
Financing costs	–	9	1
Construction costs	5	5	2
Vacancy rate	–	2	1
Sales delay	–	3	–
Inflation rate	–	2	–
Stock of housing	1	2	2
Price of agricultural land	1	1	1
Regulation	–	2	1

dance with theory, while one of the studies finds out that the variable is not statistically significant.

The variable sales delay is included only in three studies. However its impact on housing supply is negative and statistically significant in all the papers reviewed, which is theoretically consistent: if the houses take a very long time to sell the consequence is less housing starts. It is also worthwhile to note that the magnitude of the impact of sales delay is quite big in the three studies that include this variable.

The two studies that include inflation rate as a regressor, reveal a significant and negative effect on housing starts, which is also consistent with theory.

The evidence regarding the impact of the stock of housing (normally with a lag) on housing starts is inconclusive: 2 articles reveal a negative impact, 2 studies show a non-significant impact and 1 study finds a positive impact. Similarly, the effect of the price of agricultural land is also not clear as the three studies that include this variable reach completely different results.

In the category of regulation we include the planning controls, which is used in two papers. The reason why this type of regressor is not used more often is probably related with the lack of information, in particular it is difficult to have a time series regarding this variable. In theory, places where the regulatory controls are more restrictive have less housing starts, hence the sign of the coefficient should be negative. Two of the three studies that include regulation show the expected theoretical result whereas in one study the variable is not statistically significant. It is also important to mention that Green, Malpezzi and Mayo (2005) concludes that metropolitan areas that were more regulated have lower PES.

2.5 Strategic interaction models

In the three previous sections we revised the empirical literature on housing supply. However, within the housing supply literature, there are other studies that we would like to highlight. We want to review also the application of game theory/industrial organization to model housing supply. Unfortunately, has we will show, there are very few studies in this area.[1]

One of the most important application of game theory that we found was Baudewyns (2000). This article focus on the strategic interactions of land developers, analyzing the decisions made by two land developers that decide independently two variables: price and quality. He assumes that one firm is at the Central Business District (CBD) and builds houses in this location while the other builds in a more decentralized area. In his article, he considers a first stage in which the duopolists choose the level of housing quality, where the quality is defined as a function of accessibility and housing quality. In the second and last stage, the two firms simultaneously compete in prices to attract potential

[1] Strategic interaction models have also been used in other related areas. For instance, Firoozi, Hollas, Rutherford, and Thomson (2006) present a game theoretic model of property tax assessment and provide evidence of asymmetric information in residential property assessments. Similarly, Anglin and Arnott (1991), analyze the terms of the brokerage contract between a house seller and his agent, applying the literature on the principal-agent problem.

clients. Baudewyns concludes that the decentralized developer can adopt two kinds of strategies depending on the distance and the anticipated level of quality at the CBD. If the centralized land developer offers high quality apartments, then the decentralized developer offers low-quality housing units in the CBD, the idea of the decentralized developer is to differentiate its residential quality to soften price competition in the second stage, in the suburban areas, it offers a higher quality of housing but the residential quality is lower, because of the transportation costs.

Ong, Sing and Choo (2004) apply a game theoretic Nash equilibrium approach to the issue of planning flexibility within the land use zoning. This work is based in the land use planning in Singapore, and in the example of the "white sites" programme in that country. The authors refer that flexibility in land use may be valuable, but it potentially introduces a supply inefficiency through the uncertainty in the development decision-making process. The main proposition is that interaction between developers of proximate sites may result in a suboptimal supply situation. The authors demonstrate that a first-mover advantage exists such that subsequent "white sites" released shortly after the first "white sites" are likely to fetch lower land prices.

Wang and Zhou (2000), study one well documented problem in the real estate markets literature – the excess vacancy or overbuilding in the market. The article models overbuilding as a two-stage infinite-horizon non-cooperative game between land developers. The game is divided into two stages. In the first stage each developer simultaneously and independently decides to build a certain number of real properties to meet the demand level. In the second stage given the available supply and demand of the market, developers select the optimal rental price for their properties. The authors conclude that it is natural to observe oversupply in real estate markets, developers have the incentive to build once they find a development opportunity. As consequence, developers as a whole, will supply more houses into the market than the level of demand. After the oversupply, developers will stop building until the demand absorb the existing supply. Their model explains the long-lasting overbuilding in real estate markets without some traditional explanations such as agency costs, irrational behavior or uncertainty of demand.

Chu and Sing (2007) incorporate strategic interaction in the modeling of optimal timing decision for real estate development projects. In their article they examine the subgame perfect equilibrium strategies for a duopoly real option model, with two firms with asymmetric demand functions. In the presence of preemptive threat, firms may forgo the waiting options, and invest earlier than what the monopolistic real option models would predict. In their symmetric duopoly model firms are identical and products are homogeneous. So there are no relative advantages in the price function of the first mover over the next. Short bursts and recession induced overbuilding are two outcomes in the authors model. The model predicts that those two phenomena occur in earlier phases of market cycles, and not in the state of recession. In a recessed market with high volatility, the two firms will choose the waiting strategies.

Other important articles on housing supply examine the home building industry, its structure and industrial organization. Somerville (1999) states that his article is the first analytical treatment of the industrial organization of housing supply. He says that traditional studies of housing markets assume house building as a perfectly competitive industry. This study uses metropolitan area level data on the average size of homebuilder firms and homebuilder market concentration, to analyze the market structure of the industry. He concludes that there is a systematic variation across metropolitan areas on the housing market, this variation occurs in the average size of builders and in the market share for the largest builders. So he argues that the results are more consistent with treating the industry as monopolistically competitive with a differentiated product. He also concludes that home builders are larger in more active housing markets, and they are also larger where there is a bigger supply of developed land adequate for larger developments. He argues that the type of regulating jurisdiction that establishes land-use regulation has influence on the builder size and market concentration.

Ball (2003) in his paper examines the way that the housebuilding industry is organized and tries to identify some implications for the wider operation of housing markets. He argues that there are several characteristics of the industry that seem to reject the idea of a competitive industry. First, there are different institutional forms within and across countries, housebuilding industrial structures vary considerably. Second, firms adopt strategies and they know, from experience, that they are important in determining profit. He states that strategic behavior can not have effect on market outcomes in a competitive model. The article analysis potential economies of scale, market factors, information asymmetries, regulation and risk. Ball argues that the great variety of ways in which housing is built, is not the reason that explain its industrial organization. Things like market instability, locational specificity, the markets where the houses are sold, information, strategic behavior, regulation in labour markets, land availability and the regulation, are factors that affect the size of firms. The author states also that strategic behavior is important in this industry, particularly through behavior with regard to the land market and residential development strategies.

2.6 Conclusion

Along the years, various empirical studies have been undertaken. Although there are some studies using cross section or panel data sets for metropolitan areas, the great majority of the studies use aggregated time series data. In spite of the differences regarding the type of data and econometric estimation methods, the main results are quite consistent across studies.

Excluding some earlier studies like Muth (1960) and Follain (1979), we can reject a perfectly elastic supply of housing. Most studies find an elastic housing supply but there are some studies that obtain below unit elasticities. The studies that distinguish between short run and long run elasticities reveal that price elasticity of housing supply is lower in the short run. Moreover, the studies that allow comparisons across countries or regions show that there are significant differences in supply elasticities between countries and regions. For instance, the values of the price elasticity of supply are higher in USA than in the UK.

Regarding the other determinants of housing supply, most empirical results are according to the theoretical predictions. For instance, financial costs, inflation and sales delay influence negatively the housing supply. However there are also some results which are unexpected, namely the inconclusive results with respect to the impact of construction costs. One possible explanation for this inconclusive results is the difficulty in measuring accurately the construction costs.

Our review on the articles that use game theory/ industrial organization models of housing supply shows that the strategic interaction between land developers or constructors is still understudied and hence there is a lot of potential in exploring this type of models.

We believe that there is a need to increase our understanding of the behavior of constructors and land developers. This deeper understanding can come from the development of theoretical models predicting their decisions in a context where there exists strategic interactions between land developers and the estimation of empirical models based on micro data. Strategic interaction models of housing supply may allow us to understand how land developers make their decisions regarding the house location and house quality, may allow us to explore the market structure of the housing market and test if the market is competitive or if the land developers have some oligopolistic power. By using data where the unit of analysis is the land developer, we may be able to resolve some contra-intuitive results such as those obtained with respect to the impact of construction costs.

Chapter 3

Duopoly price competition in the housing market

3.1 Introduction

The vast majority of the literature on housing markets assumes that the housing industry is perfectly competitive, with a few exceptions like Arnott (1987), Arnott and Igarashi (2000) and Baudewyns (2000). However the existence of differences in the housing quality, differences in housing accessibility, differences in households tastes, just to mention a few, can be sources of market power and lead to strategic interactions between the urban land developers (ULD).

This chapter and the next one apply game theory and industrial organization tools to model housing supply. In these two chapters we discuss a dynamic duopoly game with two stages. In the first stage, the two ULD simultaneously choose the quality of housing and, in the second stage, the ULD simultaneously choose prices. We assume that one of the urban land developers is located at the central business district (CBD) while the other is located at a more peripheral area.

Our model is naturally related to the vertical differentiation literature. Our model assumes that a quality improvement has fixed costs but it also increases the marginal production costs. Thus a quality improvement has cost implications both for the price-stage game as well as for the quality-stage game. This is a contribution to the literature on vertical differentiation, since none of the existent studies incorporates simultaneously these two types of costs of increasing quality. Moreover, our study is more complete than most in the literature because we study whether in the subgame perfect equilibrium there is partial coverage of the market or full coverage of the market (instead of assuming one or the other).

In this chapter the emphasis is on the second stage price competition game, considering the quality levels as given. We compute analytically the Nash equilibrium of the price game for many possible combinations of the qualities of the two urban land developers. Our analysis covers cases where both urban land developers operate (with full or with partial coverage) as well as cases where only one of the ULD operates (again, with full or partial coverage). Next we characterize the type of equilibrium obtained, using a numerical analysis. Chapter 4 completes the analysis of our model by looking at the choice of the quality levels in the first stage, for various parameters values.

This chapter is organized as follows. In the next section we present a literature review on vertical differentiation. Section 3 describes the model. In section 4, we start by imposing the necessary conditions on the two ULD quality levels, such that, their demand is positive, this permits to restrict our analysis of the Nash equilibrium, for cases where at least one ULD has positive demand. In this section we define also some cut-off valuations that permit us to simplify the exposition. In section 5 we study the Nash equilibrium of the price game, obtaining analytically the equilibrium prices for the second stage, assuming given quality levels, considering all the cases that can occur. In section 6 we perform numerical analysis to characterize the Nash equilibrium of the price game. For each quality levels, we compute the equilibrium prices, profit and type of equilibria. Finally, section 7 summarizes our main conclusions of the chapter.

3.2 Literature review on vertical differentiation

There is a wide literature in industrial economics about vertical product differentiation, including models with endogenous quality choice. However most of the literature that we have reviewed consists of general theoretical models, that do not take into account the specificities of the housing market. In particular, one feature that is important in the housing market is the location. In addition, differences in quality are likely to affect the marginal production costs. These characteristics are taken into account in our model.

The literature on vertical product differentiation models, specifically with endogenous quality choice, can be divided according to the assumption that is made about the nature of the costs of quality improvement. As Motta (1993) explains, some authors like Shaked and Sutton (1982), Bonanno (1986), Aoki and Prusa (1997), Lehman-Grube (1997) or Lambertini and Tampieri (2012) assume that there are fixed costs of quality improvement while variable costs do not change with quality. This assumption is reasonable when producers improve quality by advertising or by research and development. The authors that assume variable costs of quality improvement like Mussa and Rosen (1978) or Lambertini and Tedeschi (2007a) argue that higher quality requires more expensive inputs or a more specialized labour force. Motta (1993) compares the two assumptions about the nature of the costs of quality improvement in the same vertical product differentiation model. He concludes that in both cases differentiation always exists at equilibrium. The author states that firms choose to differentiate products in the first stage in order to smooth the competition on prices in the second stage.

Shaked and Sutton (1982) is one of the earliest studies about how product differentiation relaxes price competition. In their model with three stages, first firms choose to enter or not in the industry, second the firms choose the quality of the product and, in the third stage of the game, firms compete in prices. They conclude that when there are two firms in the market, they choose to differentiate the product, and the two firms have positive profit at equilibrium, as the qualities are closer there is more competition in prices and the profits get smaller for both firms. They also conclude that, if there are more than two firms in the market, all the firms choose the maximum level of quality possible and profits become zero. They state that when a small cost of entry is introduced, the only perfect equilibrium in the game is the one in which two firms enter in the market, in this case, they differentiate the product and have positive profits.

Aoki and Prusa (1997) consider a vertical quality differentiation model, where they analyze the effect of the timing of investment decisions on the levels of quality chosen by producers. The authors compare sequential with simultaneous quality decisions, and they conclude that sequential choice of quality drives both firms to make smaller quality investments than they would make if their decisions were simultaneous. They assume that marginal cost of production is zero, but they state that if the marginal production cost depends on quality, and if with quality improvement the marginal production costs increases a lot then such quality improvements should not be undertaken.

Lambertini (1999) also discusses the timing and the choice of quality in a differentiated oligopoly. The author extends the work of Aoki and Prusa (1997) assuming that quality improvements has fixed costs.

Lambertini and Tedeshi (2007b) studied a market of vertically differentiated goods with sequential entry. Firms enter in the market after having developed innovations that imply different quality levels. They conclude that the time and quality dimensions interact in the formation of the industry. They also conclude that by imposing quality improvements on later entrants implies the persistence of monopoly, and that when a second innovator is allowed to produce an inferior quality good and the patent protection is not too long then emerges a duopoly equilibrium.

Lambertini and Tedeschi (2007a) in a two period duopoly model of vertical product differentiation, proves that there exists a unique subgame perfect equilibrium where the first entrant offers a lower quality good and gains higher profits than the last entrant, he also proves that this sequential entry is socially efficient. In a more recent article, in the same field, Lambertini and Tampieri (2012) assumes partial market coverage and fixed costs of increasing quality.

Aoki (2003) analyses the effect of credible quality investment, comparing Bertrand

and Cournot competition. The author concludes that with Bertrand competition the equilibrium qualities are lower with credible commitment and the competition is softer while with Cournot competition the equilibrium qualities are higher.

Liao (2008) analyses the issue of market coverage in a vertical differentiation model with fixed costs. He finds out that a covered market with an interior solution in the price stage is not a Nash equilibrium.

To the best of our knowledge, Baudewyns (2000) is the only paper that considers vertical differentiation among land developers. The paper analyzes the decisions made by two land developers that decide independently two variables: price and quality. He assumes that one firm is at the CBD and builds houses in this location while the other builds both in the CBD and in a more decentralized location. He considers a first stage in which the duopolists choose the level of housing quality, where the quality is defined as a function of accessibility and housing quality. In the second stage, the two firms simultaneously compete in prices. The author concludes that the decentralized developer can adopt two kinds of strategies depending on the distance and the anticipated level of quality at the CBD. If the centralized land developer offers high quality apartments, then the decentralized developer offers low-quality housing units in the CBD, the idea of the decentralized developer is to differentiate its residential quality to soften price competition in the second stage. In the suburban areas, it offers a higher quality of housing but the residential quality is lower, because of the transportation costs.

3.3 The model

Let us consider a standard model of vertical differentiation. There are two urban land developers indexed by $i = 1, 2$. The first urban land developer (ULD 1) stays at the CBD while the second urban land developer (ULD 2) builds houses at a more peripheral location. In the first stage, each ULD decides the quality of its houses. In the second stage of the game, each ULD decides its housing price.

The consumer net utility if he buys a house[1] from urban developer i, is given by:

$$U = \theta k_i - td_i - p_i$$

where k_i represents the quality of the house sold by developer i and p_i is the corresponding price. The parameter θ is a taste parameter that reflects how much the consumer values quality. This parameter is uniformly distributed across the population between $\underline{\theta}$ and $\overline{\theta} = \underline{\theta} + 1$, $\theta \in [\underline{\theta}, \underline{\theta} + 1]$ which implies that the density function is equal to 1. Finally, d_i is the distance from the urban developer i's house to the central business district (CBD) and t is the transportation cost by unit of distance. Since the ULD 1 is located at the CBD, $d_1 = 0$. For simplicity we assume that $d_2 = 1$.

It should be noted that when a consumer buys a house he is also choosing his own location (where he wants to live). If we assume that jobs and shops are located in the CBD (like in the traditional monocentric city model), a consumer who buys a house in a peripheral location has to move whenever he goes to work or shopping, which explains the inclusion of the transportation cost in the utility function. On the contrary, a consumer who buys a house in the CBD (from ULD 1) does not incur transportation costs.

Considering the location of the two urban land developers, the net utility of the consumer is:

$$\begin{cases} \theta k_1 - p_1 & \text{if he buys from ULD 1} \\ \theta k_2 - t - p_2 & \text{if he buys from ULD 2} \\ 0 & \text{if he does not buy} \end{cases}$$

Among these three options, the consumer chooses the alternative that gives him the highest net utility. Note that the two ULD are not in a symmetric position, unless the transportation costs are nil. For positive transportation costs, if the two ULD offer the same quality and the same price, the consumer prefers ULD 1 house to ULD 2 house. On the other hand, when $t = 0$, our model is similar to the traditional model of vertical product differentiation.

[1] Thus we are assuming that each consumer has a unit demand.

This is a two-stage dynamic game with imperfect information. In the first stage the duopolists simultaneously choose the quality of housing and in the second stage they simultaneously choose prices. In the price game, we assume that the urban land developers have constant marginal production costs that depend on the quality chosen in the first stage. That is the production costs are given by:

$$C(q_i) = \frac{k_i^2}{2} \cdot q_i$$

where q_i is the quantity of houses produced.

In addition, we assume that in the first stage of the game there is an investment cost of quality given by:

$$I(k_i) = \begin{cases} 0 & \text{if } k_i = 0 \\ F & \text{if } k_i > 0 \end{cases}$$

where F is a positive constant.[2]

In order to find the subgame perfect equilibrium, as we have a two stage game, we need to find first the Nash equilibrium of the second stage of the game (the simultaneous choice of prices), then go back to the first stage and find the solution of the complete game.

In section 3.5 we analyze the equilibrium in the second stage game, considering the housing quality of the two ULD as given. The Nash equilibrium of the game depends on the vector of qualities (k_1, k_2) chosen in the first stage of the game. If the game was symmetric we could, without loss of generality, assume that $k_1 \geq k_2$ and compute the corresponding Nash equilibria, knowing that if the assumption was the reverse one we would have similar equilibria but with the roles of the two firms reversed. However, when the transportation costs are positive, our model is not symmetric (ULD 2 has a disadvantage because it is not located in the CBD). Thus, in our model it is important to explore the

[2] This assumption is not relevant for the second stage of the game but influences the determination of the equilibrium qualities in the first stage. The numerical solution presented in the next chapter could be changed easily to study other functional forms of the investment cost function. However time constraints did not allow us to explore the implications of other assumptions.

cases where $k_1 > k_2$, $k_1 = k_2$ and $k_2 > k_1$. In all these cases we may have different types of equilibrium, depending on the housing quality difference, the transportation cost and on the quality taste parameter, $\underline{\theta}$. In particular, we may have cases where only one firm operates (with partial or full coverage) and cases where both firms operate (with partial or full coverage). Before we compute the Nash equilibrium it is useful to derive some preliminary results.

3.4 Some preliminary results

We start by imposing necessary conditions on the two ULD quality levels for their demand to be positive. This allows us to restrict our analysis of the Nash equilibrium for vector of qualities (k_1, k_2) where at least one of the ULD has positive demand. Moreover, in order to simplify the exposition it is also useful to define some cut-off valuations.

3.4.1 Necessary conditions for demand to be positive

A necessary condition for ULD 1 to have positive demand is that the consumer with the highest quality valuation, $\theta = \underline{\theta} + 1$, has a positive net utility if he buys from ULD 1 at a price equal to its marginal cost, $p_1 = c_1$. In other words:

$$(\underline{\theta} + 1)k_1 - \frac{k_1^2}{2} > 0$$

which is equivalent to:

$$k_1 < 2(\underline{\theta} + 1). \tag{3.1}$$

If k_1 is equal or greater than $2(\underline{\theta} + 1)$, ULD 1 has zero demand even if it charges a price equal to its marginal cost. What happens is that for $k_1 > 2(\underline{\theta} + 1)$, quality is too high. Since the marginal costs of production are increasing with quality, for those levels of quality the marginal cost of production is so high that even the consumer who values most quality would prefer not to buy than to buy the house.

Similarly, a necessary condition for ULD 2 to have a positive demand, is that the consumer with the highest quality valuation, $\theta = \underline{\theta} + 1$, has a positive net utility if he buys from ULD 2 at a price equal to the marginal cost, $p_2 = c_2$. In other words:

$$(\underline{\theta} + 1)k_2 - t - \frac{k_2^2}{2} > 0.$$

Since the second order derivative with respect to k_2 is negative, we have an inverted parabola, which means that the expression is positive between the roots of the quadratic equation:

$$(\underline{\theta} + 1)k_2 - t - \frac{k_2^2}{2} = 0.$$

The intuition is that, if the quality, k_2, is too low no one wants to buy the house from ULD 2 for a price equal to the marginal cost due to the transportation costs. If the quality is too high, it also happens that no one wants to buy because the corresponding price would be too high, even if the price was equal to the marginal cost. Thus, in order for ULD 2 to have a positive demand, its quality must satisfy the following condition :

$$(\underline{\theta} + 1) - \sqrt[2]{(\underline{\theta} + 1)^2 - 2t} < k_2 < (\underline{\theta} + 1) + \sqrt[2]{(\underline{\theta} + 1)^2 - 2t} \tag{3.2}$$

In order to have real roots, the radicand must be positive. Therefore $(\underline{\theta} + 1)^2 - 2t \geqslant 0$, or equivalently:

$$t \leq \frac{(\underline{\theta} + 1)^2}{2}.$$

Thus, in order for ULD 2 to have a positive demand, the transportation costs cannot be too high with respect to the highest quality valuation, $\underline{\theta} + 1$. Otherwise, even the highest valuation consumer would prefer not to buy than to buy a house from ULD 2. In what follows we restrict the analysis to the case where $t \leq \frac{(\underline{\theta}+1)^2}{2}$, since otherwise we would never have a duopoly.

3.4.2 Some important cut-off valuations and preliminary results

The consumer's choice between buying from ULD 1 or not buying depends on whether buying from ULD 1 gives the consumer a positive net utility or not. The consumer prefers to buy from ULD 1 than not buy, if and only if:

$$U_1(\theta) = \theta k_1 - p_1 \geq 0$$

Let $\widehat{\theta}_1$ be the quality valuation of the consumer for which the previous condition is satisfied in equality. In other words:

$$\widehat{\theta}_1 = \frac{p_1}{k_1}. \tag{3.3}$$

The consumer with valuation $\widehat{\theta}_1$ is indifferent between buying from ULD 1 or not buying at all. Note that all consumers with $\theta > \widehat{\theta}_1$ strictly prefer to buy from ULD 1 than not to buy, whereas all consumers with $\theta < \widehat{\theta}_1$ prefer not buy than to buy from ULD 1.

Depending on k_1 and p_1, $\widehat{\theta}_1$ may be below $\underline{\theta}$, between $\underline{\theta}$ and $\underline{\theta}+1$ or above $\underline{\theta}+1$. If $\widehat{\theta}_1 < \underline{\theta}$ then all the consumers prefer to buy from ULD 1 than not buy, which means that all consumers are served. On the contrary, if $\widehat{\theta}_1 > \underline{\theta}+1$, none of the consumers wants to buy from ULD 1.

Similarly, one can find the consumers who prefer to buy from ULD 2 than not buying, by solving:

$$U_2(\theta) = \theta k_2 - t - p_2 \geq 0$$

Let $\widehat{\theta}_2$ be the indifferent consumer between buying from ULD 2 or not buying at all. In other words:

$$\widehat{\theta}_2 = \frac{p_2 + t}{k_2}. \tag{3.4}$$

Again the consumers with valuations above $\widehat{\theta}_2$ strictly prefer to buy from ULD 2 than not buying. Depending on k_2, t and p_2, $\widehat{\theta}_2$ may be below $\underline{\theta}$, between $\underline{\theta}$ and $\underline{\theta}+1$ or above $\underline{\theta}+1$. If $\widehat{\theta}_2 < \underline{\theta}$ then all the consumers prefer to buy from ULD 2 than not buy, which means that all consumers are served. On the contrary, if $\widehat{\theta}_2 > \underline{\theta}+1$, none of the consumers wants to buy from ULD 2.

Figure 3.1 shows the utility of buying a house from ULD 1 (in the left) and from ULD 2 (in the right) in a case where some lower valuation consumers prefer not to buy any of the houses, and thus the indifferent consumers, $\widehat{\theta}_1$ and $\widehat{\theta}_2$, are above $\underline{\theta}$. From the figure it is clear that all consumers to the right of $\widehat{\theta}_i$ strictly prefer to buy a house from ULD i than not to buy a house whereas all consumers to the left of $\widehat{\theta}_i$ strictly prefers not to buy a house from ULD i.

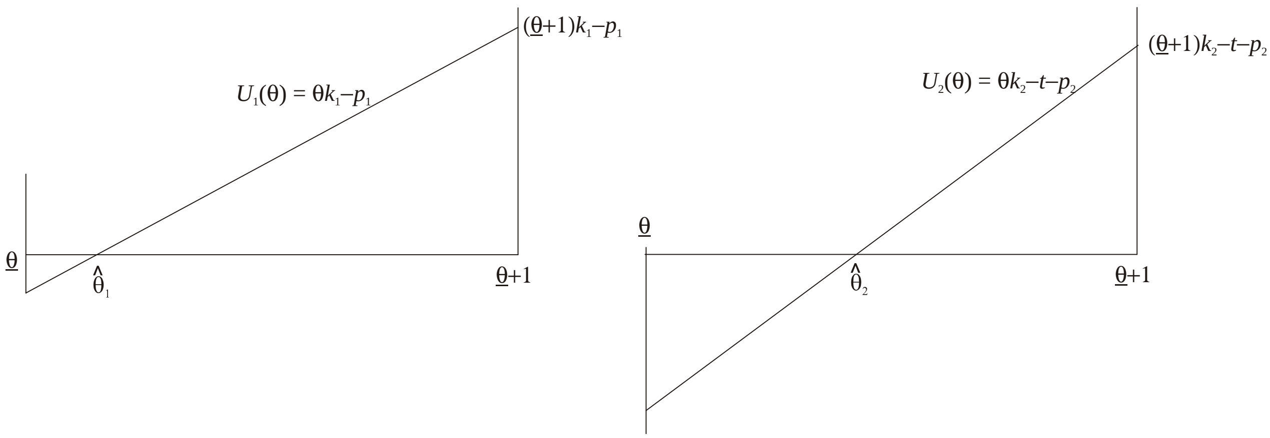

Figure 3.1: The indifferent consumer between buying and not buying from ULD 1 and from ULD 2, respectively.

Note that the slope of the utility function of buying from ULD i is equal to k_i and therefore for positive qualities the utility is increasing with θ. This has implications on the way the consumers choose between the two ULD. Some results are easy to show:

Lemma 3.1 *If the highest valuation consumer,* $\underline{\theta} + 1$*, prefers the house with the lower quality, then all the consumers prefer the house of lower quality.*

Proof. Assume that $k_2 > k_1$, the difference in utilities, $U_1(\theta) - U_2(\theta)$, for type θ is positive if $(\theta k_1 - p_1) - (\theta k_2 - t - p_2) > 0$, or equivalently, $p_2 + t - p_1 > \theta\,(k_2 - k_1)$. Since the right hand side of the previous expression is increasing with θ, that implies that if the condition holds for $(\underline{\theta} + 1)$ then it holds for any $\theta < \underline{\theta} + 1$. A similar proof holds in the case of $k_1 > k_2$. ■

A similar results holds when the lower valuation consumer prefers the high quality house:

Lemma 3.2 *If the lowest valuation consumer, $\underline{\theta}$, prefers the house with the higher quality, then all the consumers prefer the house of higher quality.*

Proof. Assume that $k_2 > k_1$, the difference in utilities $U_2(\theta) - U_1(\theta)$ for type θ is positive if $(\theta k_2 - t - p_2) - (\theta k_1 - p_1) > 0$, or equivalently, $p_2 + t - p_1 < \theta (k_2 - k_1)$. Since the right hand side of the previous expression is increasing with θ, that implies that if the condition holds for $\underline{\theta}$ then it holds for any $\theta > \underline{\theta}$. A similar proof holds in the case of $k_1 > k_2$. ■

The proofs show that, for a given quality differential, the consumer decision depends on the difference between the total prices, $p_2 + t - p_1$, where $p_2 + t$ is the total price of firm 2 and p_1 is the total price of firm 1. Note that, in the two previous cases, only one of the ULD has positive demand.

In order for both firms to have positive demand, the price differential cannot be too high since otherwise all consumers would prefer the low quality house. On the other hand the price differential cannot be too low, otherwise all the consumers would prefer the high quality house.

Lemma 3.3 *If prices are such that both ULD have positive demand, the higher quality ULD serves the higher valuation consumers whereas the lower quality ULD serves the lower valuation consumers.*

Proof. Assume that $k_2 > k_1$, then from lemma 3.1 we know that ULD 2 can only have positive demand if the highest valuation consumer prefers the higher quality house. Moreover, from lemma 3.2 we know that ULD 1 can only have positive demand if the lowest valuation consumer prefers the lower quality house. This implies that $U_2(\theta) - U_1(\theta) = (\theta k_2 - t - p_2) - (\theta k_1 - p_1)$ is negative at $\underline{\theta}$ but positive at $\underline{\theta}+1$. Since the function is continuous in θ, there exists an intermediate value of θ, θ^*, where $(\theta k_2 - t - p_2) - (\theta k_1 - p_1) = 0$. Moreover since $U_2 - U_1$ is increasing in θ, then all consumers to the right of θ^* prefer to buy the house from ULD 2, whereas all consumers to the left of θ^* prefer to buy from ULD 1. A similar proof holds in the case where $k_1 > k_2$. ■

This shows that we can never have two local monopolies. Either only the high quality ULD operates, only the low quality ULD operates, or both ULD operate.

The consumer choice between the two urban lands developers depends on whether $k_1 = k_2$, $k_1 > k_2$ or $k_2 > k_1$. If the two ULD have the same quality, the utility functions of buying from the two ULD have the same slope and either ULD 1 is strictly preferred to ULD 2 for all consumers, or the reverse, or all consumers are indifferent between buying from ULD 1 and buying from ULD 2.

When $k_2 > k_1$, consumers prefer to buy from ULD 1 than from ULD 2 if:

$$\theta k_1 - p_1 \geq \theta k_2 - t - p_2$$

which is equivalent to:

$$\theta \leq \frac{p_2 - p_1 + t}{k_2 - k_1}$$

Let θ^* be the value of θ such that previous expression holds in equality:

$$\theta^* = \frac{p_2 - p_1 + t}{k_2 - k_1}$$

In other words, θ^* is the indifferent consumer between buying from ULD 1 or buying from ULD 2. The consumers to the right of θ^* strictly prefer to buy from ULD 2 whereas the consumers to the left of θ^* strictly prefer to buy from ULD1 than from ULD2. Therefore, the higher valuation consumers buy from the higher quality ULD while the lower valuation consumers buy from the lower quality ULD.

Similarly, when $k_1 > k_2$, consumers prefer to buy from ULD 1 than from ULD 2 if:

$$\theta k_1 - p_1 \geq \theta k_2 - t - p_2$$

which is equivalent to:

$$\theta \geq \frac{p_1 - p_2 - t}{k_1 - k_2}$$

Let θ^* be the value of θ such that previous expression holds in equality, in other words θ^* is the indifferent consumer between buying from ULD 1 or buying from ULD 2. The consumers to the right of θ^* strictly prefer to buy from ULD 1 whereas the consumers to the left of θ^* strictly prefer to buy from ULD2 than from ULD1. Therefore, the higher valuation consumers buy from the higher quality ULD while the lower valuation consumers buy from the lower quality ULD.

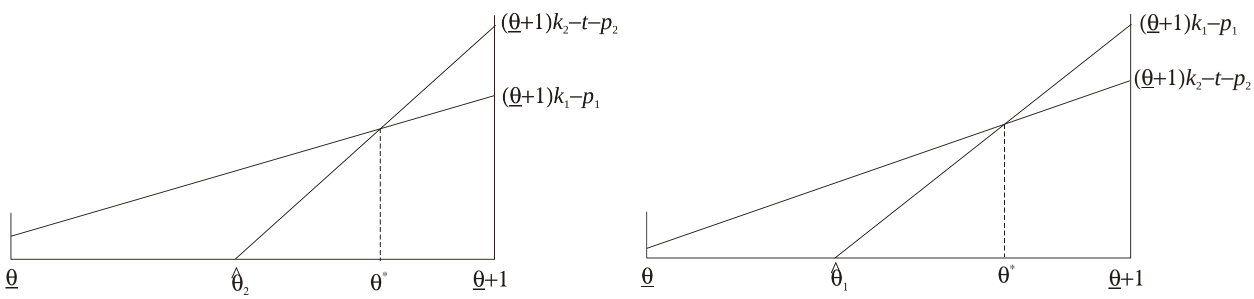

Figure 3.2: The indifferent consumer between buying from ULD 1 and ULD 2 when $k_2 > k_1$ (left) and when $k_1 > k_2$ (right).

Using the results in this section and the previous one, it is relatively easy, although slightly boring, to derive the demand functions when $k_2 = k_1$, when $k_2 > k_1$ and when $k_1 > k_2$. The detailed calculations are presented in Appendix A - Demand functions.

3.5 Nash equilibrium of the price game

Considering that any price below the marginal cost is weakly dominated by charging a price equal to marginal cost, to derive the Nash equilibrium we restrict the analysis to prices $p_i \geq c_i$. To organize our results, it is helpful to think about the maximum utility that a firm could offer to the consumers, without charging a price below marginal cost.

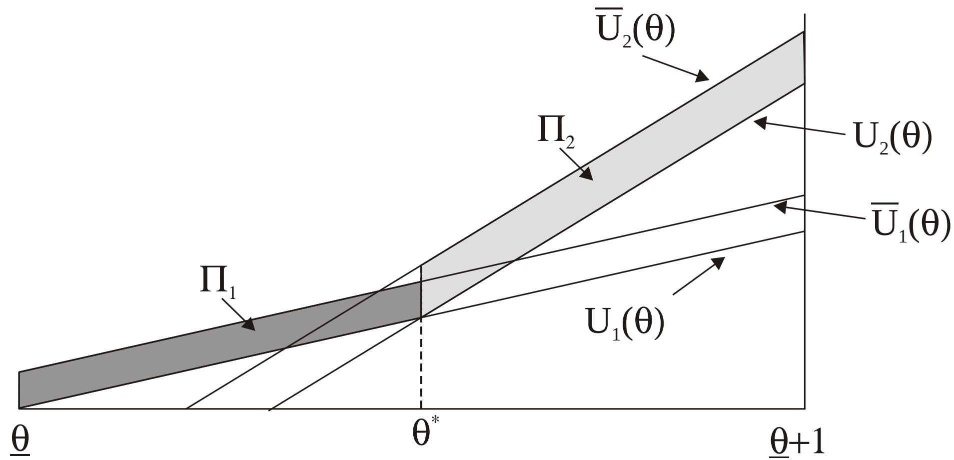

Figure 3.3: The ULD profit represented as the difference between the maximal surplus of firm i and the surplus at p_i

This maximal surplus is given by:

$$\overline{U}_1(\theta) = \theta k_1 - \frac{k_1^2}{2} \quad \text{and} \quad \overline{U}_2(\theta) = \theta k_2 - t - \frac{k_2^2}{2}$$

An interesting result is:

Lemma 3.4 *If ULD i serves consumer with valuation θ, the profit obtained with this consumer is given by $\overline{U}_i(\theta) - U_i(\theta)$.*

Proof. Consider ULD 1, $\overline{U}_1(\theta) - U_1(\theta) = \theta k_1 - \frac{k_1^2}{2} - (\theta k_1 - p_1) = p_1 - \frac{k_1^2}{2} = p_1 - c_1$. A similar proof can be done for ULD 2. ■

This results allows us to get a nice representation of the profit obtained by the ULD i, since it will be given by the area between $\overline{U}_i(\theta)$ and $U_i(\theta)$ in the market area of ULD i.

Moreover this result also helps organizing the presentation of the Nash equilibrium as that will depend on which firm offers a higher quality but it will also depend on which firm (if any) has a natural advantage in terms of the maximum surplus it can offer. In particular, if the maximum surplus a firm can offer is always non-positive, the firm has zero demand as long as $p_i \geq c_i$. In this case the other firm has a guaranteed monopoly position. It is also possible that the maximum surplus that one ULD can offer is always above the maximal surplus that the other ULD can offer. In this case, the firm with higher maximal surplus has the possibility of being a monopolist by charging a price low enough so that for the other ULD is not profitable to build any house. However the ULD that has a «natural advantage» may prefer to charge a higher price and share the market with the other ULD (it all depends on which of these options is more profitable). Finally, it is also possible that when we look at the maximal surplus offered by each ULD, both firms have a «natural market». In this case, both firms will operate in equilibrium.

Another interesting conclusion that follows from the analysis of the maximum surplus expression, $\overline{U}_i(\theta)$, is that it is a quadratic function with a maximum at $k_i = \theta$. In other words, increasing quality above θ has a negative impact on the maximum surplus that can be offered to the consumer with valuation θ. Thus choosing a quality above $\underline{\theta}+1$ decreases the maximum surplus that can be offered to all the consumers and hence it is detrimental in terms of the firm potential demand.

3.5.1 One of the ULD has a guaranteed monopoly

Firm 1 has a guaranteed monopoly

As explained in section 3.4.1 no consumer will ever buy from firm 2 if k_2 is outside the limits in expression (3.2). In this case, if $k_1 < 2(\underline{\theta}+1)$ firm 1 has a guaranteed monopoly.

If ULD 1 operates in the market with partial coverage, its demand is given by all the consumers above $\widehat{\theta}_1$. Figure 3.4 illustrates this case.

Figure 3.4: Only ULD 1 operates with partial coverage.

ULD 1 solves the following problem:

$$\max_{p_1} \Pi_1 = \left(\underline{\theta} + 1 - \frac{p_1}{k_1}\right)(p_1 - c_1) \qquad \text{subject to } p_1 \geq k_1\underline{\theta}$$

If we solve the unconstrained problem, the first order condition is

$$\frac{d\Pi_1}{dp_1} = -\frac{p_1 - c_1}{k_1} + \underline{\theta} + 1 - \frac{p_1}{k_1} = 0$$

Solving with respect to p_1, we obtain:

$$p_1^* = \frac{c_1 + k_1(\underline{\theta} + 1)}{2} \tag{3.5}$$

If the previous price is equal or above $k_1\underline{\theta}$, then p_1^* is the equilibrium price. Otherwise ULD 1 covers the whole market and it solves:

$$\max_{p_1} \Pi_1 = (p_1 - c_1) \qquad \text{subject to } p_1 \leq k_1\underline{\theta}$$

Since the profit function increases linearly with p_1, it is optimal to charge the highest price possible. That is, with full coverage, $p_1^* = k_1\underline{\theta}$.

Therefore, the equilibrium price of ULD 1 when it has a guaranteed monopoly is:

Proposition 3.5 *If* $k_2 < (\underline{\theta} + 1) - \sqrt[2]{(\underline{\theta} + 1)^2 - 2t}$ *or* $k_2 > (\underline{\theta} + 1) + \sqrt[2]{(\underline{\theta} + 1)^2 - 2t}$ *and* $k_1 < 2(\underline{\theta} + 1)$*, ULD 1 has a guaranteed monopoly and its optimal price is:*

$$p_1^* = \max\left[k_1\underline{\theta}, \frac{\frac{k_1^2}{2} + k_1(\underline{\theta} + 1)}{2}\right]$$

The market is partially covered whenever $\underline{\theta} \leq 1$ *or when* $\underline{\theta} > 1$ *and* $k_1 > 2(\underline{\theta} - 1)$*. If* $\underline{\theta} > 1$ *and* $k_1 < 2(\underline{\theta} - 1)$ *the market is fully covered.*

Proof. The first part of the proof is an immediate consequence of ULD 1 profit maximization problem and from the substitution of $c_1 = \frac{k_1^2}{2}$ in expression (3.5). The second part of the proof follows from the comparison between terms in the maximum function. The condition

$$\frac{\frac{k_1^2}{2} + k_1(\underline{\theta} + 1)}{2} > k_1\underline{\theta} \quad \Leftrightarrow \quad k_1\left(k_1 + 2(1 - \underline{\theta})\right) > 0$$

which holds for every $k_1 > 0$ when $\underline{\theta} \leq 1$ and it also holds for $k_1 > 2(\underline{\theta} - 1)$ for $\underline{\theta} > 1$. Obviously, the reverse condition holds for $\underline{\theta} > 1$ and $k_1 < 2(\underline{\theta} - 1)$. ■

The intuition for this result is that when the lowest valuation consumer has a very low valuation of quality, the monopolist is better off by not covering the whole market, since full coverage would imply a too low price (in the limit case of $\underline{\theta} = 0$, the price would have to be 0 to have full coverage). On the other hand, when the lowest valuation consumer is high, the monopolist is also better off covering only partially the market if the quality is very high. The reason is that, for a high quality the marginal production costs are also very high, which implies very high prices. But then the lower valuation consumers do not want to buy and hence the market is not fully covered.

The previous results also tells us that the monopolist ULD 1 only covers the market completely for higher valuations and not too high quality.

If ULD 1 has a guaranteed monopoly, its price is increasing with its quality. It increases linearly when the market is fully covered. It increases at an increasing rate if the market is only partially covered (due to the shape of marginal costs). Moreover the optimal price does not depend on t because the demand of ULD 1 is not a function of t.

Firm 2 has a guaranteed monopoly

As we can see in section 3.4.1, ULD 1 has zero demand if $k_1 > 2(\underline{\theta} + 1)$. In this case, ULD 2 has a guaranteed monopoly as long as k_2 satisfies condition (3.2). If the market is partially covered, the demand of ULD 2 is given by all the consumers above $\widehat{\theta}_2$. Figure 3.5 illustrates this case.ULD 2 solves the following problem:

Figure 3.5: Only ULD 2 operates with partial coverage.

$$\max_{p_2} \Pi_2 = \left(\underline{\theta} + 1 - \frac{p_2 + t}{k_2}\right)(p_2 - c_2) \quad \text{subject to } p_2 > k_2\underline{\theta} - t$$

If we solve this problem ignoring the constraint, the first order condition is:

$$\frac{d\Pi_2}{dp_2} = -\frac{p_2 - c_2}{k_2} + \underline{\theta} + 1 - \frac{p_2 + t}{k_2} = 0$$

Then solving with respect to p_2, we get:

$$p_2^* = \frac{c_2 + k_2(\underline{\theta} + 1) - t}{2}$$

If the previous price is higher than $k_2\underline{\theta} - t$, then p_2^* is the equilibrium price. Otherwise ULD 2 covers the whole market in which case it wants to charge the highest price that guarantees full coverage, thus

$$p_2^* = k_2\underline{\theta} - t.$$

Therefore, the equilibrium price of ULD 2 when it has a guaranteed monopoly is:

Proposition 3.6 *If* $k_1 \geq 2(\underline{\theta} + 1)$ *and* $(\underline{\theta} + 1) - \sqrt[2]{(\underline{\theta} + 1)^2 - 2t} < k_2 < (\underline{\theta} + 1) + \sqrt[2]{(\underline{\theta} + 1)^2 - 2t}$ *then ULD 2 has a guaranteed monopoly and its optimal price is*

$$p_2^* = \max\left[k_2\underline{\theta} - t, \frac{\frac{k_2^2}{2} + k_2(\underline{\theta} + 1) - t}{2}\right]$$

Proof. It is immediate from the solution of ULD 2 profit maximization problem. ■

The optimal price of ULD 2 when it has a guaranteed monopoly is an increasing function of k_2 (the intuition is the same than in the previous subsection). Moreover the optimal price of firm 2 is decreasing with t. This last result is an immediate consequence of t having a negative impact on the firm demand (if the market is partially covered) and a negative impact on the price that can be charged to the lowest valuation consumer (if the market is fully covered).

3.5.2 Case where $k_1 = k_2$

When $k_1 = k_2 = k$, the two ULD are offering precisely the same quality, thus there is no differentiation and we have a traditional Bertrand model. However, if $t > 0$, ULD 2 has a disadvantage and the ULDs demand are discontinuous at $p_1 = p_2 + t$.

In the Nash equilibrium one cannot have p_2 above $c_2 = \frac{k^2}{2}$ or p_1 above $\frac{k^2}{2} + t$, since if that happened there would be an incentive for each firm to slightly undercut the price so has to capture the whole demand.

If $t = 0$ the two ULD will share equally the market, but both have a zero operating profit. This implies that if there are fixed cost to improve quality, the ULD get a negative payoff in the complete game. If $t > 0$, ULD 1 has an advantage over ULD 2 and will be the only ULD operating in the market by selling at price below $\frac{k^2}{2} + t$. For t high, ULD 1 may be able to charge its optimal monopoly price (as long as this price is below $\frac{k^2}{2} + t$), while for t small the constraint $p_1 \leq \frac{k^2}{2} + t$ will be binding.

From section 3.5.1 we already know the optimal monopoly price of ULD 1 depending on whether the market is fully covered or not. But now ULD 1 is constrained to charge a price below $\frac{k^2}{2} + t$. For t high the constraint $p_1 \leq \frac{k^2}{2} + t$ is not binding and either we get full coverage or partial coverage according the result in section 3.5.1. On the other hand, for t low the constraint is binding and ULD 1 will have to charge a lower price to match the surplus offered by ULD 2. Thus, for t low ULD 1 is a constrained monopoly (with partial or full coverage).

Therefore when the two ULD offer the same quality the Nash equilibrium of the price game is as follows:

Proposition 3.7 *When $k_1 = k_2 = k$, if $t = 0$, then $p_1^* = p_2^* = \frac{k^2}{2}$. When $t > 0$, then $p_2^* = \frac{k^2}{2}$ and*

$$p_1^* = \min\left[\frac{k^2}{2} + t, \max\left[\underline{\theta}k, \frac{\frac{k^2}{2} + k(\underline{\theta}+1)}{2}\right]\right]$$

When $t \leq k\left(\underline{\theta} - \frac{k}{2}\right)$ the market is always fully covered and $p_1^* = \frac{k^2}{2} + t$. When $t > k\left(\underline{\theta} - \frac{k}{2}\right)$, $\underline{\theta} > 1$ and $k_1 < 2(\underline{\theta} - 1)$, then the market is fully covered and $p_1^* = \underline{\theta}k$. If $t > k\left(\underline{\theta} - \frac{k}{2}\right)$ and either $\underline{\theta} \leq 1$ or $\underline{\theta} > 1$ and $k_1 > 2(\underline{\theta} - 1)$, the market is partially covered and $p_1^* = \min\left[\frac{\frac{k^2}{2} + k(\underline{\theta}+1)}{2}, \frac{k^2}{2} + t\right]$.

Proof. The first part of the result follows from the solution of ULD 1 profit maximization problem subject to the constraint $p_1 \leq \frac{k^2}{2} + t$. If the constraint is not binding the ULD 1 optimal price as given in 3.5.1, otherwise $p_1 = \frac{k^2}{2} + t$. If we have $t \leq k\left(\underline{\theta} - \frac{k}{2}\right)$, then $\frac{k^2}{2} + t \leq \underline{\theta}k$ and therefore $p_1^* = \frac{k^2}{2} + t$. Since $p_1^* \leq \underline{\theta}k$ the market is fully covered.

When $t > k\left(\underline{\theta} - \frac{k}{2}\right)$, we have $\frac{k^2}{2} + t > \underline{\theta}k$. If $\frac{\frac{k^2}{2} + k(\underline{\theta}+1)}{2} \leq \underline{\theta}k$ then $p_1^* = \underline{\theta}k$ and the market is fully covered. This case holds for $\underline{\theta} > 1$ and $k_1 < 2(\underline{\theta} - 1)$. However if $t > k\left(\underline{\theta} - \frac{k}{2}\right)$ and $\frac{\frac{k^2}{2} + k(\underline{\theta}+1)}{2} > \underline{\theta}k$ the market is partially covered and $p_1^* = \min\left[\frac{\frac{k^2}{2} + k(\underline{\theta}+1)}{2}, \frac{k^2}{2} + t\right]$. ■

The previous result tells that when t is low, $t \leq k\left(\underline{\theta} - \frac{k}{2}\right)$, ULD 1 covers the whole market and charges $p_1^* = \frac{k^2}{2} + t$ (in reality p_1^* has to be slightly below this). Consequently, the profit is:

$$\Pi_1^* = \frac{k^2}{2} + t - c_1 = \frac{k^2}{2} + t - \frac{k^2}{2} = t$$

In this case, the lowest valuation consumer gets a positive surplus, the same surplus it would be able to get buying from ULD 2 at $p_2 = \frac{k^2}{2}$.

For t high, $t > k\left(\underline{\theta} - \frac{k}{2}\right)$, the market will be totally covered for $\underline{\theta} > 1$ and $k_1 < 2(\underline{\theta} - 1)$ at price $p_1^* = \underline{\theta}k$ and the corresponding profit is:

$$\Pi_1^* = \underline{\theta}k - c_1 = \underline{\theta}k - \frac{k^2}{2}.$$

Note that in this case the lowest valuation consumer gets a zero surplus.

In the remaining cases the market will be partially covered. The consumers covered may be the same than under unconstrained monopoly (for very high t), but for t not too high, more consumers will be served due to existence of ULD 2 (when matching the surplus offered by ULD 2 is a binding constraint).

Looking at the equilibrium prices when $k_1 = k_2 = k$, one observes that the prices are increasing with the quality and are increasing with t whenever the constraint $p_1^* \leq \frac{k^2}{2} + t$ is binding (that is for low t). For higher t the equilibrium prices depend only on the quality.

3.5.3 Case where $k_2 > k_1$

Both ULDs have a «natural market»

Suppose that if ULD 2 charges $p_2 = \frac{k_2^2}{2}$ and ULD 1 charges $p_1 = \frac{k_1^2}{2}$, then the highest valuation consumer prefers to buy from ULD 2, while the lowest valuation consumer who buys a house prefers to buy a house from ULD 1. In other words, suppose that $\overline{U}_2(\theta)$ and $\overline{U}_1(\theta)$ intersect at some θ between $\max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right]$ and $(\underline{\theta}+1)$, where $\widehat{\theta}_1(c_1) = \frac{k_1}{2}$ is the indifferent consumer between buying from ULD 1 and not buying if this firm charges $p_1 = c_1$. Then with marginal cost pricing both firms have a strictly positive demand. This is illustrated in Figure 3.6. This case holds when the following conditions are both true:

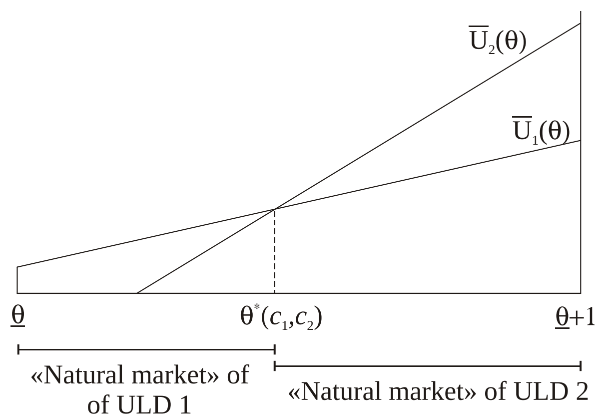

Figure 3.6: The «natural markets» of ULD 1 and ULD 2.

$$\left((\underline{\theta}+1)k_2 - t - \frac{k_2^2}{2}\right) - \left((\underline{\theta}+1)k_1 - \frac{k_1^2}{2}\right) > 0$$
$$\text{and } \left(\max\left[\underline{\theta}, \frac{k_1}{2})\right] k_2 - t - \frac{k_2^2}{2}\right) - \left(\max\left[\underline{\theta}, \frac{k_1}{2}\right] k_1 - \frac{k_1^2}{2}\right) < 0$$

which is equivalent to:

$$(k_2 - k_1)\left(\max\left[\underline{\theta}, \frac{k_1}{2}\right] - \frac{(k_1+k_2)}{2}\right) < t < (k_2 - k_1)\left((\underline{\theta}+1) - \frac{(k_1+k_2)}{2}\right)$$

Note that if $\max\left[\underline{\theta}, \frac{k_1}{2}\right] = \frac{k_1}{2}$ the left hand side limit is necessarily negative, and thus only the right hand side limit is relevant. Intuitively t cannot be too high for both firms to operate, since with t very high only firm 1 would operate. On the other hand, if $\max\left[\underline{\theta}, \frac{k_1}{2}\right] = \underline{\theta}$, which happens for low levels of k_1, $k_1 < 2\underline{\theta}$, the left hand side limit may be positive (if k_2 is not very high) in which case for low values of t all consumers prefer ULD 2. To summarize, when $k_2 > k_1$, the two firms operate if t is not too high and k_1 is not too low (and k_2 is not very high).

Under this circunstances one can show the following:

Lemma 3.8 *If* $(k_2 - k_1)\left(\max\left[\underline{\theta}, \frac{k_1}{2}\right] - \frac{(k_1+k_2)}{2}\right) < t < (k_2 - k_1)\left((\underline{\theta} + 1) - \frac{(k_1+k_2)}{2}\right)$*, then in equilibrium both firms operate.*

Proof. By contradiction, suppose that one of the ULDs does not operate in equilibrium (let us assume it is ULD 1). If ULD 2 is charging a price above the marginal cost, that would mean that some consumers would buy from ULD 1 if it charges a price slightly above its marginal cost, which would imply a positive profit. Thus ULD 1 has an incentive to deviate. On the other hand, if ULD 2 was charging $p_2 \leq c_2$, this ULD would gain by deviating to a price slightly above c_2. Thus, if one of the firms was not operating, at least one of them would gain by deviating. Hence in equilibrium one cannot have one of the firms without operating. ■

When both firms operate in the market with full coverage, we already know that ULD 2 covers the consumer with higher valuation while the ULD 1 covers the consumers with lower valuation as shown in Figure 3.7.The demands of the two ULD are given by:

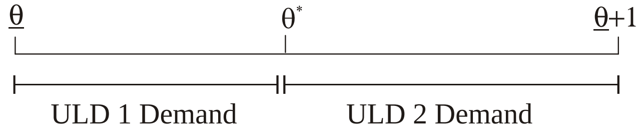

Figure 3.7: Both firms operate and the whole market is covered when $k_2 > k_1$.

$$D_1 = \frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta} \qquad \text{and} \qquad D_2 = \underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}$$

Then the profit functions for the two ULD are given by the following expressions:

$$\Pi_1 = \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta}\right)(p_1 - c_1)$$
$$\Pi_2 = \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}\right)(p_2 - c_2)$$

The first order conditions of the two ULD profit maximization problems are:

$$\frac{\partial \Pi_1}{\partial p_1} = -\frac{p_1 - c_1}{k_2 - k_1} + \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta}\right) = 0$$
$$\frac{\partial \Pi_2}{\partial p_2} = -\frac{p_2 - c_2}{k_2 - k_1} + \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}\right) = 0$$

Solving this system with respect to p_1 and p_2, we obtain the equilibrium prices:

$$p_1^* = \frac{(1 - \underline{\theta})(k_2 - k_1) + 2c_1 + c_2 + t}{3}$$
$$p_2^* = \frac{(\underline{\theta} + 2)(k_2 - k_1) + 2c_2 + c_1 - t}{3}$$

It should be noted that in order for this to be the equilibrium, we have to have $\underline{\theta} \leq \theta^*(p_1^*, p_2^*) \leq \underline{\theta} + 1$ (so that both firms operate) and the consumer with lowest quality valuation, $\underline{\theta}$, has to have a non-negative net utility buying the house of ULD 1 at price p_1^*. In other words, full coverage holds if:

$$\underline{\theta} k_1 \geq \frac{(1 - \underline{\theta})(k_2 - k_1) + k_1^2 + \frac{k_2^2}{2} + t}{3} \tag{3.6}$$

Note that for very low values of $\underline{\theta}$ the previous condition does not hold (it does not hold for $\underline{\theta}$ and hence, by continuity it does not hold in a neighborhood of $\underline{\theta} = 0$. Moreover, the condition is easier to be satisfied for higher $\underline{\theta}$ (since the left hand side of the expression is increasing and the right hand side is decreasing with $\underline{\theta}$). The previous result can be summarized as follows:

Proposition 3.9 *When $k_2 > k_1$ and both ULD operate in equilibrium and there is full* *coverage, the equilibrium prices are:*

$$p_1^* = \frac{(1-\underline{\theta})(k_2-k_1)+k_1^2+\frac{k_2^2}{2}+t}{3}$$
$$p_2^* = \frac{(\underline{\theta}+2)(k_2-k_1)+k_2^2+\frac{k_1^2}{2}-t}{3}$$

Proof. It follows from the solution of the system of first order conditions of the two ULD profit maximization problems assuming full coverage and substituting c_i by $\frac{k_i^2}{2}$. ■

The equilibrium prices depend on the quality differential $(k_2 - k_1)$, depend on the marginal costs which are a quadratic function of quality and depend on t. Note that the shape of the equilibrium prices as a function of the qualities is influenced a lot by the shape of the marginal costs.

Under full coverage, increasing t influences positively the equilibrium price of ULD 1 and negatively the equilibrium price of ULD 2. This happens because, increasing t increases the demand of ULD 1 and decreases the demand of ULD 2. As a consequence it is optimal for ULD 1 to charge a higher price (its best response shifts to the right) whereas for ULD 2 it is optimal to decrease it price (its best response shifts down).

The impact of changes in the firm house quality on its own price is clearly positive for the higher quality ULD. For the lower quality ULD the sign is positive if $\underline{\theta} \geq 1$, but may be negative otherwise.

If at the previous equilibrium prices, condition 3.6 does not hold, then it means that in the Nash equilibrium we cannot have a duopoly with full coverage. Figure 3.8 illustrates what happens if both firms operate with partial coverage.

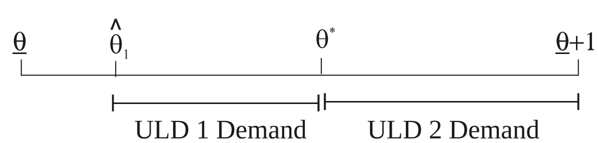

Figure 3.8: Both firms operate but the market is only partially covered when $k_2 > k_1$.

Thus, if the equilibrium involves a duopoly with partial coverage demands are:

$$D_1 = \frac{p_2 - p_1 + t}{k_2 - k_1} - \frac{p_1}{k_1} \qquad \text{and} \qquad D_2 = \underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}$$

The profit functions for the two ULD are given by the following expressions:

$$\begin{aligned} \Pi_1 &= \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \frac{p_1}{k_1} \right) (p_1 - c_1) \\ \Pi_2 &= \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1} \right) (p_2 - c_2) \end{aligned}$$

The first-order conditions of the two ULD profit maximization problems are:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial p_1} &= -\frac{p_1 - c_1}{k_2 - k_1} - \frac{p_1 - c_1}{k_1} + \frac{p_2 - p_1 + t}{k_2 - k_1} - \frac{p_1}{k_1} = 0 \\ \frac{\partial \Pi_2}{\partial p_2} &= -\frac{p_2 - c_2}{k_2 - k_1} + \underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1} = 0 \end{aligned}$$

Solving the system of equations with respect to p_1 and p_2, we obtain the equilibrium prices:

$$\begin{aligned} p_1^* &= \frac{k_1(\underline{\theta} + 1)(k_2 - k_1) + 2c_1 k_2 + c_2 k_1 + t k_1}{4k_2 - k_1} \\ p_2^* &= \frac{2k_2(\underline{\theta} + 1)(k_2 - k_1) + c_1 k_2 + 2c_2 k_2 - t(2k_2 - k_1)}{4k_2 - k_1} \end{aligned}$$

In order for this to be a Nash equilibrium it has to happen that, considering the equilibrium price p_1^*, the indifferent consumer between buying from ULD 1 and not buying, $\hat{\theta}_1(p_1^*)$, is between $\underline{\theta}$ and $\theta^*(p_1^*, p_2^*)$ and that $\theta^*(p_1^*, p_2^*)$ is smaller than $(\underline{\theta}+1)$, since otherwise all the consumers would prefer ULD 1.

Therefore if both ULD operate with partial coverage, the equilibrium is as follows:

Proposition 3.10 *When $k_2 > k_1$ and both ULD operate in equilibrium and there is* *partial coverage, the equilibrium prices are:*

$$p_1^* = \frac{k_1(\underline{\theta}+1)(k_2-k_1)+k_1^2k_2+0.5k_2^2k_1+tk_1}{4k_2-k_1}$$
$$p_2^* = \frac{2k_2(\underline{\theta}+1)(k_2-k_1)+0.5k_1^2k_2+k_2^3-t(2k_2-k_1)}{4k_2-k_1}$$

Proof. It follows from the solution of the system of first order conditions of the two ULD profit maximization problems assuming partial coverage and substituting c_i by $\frac{k_i^2}{2}$. ■

ULD 2 has a natural advantage

ULD 2 has a natural advantage if $\overline{U}_2(\theta) > \overline{U}_1(\theta)$ for all consumers who buy a house at marginal cost pricing. This is illustrated in Figure 3.9. ULD 2 has a natural advantage

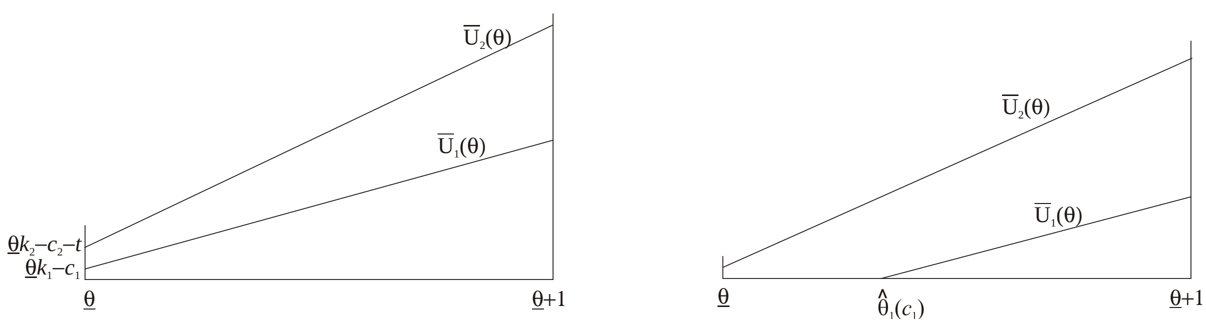

Figure 3.9: ULD 2 has a «natural advantage» and can be a monopolist.

if:

$$\overline{U}_2(\theta) - \overline{U}_1(\theta) = \left(\theta k_2 - t - \frac{k_2^2}{2}\right) - \left(\theta k_1 - \frac{k_1^2}{2}\right) > 0 \text{ for all } \theta \geq \max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right]$$

which is equivalent to:

$$\frac{1}{2}(k_2-k_1)(2\theta-(k_1+k_2)) > t \quad \text{for all } \theta \geq \max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right]$$

Note that for this condition to hold the sum of the qualities cannot be too high, $k_1 + k_2 < 2\max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right]$. Moreover the condition is easier to be satisfied for higher quality differentials, i.e. higher $k_2 - k_1$ (for a constant $k_1 + k_2$) and for smaller t.

When ULD 2 has a natural advantage, this firm can be a monopolist by charging a price that guarantees consumers at least the same surplus they get from ULD 1 at price $p_1 = c_1$. However such behavior may not be optimal if it implies a very low price since ULD 2 may be better off by charging a higher price, thus gaining a higher mark-up, even if that implies loosing some customers to ULD 1. Intuitively, ULD 2 will only prefer to be a monopolist if it has a very big advantage ($\overline{U}_2(\theta)$ is much higher than $\overline{U}_1(\theta)$). Otherwise sharing the market will be a better alternative and we obtain the equilibrium prices derived in section 3.5.3.

From section 3.5.1 we already know the optimal price charged by ULD 2 if it was an unconstrained monopoly. To be a monopolist, now ULD 2 maximizes its profit subject to constraint that the lowest valuation consumer who buys a house, if he buys from ULD 2 gets a surplus at least as high as the surplus he would get from buying a house from ULD 1. This surplus constraint can be written as follows:

$$\max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right] k_2 - t - p_2 \geq \max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right] k_1 - c_1 \quad \Leftrightarrow \quad p_2 \leq \max\left[\underline{\theta}, \widehat{\theta}_1(c_1)\right] (k_2 - k_1) + c_1 - t$$

If the lowest valuation consumer, $\underline{\theta}$, gets a positive surplus if he buys from ULD 1 at $p_1 = c_1$, $\widehat{\theta}_1(c_1) \leq \underline{\theta}$, then in order to be a monopolist ULD 2 has to cover the whole market and offer consumer $\underline{\theta}$ at least the same surplus he would get from ULD 1. This case is illustrate in the left side of Figure 3.9. The surplus constraint is given by:

$$\underline{\theta} k_2 - t - p_2 \geq \underline{\theta} k_1 - c_1 \quad \Leftrightarrow \quad p_2 \leq \underline{\theta}\,(k_2 - k_1) + c_1 - t$$

Thus, in this case $p_2^* = \underline{\theta}\,(k_2 - k_1) + c_1 - t$.

On the other hand, if $\widehat{\theta}_1(c_1) > \underline{\theta}$ (illustrated in the right side of Figure 3.9), the surplus constraint is given by:

$$\widehat{\theta}_1(c_1)k_2 - t - p_2 \geq \widehat{\theta}_1(c_1)k_1 - c_1 \quad \Leftrightarrow \quad p_2 \leq c_1\frac{k_2}{k_1} - t$$

In this case, ULD 2 can be a monopolist covering the whole market or not, depending on which of these alternatives is most profitable. If the market is fully covered, the constraint is not binding and the price is $p_2^* = k_2\underline{\theta} - t$. If the market is partially covered the constraint may or not be binding and the price is $p_2^* = \min\left[c_1\frac{k_2}{k_1} - t, \frac{c_2+k_2(\underline{\theta}+1)-t}{2}\right]$.

To summarize, if ULD 2 has a natural advantage, and wants to be a monopolist, its optimal price is:

Proposition 3.11 *If ULD 2 has a natural advantage and wants to be a monopolist, its optimal price is:*

$$p_2^* = \min\left[\max\left[\underline{\theta}, \frac{k_1}{2}\right](k_2 - k_1) + c_1 - t, \max\left[k_2\underline{\theta} - t, \frac{\frac{k_2^2}{2} + k_2(\underline{\theta}+1) - t}{2}\right]\right]$$

Proof. It follows from the solution of the constrained profit maximization problem of ULD 2. If solving the unconstrained problem p_2^* is lower or equal than the upper limit imposed by the constraint, then ULD 2 can behave as an unconstrained monopoly and p_2^* is as given in section 3.5.1. If the unconstrained optimal price does not satisfy the surplus constraint, then the constraint is binding and $p_2^* = \max\left[\underline{\theta}, \frac{k_1}{2}\right](k_2 - k_1) + c_1$. ■

However, when the constraint is binding, ULD 2 may be better off by not being a monopolist since that position is achieved only by charging a lower price than the monopolist would wish. When that happens the equilibrium prices will be the ones given in section 3.5.3.

ULD 1 has a natural advantage

ULD 1 is offering the lower quality house. We know that if the highest valuation consumer prefers to buy from ULD 1, then everyone else also prefers to buy from ULD 1. Considering this, if $\overline{U}_1(\underline{\theta}+1) > \overline{U}_2(\underline{\theta}+1)$, ULD 1 has a natural advantage. This case is illustrated in Figure 3.10.

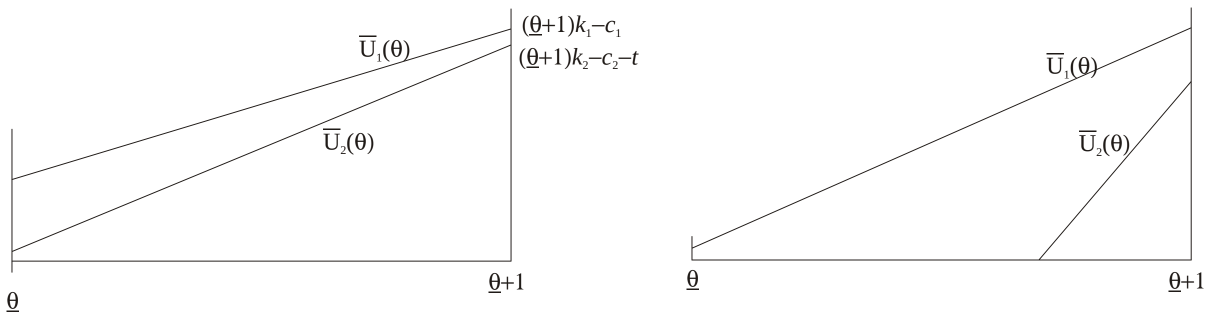

Figure 3.10: ULD 1 has a «natural advantage» and can be a monopolist.

This scenario holds when:

$$\left((\underline{\theta}+1)k_1 - \frac{k_1^2}{2}\right) - \left((\underline{\theta}+1)k_2 - t - \frac{k_2^2}{2}\right) > 0$$

which is equivalent to

$$(k_2 - k_1)\left(\frac{1}{2}(k_1+k_2) - (\underline{\theta}+1)\right) + t > 0$$

Note that a sufficient condition (but not necessary) for the previous expression to hold is $k_1 + k_2 > 2(\underline{\theta}+1)$. Thus this case is more likely to hold when the sum of the qualities is high and when the quality differential is also high.

From section 3.5.1 we already know the optimal price charged by ULD 1 if it was an unconstrained monopoly. To be a monopolist, now ULD 1 maximizes its profit subject to constraint that the highest valuation consumer gets a higher surplus if he buys from ULD 1 than if he buys ULD 2 at $p_2 = c_2$. In other words, the surplus constraint is:

$$(\underline{\theta}+1)k_1 - p_1 \geq (\underline{\theta}+1)k_2 - t - c_2 \quad \Leftrightarrow \quad p_1 \leq c_2 + t - (\underline{\theta}+1)(k_2 - k_1)$$

If the previous constraint is satisfied when we solve the unconstrained problem of profit maximization, the optimal price of ULD 1 is the one given in section 3.5.1. Otherwise the surplus constraint is binding, and to be a monopolist, ULD has to charge $p_1^* = c_2 + t - (\underline{\theta} + 1)(k_2 - k_1)$.

To summarize, if ULD 1 has a natural advantage and wants to be a monopolist, its optimal price is:

Proposition 3.12 *If ULD 1 has a natural advantage and wants to be a monopolist, its optimal price is:*

$$p_1^* = \min\left[\frac{k_2^2}{2} + t - (\underline{\theta}+1)(k_2 - k_1), \max\left[k_1\underline{\theta}, \frac{\frac{k_1^2}{2} + k_1(\underline{\theta}+1)}{2}\right]\right]$$

Proof. It follows from the solution of the constrained profit maximization problem of ULD 1 and the result in section 3.5.1. ■

If the optimal monopoly price is such that the surplus constraint is binding, ULD 1 may be better off by not decreasing the price so much and share the market with ULD 2. In that case, the equilibrium prices will be the ones presented in section 3.5.3.

3.5.4 Case where $k_1 > k_2$

Both ULDs have a «natural market»

Now ULD 1 is offering the higher quality house. Therefore, if both firms operate, ULD 1 serves the higher valuation consumers while ULD 2 serves the lower valuation ones. Similarly to section 3.5.3, we can show that if $\overline{U}_1(\theta)$ and $\overline{U}_2(\theta)$ intersect at some θ between $\max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right]$ and $(\underline{\theta}+1)$, where $\widehat{\theta}_2(c_2)$ is the indifferent consumer between buying from ULD 2 and not buying if this firm charges $p_2 = c_2$, then with marginal cost pricing both firms have a strictly positive demand and in equilibrium both firms operate.

If both ULD operate and the market is fully covered, consumers with $\theta \leq \theta^*$are served by ULD 2 whereas consumers with $\theta \geq \theta^*$ are served by ULD 1 (see Figure 3.11).

Hence the demands of the two ULD are given by:

$$D_1 = \underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2} \qquad \text{and} \qquad D_2 = \frac{p_1 - p_2 - t}{k_1 - k_2} - \underline{\theta}$$

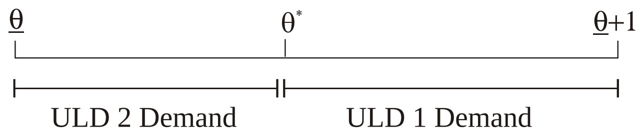

Figure 3.11: Both ULD operate and cover the whole market when $k_1 > k_2$.

Thus the profit functions for the two ULD are given by the following expressions:

$$\begin{aligned}\Pi_1 &= \left(\underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2}\right)(p_1 - c_1) \\ \Pi_2 &= \left(\frac{p_1 - p_2 - t}{k_1 - k_2} - \underline{\theta}\right)(p_2 - c_2)\end{aligned}$$

The first order conditions of the profit maximization problems are:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= -\frac{p_1 - c_1}{k_1 - k_2} + \underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2} = 0 \\ \frac{\partial \Pi_2}{\partial p_2} &= -\frac{p_2 - c_2}{k_1 - k_2} + \frac{p_1 - p_2 - t}{k_1 - k_2} - \underline{\theta} = 0\end{aligned}$$

Solving this system of equations with respect to p_1 and p_2 we obtain the equilibrium prices:

$$\begin{aligned}p_1^* &= \frac{(\underline{\theta} + 2)(k_1 - k_2) + 2c_1 + c_2 + t}{3} \\ p_2^* &= \frac{(1 - \underline{\theta})(k_1 - k_2) + 2c_2 + c_1 - t}{3}\end{aligned}$$

Consequently, when both firms operate with full coverage, the equilibrium is as follows:

Proposition 3.13 *When $k_1 > k_2$ and both ULD operate in equilibrium and there is full coverage, the equilibrium prices are:*

$$\begin{aligned}p_1^* &= \frac{(\underline{\theta} + 2)(k_1 - k_2) + k_1^2 + \frac{k_2^2}{2} + t}{3} \\ p_2^* &= \frac{(1 - \underline{\theta})(k_1 - k_2) + k_2^2 + \frac{k_1^2}{2} - t}{3}\end{aligned}$$

Proof. It follows from the solution of the system of first order conditions of the two ULD profit maximization problems assuming full coverage and substituting c_i by $\frac{k_i^2}{2}$. ■

It should be noted that in order for this to be the equilibrium, we have to have $\underline{\theta} \leq \theta^* \leq \underline{\theta} + 1$ (so that both firms operate) and the consumer with lowest quality valuation, $\underline{\theta}$, has to have a non-negative net utility buying the house of ULD 2 at price p_2^*. If at the previous equilibrium prices the last condition does not hold, then it means that in the Nash equilibrium we cannot have a duopoly with full coverage.

If the two ULD operate in the market with partial coverage, the lower valuation consumers prefer not to buy the house and demands are as given in Figure 3.12.That is,

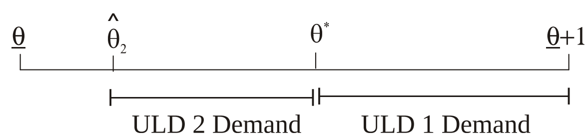

Figure 3.12: Both ULD operate but the market is only partially covered when $k_1 > k_2$.

the demand functions are given by:

$$D_1 = \underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2} \qquad \text{and} \qquad D_2 = \frac{p_1 - p_2 - t}{k_1 - k_2} - \frac{p_2 + t}{k_2}$$

And the profit functions are:

$$\begin{aligned}
\Pi_1 &= \left(\underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2}\right)(p_1 - c_1) \\
\Pi_2 &= \left(\frac{p_1 - p_2 - t}{k_1 - k_2} - \frac{p_2 + t}{k_2}\right)(p_2 - c_2)
\end{aligned}$$

The first-order conditions are given by:

$$\begin{aligned}
\frac{\partial \Pi_1}{\partial p_1} &= -\frac{p_1 - c_1}{k_1 - k_2} + \underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2} = 0 \\
\frac{\partial \Pi_2}{\partial p_2} &= -\frac{p_2 - c_2}{k_1 - k_2} - \frac{p_2 - c_2}{k_2} + \frac{p_1 - p_2 - t}{k_1 - k_2} - \frac{p_2 + t}{k_2} = 0
\end{aligned}$$

After we solve with respect to p_1 and p_2 we get the following equilibrium prices:

$$p_1^* = \frac{2k_1(\underline{\theta}+1)(k_1-k_2)+k_1(2c_1+c_2+t)}{4k_1-k_2}$$
$$p_2^* = \frac{k_2(\underline{\theta}+1)(k_1-k_2)+2k_1c_2+k_2c_1-t(2k_1-k_2)}{4k_1-k_2}$$

Therefore if both ULD operate with partial coverage, the equilibrium is as follows:

Proposition 3.14 *When $k_1 > k_2$ and both ULD operate in equilibrium and there is partial coverage, the equilibrium prices are:*

$$p_1^* = \frac{2k_1(\underline{\theta}+1)(k_1-k_2)+k_1\left(k_1^2+\frac{k_2^2}{2}+t\right)}{4k_1-k_2}$$
$$p_2^* = \frac{k_2(\underline{\theta}+1)(k_1-k_2)+k_1k_2^2+k_2\frac{k_1^2}{2}-t(2k_1-k_2)}{4k_1-k_2}$$

Proof. It follows from the solution of the system of first order conditions of the two ULD profit maximization problems assuming partial coverage and substituting c_i by $\frac{k_i^2}{2}$. ■

ULD 2 has a natural advantage

ULD 2 is offering the lower quality house. We know that if the highest valuation consumers prefers to buy from ULD 2, then everyone else also prefers to buy from ULD 2. Considering this, if $\overline{U}_2(\underline{\theta}+1) > \overline{U}_1(\underline{\theta}+1)$, then ULD 2 has a natural advantage.

From section 3.5.1 we already know the optimal price charged by ULD 2 if it was an unconstrained monopoly. To be a monopolist, now ULD 2 maximizes its profit subject to the surplus constraint is:

$$(\underline{\theta}+1)k_2 - t - p_2 \geq (\underline{\theta}+1)k_1 - c_1 \quad \Leftrightarrow \quad p_2 \leq c_1 - t - (\underline{\theta}+1)(k_1-k_2)$$

If the previous constraint is satisfied when we solve the unconstrained problem of ULD 2 profit maximization, the optimal price of ULD 2 is the one given in section 3.5.1.

Otherwise the surplus constraint is binding, and to be a monopolist, ULD 2 has to charge $p_2^* = c_1 - t - (\underline{\theta} + 1)(k_1 - k_2)$.

To summarize, if ULD 2 has a natural advantage and wants to be a monopolist, its optimal prices would be:

Proposition 3.15 *If ULD 2 has a natural advantage and wants to be a monopolist, its optimal price is:*

$$p_2^* = \min\left[\frac{k_1^2}{2} - t - (\underline{\theta} + 1)(k_1 - k_2), \max\left[k_2\underline{\theta} - t, \frac{\frac{k_2^2}{2} + k_2(\underline{\theta} + 1) - t}{2}\right]\right]$$

Proof. It follows from the solution of the constrained profit maximization problem of ULD 2 and the result in section 3.5.1. ■

If the optimal monopoly price is such that the surplus constraint is binding, ULD 2 may be better off by not decreasing the price so much and share the market with ULD 1. In that case, the equilibrium prices will be the ones presented in section 3.5.4.

ULD 1 has a natural advantage

ULD 1 has a natural advantage if $\overline{U}_1(\theta) > \overline{U}_2(\theta)$ for all consumers who buy a house at marginal cost pricing. In other words, $\overline{U}_1(\theta) > \overline{U}_2(\theta)$ for all $\theta \geq \max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right]$.

From section 3.5.1 we already know the optimal price charged by ULD 1 if it was an unconstrained monopoly. To be a monopolist, now ULD 1 maximizes its profit subject to constraint that the even lowest valuation consumer who buys a house, if he buys from ULD 1 gets a surplus at least as high as the surplus he would get from buying a house from ULD 2 at price $p_2 = c_2$. The surplus constraint can be written as follows:

$$\max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right] k_1 - p_1 \geq \max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right] k_2 - t - c_2 \quad \Leftrightarrow \quad p_1 \leq \max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right](k_1 - k_2) + c_2 + t$$

If this constraint is not binding when ULD 1 solves its unconstrained profit maximization problem, then its optimal price is the one given in section 3.5.1. Otherwise, to be a monopolist, ULD 1 has to charge $p_1^* = \max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right](k_1 - k_2) + c_2 + t$. To summarize:

Proposition 3.16 *If ULD 1 has a natural advantage and wants to be a monopolist, its optimal price is:*

$$p_1^* = \min\left[\max\left[\underline{\theta}, \widehat{\theta}_2(c_2)\right](k_1 - k_2) + \frac{k_2^2}{2} + t, \max\left[k_1\underline{\theta}, \frac{\frac{k_1^2}{2} + k_1(\underline{\theta}+1)}{2}\right]\right]$$

Proof. It follows from the solution of the constrained profit maximization problem of ULD 1 and the result in section 3.5.1. ■

If the optimal monopoly price is such that the surplus constraint is binding, ULD 1 may be better off by not decreasing the price so much and share the market with ULD 2. In that case, the equilibrium prices will be the ones presented in this section.

3.6 Numerical analysis

In the previous section we analyzed the Nash equilibrium of the price competition game, considering all the cases that can potentially occur. For a given case, that analysis allowed us to characterize the equilibrium prices and how they change with the quality levels of the two firms. However it does not provide a global view of how equilibrium prices change with the vector of quality levels $(k_1,\ k_2)$. In this section, we use numerical analysis to get a more global perspective of how the equilibrium prices and profits change with the quality levels $(k_1,\ k_2)$ and what are the types of equilibria that occur.

In order to do numerical analysis we used the software GAUSS. We developed a program that computes the Nash Equilibrium of the price game for given quality levels (k_1, k_2). For each vector of quality levels the program computes the equilibrium prices, the equilibrium profits and the type of equilibria.

3.6.1 Cases where the unit cost of transportation is nil

In this section we consider the case where the transportation costs are nil. In this case there is symmetry between the two firms. Our numerical simulations considered $\underline{\theta} = 2$ (we chose $\underline{\theta} > 1$ to guarantee that, even in the monopoly case, we would have cases where the

Table 3.1: Type of equilibrium for several combinations of k_1 and k_2, when $t = 0$ and $\underline{\theta} = 2$.

$k_1 \mid k_2$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	NO	M2FC	M2FC	M2FC	M2FC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	NO
0.5	M1FC	DB	M2FC	DFC	DFC	DFC	DFC	DFCK	DFCK	DFCK	DFCK	DFCK	M1FC
1	M1FC	M1FC	DB	DFC	DFC	DFC	DFC	DFC	DFC	DFCK	DFCK	DFCK	M1FC
1.5	M1FC	DFC	DFC	DB	DFC	DFC	DFC	DFC	DFC	DFCK	DFCK	M1FC	M1FC
2	M1FC	DFC	DFC	DFC	DB	DFC	DFC	DFC	DFC	DFC	DFCK	M1PC	M1FC
2.5	M1PC	DFC	DFC	DFC	DFC	DB	DFC	DFC	DFC	DFC	DFCK	M1PC	M1PC
3	M1PC	DFC	DFC	DFC	DFC	DFC	DB	DFC	DFC	DFC	M1PC	M1PC	M1PC
3.5	M1PC	DFCK	DFC	DFC	DFC	DFC	DFC	DB	DFC	M1PC	M1PC	M1PC	M1PC
4	M1PC	DFCK	DFC	DFC	DFC	DFC	DFC	DFC	DB	M1PC	M1PC	M1PC	M1PC
4.5	M1PC	DFCK	DFCK	DFCK	DFC	DFC	DFC	M2PC	M2PC	DB	M1PC	M1PC	M1PC
5	M1PC	DFCK	DFCK	DFCK	DFCK	DFCK	M2PC	M2PC	M2PC	M2PC	DB	M1PC	M1PC
5.5	M1PC	DFCK	DFCK	M2FC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	DB	M1PC
6	NO	M2FC	M2FC	M2FC	M2FC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	NO

equilibrium involves full coverage). Table 3.1 shows the type of equilibrium that occurs for every combination of (k_1, k_2), when k_1 and k_2 vary from 0 to 6, with jumps of 0.5. Note that when $k_i = 0$ or $k_i = 6$ the demand of ULD i is equal to zero (quality is too low in the first case and too high in the second case).

Table 3.1 describes seven types of equilibria. When the two ULD have zero demand (which happens when $k_i = 0$ or $k_i = 6$, with $i = 1, 2$) none of the ULD operates in the market (in the table this case is denoted by NO). When $k_1 = k_2$, excluding the cases where $k_1 = k_2 = 0$ and $k_1 = k_2 = 6$, we get the Bertrand equilibrium where the two ULD charge a price equal to marginal cost and have nil profit.

When $k_2 = 0$ or $k_2 = 6$ and $0 < k_1 < 6$, only ULD 1 operates. We obtain an equilibrium where ULD 1 operates with full coverage (M1FC) or an equilibrium where ULD 1 operates with partial coverage (M1PC), the last case occurs for higher values of k_1. Symmetrically when $k_1 = 0$ or $k_1 = 6$ and $0 < k_2 < 6$, only ULD 2 operates and we obtain either the equilibrium where ULD 2 operates with full coverage (M2FC) or, for higher values of k_2, we obtain the equilibrium where ULD 2 operates with partial coverage (M2PC).

When $k_2 > k_1 > 0$, we obtain five types of equilibria: both ULD operate with full coverage (DFC); or both ULD operate with full coverage but in a kink case, where $\hat{\theta} = \underline{\theta}$ (DFCK) which means that the lowest valuation consumer gets zero surplus; or ULD 2 operate with full coverage (M2FC); or when k_2 is too high and k_1 is also high we obtain the equilibria where ULD 1 operates with partial coverage (M1PC). Finally, for very high values of k_2 and low values of k_1, ULD1 operates with full coverage (M1FC).

Symmetrically, when $k_1 > k_2 > 0$, we obtain DFC; or DFCK; or M1FC; or M1PC when k_1 is not very high. When k_1 is very high, if k_2 is also high we obtain the equilibria where ULD 2 operate with partial coverage (M2PC) whereas for lower values of k_2 we get the equilibrium where ULD 2 operates with full coverage (M2FC).

For $\underline{\theta} = 2$ and $t = 0$, we can analyze how the equilibrium prices and equilibrium profits change with the quality levels of the two ULD. Figure 3.13 shows how the equilibrium price of ULD i changes with its own quality level, k_i, for given values of the other ULD quality (considering $k_j = 0$, $k_j = 2$ and $k_j = 3$). Based on Figure 3.13, we can conclude the following:

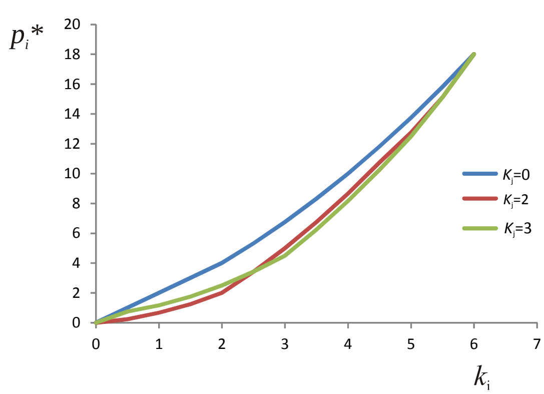

Figure 3.13: Equilibrium price of ULD$_i$, p_i^*, as a function of its quality level, k_i, given the values of k_j.

Result 3.17 *With nil transportation cost, for a given value of* k_j*, the equilibrium price of ULD* i*,* p_i^**, is an increasing and convex function of its own quality level,* k_i*.*

To explain the behavior of the equilibrium price of ULD i as a function of its own quality, there are three effects that need to be taken into account. The first effect is the impact of increasing k_i on the demand of ULD i. As its quality increases, the demand of ULD i also rises and, for a given marginal cost, it is optimal for ULD i to increase its price. The second effect is due to the fact that, as the quality of ULD i rises, its marginal cost also rises, which leads to an higher optimal price. These two effects imply a shift to the right of ULD i best response function, leading to higher equilibrium prices. The third effect is the impact of increasing k_i on the demand of ULD j. When k_i increases, the demand of ULD j decreases and thus the optimal price of firm j will be lower (the best response function of ULD j shifts downwards). The last effect alone would lead to a lower equilibrium p_i^*, since prices are strategic complements. However, the two first effects dominate the last one and, therefore, p_i^* is increasing with k_i. Moreover, the effect of increasing k_i on the marginal cost of firm i, explains the fact that equilibrium price is a convex function of k_i. This happens because the marginal costs are a quadratic function of quality and, consequently, the impact of k_i on p_i^*, through the marginal costs, becomes larger as k_i increases.

Figure 3.13 also shows the impact of changing k_j on the equilibrium price of ULD i. Note that for $k_j = 0$ (or, equivalently, for $k_j = 6$), ULD j has no demand and thus ULD i is a monopolist. We observe that the optimal prices under monopoly are above the equilibrium prices when there is competition between the two firms (which happens when $k_j = 2$ or $k_j = 3$ in the figure). Hence Figure 3.13 shows very clearly the effect of competition on the equilibrium prices.

Figure 3.14 shows the equilibrium price of ULD i, p_i^*, as a function of the quality level of the rival, k_j, (considering $k_i = 1$; $k_i = 2$ and $k_i = 3$). This figure allows us to conclude the following:

Result 3.18 *With nil transportation cost, for intermediate values of k_i, the equilibrium price of ULD i, p_i^*, is a non-monotonic function of k_j. For small values of k_j, p_i^* is decreasing with k_j, but, after a certain point, p_i^* becomes an increasing function of k_j. Finally, for very high values of k_j, p_i^* is constant. In other words, for intermediate values* *of k_i and k_j, the equilibrium price of ULD i follows a U relationship with k_j.*

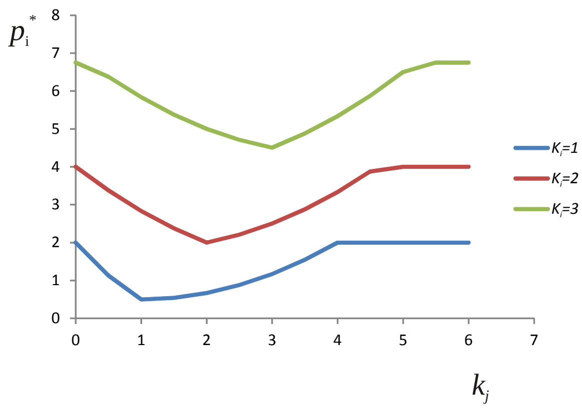

Figure 3.14: Equilibrium price of ULD$_i$, p_i^*, as a function of the quality of ULD$_j$, k_j, given the values of k_i.

How can we explain the U shaped relationship between p_i^* and k_j? When we look at the impact of k_j on the equilibrium price of ULD i, there are two effects that need to be considered. The first effect is the direct impact of k_j on the demand of ULD i. When k_j increases, for given prices, D_i decreases. This direct effect implies a lower optimal price for ULD i (the best response function of ULD i shifts to the left). This effect alone implies a lower equilibrium price for ULD i, p_i^*. However there are also indirect effect that need to be considered. When k_j increases, the demand and the marginal costs of ULD j increase, leading to an upward shift in the best response of ULD j. Since ULD j increases its price, and prices are strategic complements, in equilibrium ULD i also increases its price. Hence the two effects have opposite signs: the direct effect is negative whereas the indirect effect is positive. The previous result shows that, for small values of k_j the first effect dominates the second one and thus p_i^* decreases with k_j. However, for higher values of k_j the second effect dominates and hence p_i^* is increasing with k_j. This result is quite intuitive because, for high values of k_j, the marginal costs of ULD j increase a lot with k_j (since marginal costs are quadratic on quality), explaining why the second effect dominates.

Note that the U-shaped relationship only holds for intermediate values of k_j. As we know, when $k_j = 0$ or when k_j is too high, ULD j has no demand and therefore ULD i is a monopolist and charges its optimal price which is not a function of k_j (this explains the horizontal sections of p_i^* in figure 3.14). This figure also shows the effect of competition on the equilibrium prices, since equilibrium price is lower when k_j is intermediate and the two ULDs operate in the market.

Let us now analyze the equilibrium profits. Figure 3.15 shows the equilibrium profit of ULD i as a function of k_i, for given values of k_j. We first plot the case where ULD i has a monopoly (which happens when $k_j = 0$ or $k_j = 6$). Under monopoly, the optimal profit function is a concave function of k_i: the profit starts growing with quality, it reaches a maximum, and after that profit declines with quality till it becomes nil. Figure 3.16, shows the equilibrium profit of ULD i as a function of k_i but now considering intermediate values of k_j ($k_j = 2$ and $k_j = 3$). As it can be seen, the shape of the equilibrium profit is quite different from the one observed under monopoly.

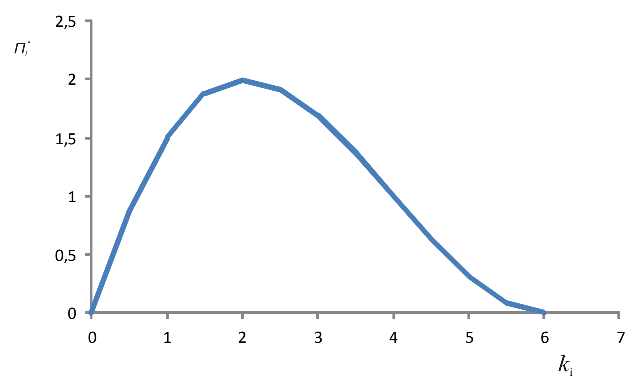

Figure 3.15: Equilibrium profit of ULD$_i$, Π_i^*, as a function of its quality, k_i, when $k_j = 0$ or $k_j = 6$.

The results in Figures 3.15 and 3.16 can be summarized as follows:

Result 3.19 *With nil transportation cost, when k_j is nil or when k_j is too high, ULD i is a monopoly and the corresponding optimal profit is a concave function of its own quality level, k_i (with a unique global maximum). For intermediate values of k_j, both ULDs operate and, for given k_j, the equilibrium profit of ULD i has two local maxima: one with $k_i < k_j$ and the other one with $k_i > k_j$.*

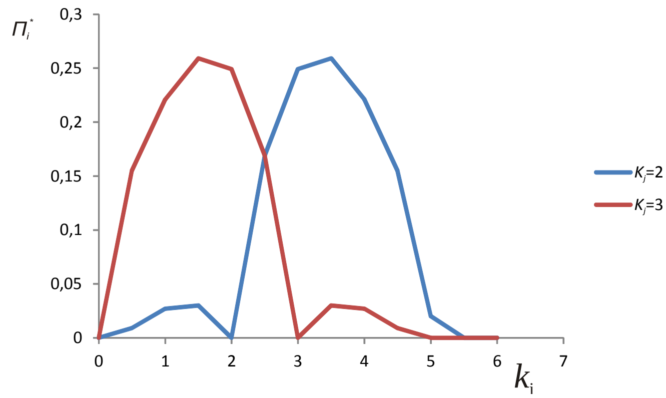

Figure 3.16: Equilibrium profit of ULD$_i$, Π_i^*, as a function of its quality, k_i, when $k_j = 2$ and when $k_j = 3$.

The shape of the equilibrium profit under duopoly shows the benefits of differentiation. For a given quality of ULD j, k_j, ULD i is always better off if he chooses a quality level different from the rival's one (choosing the same quality implies zero profit). But ULD i can differentiate either with a lower quality or with a higher quality than the rival quality. Figure 3.16 suggests that, when k_j is low, ULD i is better off if he differentiates by choosing a higher quality. However, when k_j is high, ULD i is better off by differentiating by choosing a lower quality. This Figure also suggests that the best response functions in the first stage of the game (when ULDs choose their housing qualities), are discontinuous (till a certain point it is better to choose a quality lower than the rival, after that point it is better to chose a quality higher that the rival). This intuition will be important in the next chapter.

3.6.2 Cases with positive unit transportation costs

In this subsection we analyze what happens when the unit transportation costs are positive. In this case, the two ULDs are no longer in a symmetric position and hence it is important to describe what happens in equilibrium for each of the ULDs. We analyze the different equilibria, when $t = 0.5$ and $\underline{\theta} = 2$ (the value chosen for the unit transportation costs is relatively low so as to guarantee that we have cases where both ULDs operate).

Table 3.2: Type of equilibrium for several combinations of k_1 and k_2, when $t = 0.5$ and $\underline{\theta} = 2$.

$k_1 \| k_2$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
0	NO	M2PC	M2FC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	NO
0.5	M1FC	M1FC	DFC	DFC	DFC	DFC	DFC	DFCK	DFCK	DFCK	DFCK	DFCK	M1FC
1	M1FC	M1FC	M1FC	DFC	DFC	DFC	DFC	DFC	DFCK	DFCK	DFCK	M1FC	M1FC
1.5	M1FC	M1FC	M1FC	M1FC	DFC	DFC	DFC	DFC	DFC	DFCK	DFCK	M1FC	M1FC
2	M1FC	M1FC	DFC	M1FC	M1FC	DFC	DFC	DFC	DFC	DFCK	DFCK	M1PC	M1FC
2.5	M1PC	DFC	DFC	DFC	DFC	M1FC	DFC	DFC	DFC	DFC	M1PC	M1PC	M1PC
3	M1PC	DFCK	DFC	DFC	DFC	DFC	M1FC	DFCK	DFC	M1PC	M1PC	M1PC	M1PC
3.5	M1PC	DFCK	DFC	DFC	DFC	DFC	DFC	M1FC	DFCK	M1PC	M1PC	M1PC	M1PC
4	M1PC	DFCK	DFCK	DFC	DFC	DFC	DFC	DFC	M1PC	M1PC	M1PC	M1PC	M1PC
4.5	M1PC	DFCK	DFCK	DFCK	DFCK	DFCK	DFCK	DPC	DPC	M1PC	M1PC	M1PC	M1PC
5	M1PC	DFCK	DFCK	DFCK	DFCK	DFCK	M2PC	M2PC	M2PC	M2PC	M1PC	M1PC	M1PC
5.5	M1PC	DPC	DFCK	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M1PC	M1PC
6	NO	M2PC	M2FC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	M2PC	NO

Table 3.2 shows the type of equilibrium for the various combinations of k_1 and k_2, when k_1 and k_2 vary from 0 to 6 with jumps of 0.5.

As we can see in table 3.2, we have six types of equilibria. Like before, when $k_i = 0$ or $k_i = 6$ and $k_j = 0$ or $k_j = 6$ both ULDs have no demand and thus none of them operates in the market (this case is denoted by NO in the table). However when $0 < k_1 = k_2 < 6$, we get a quite different type of equilibrium than the one with nil transportation costs. In these cases, the demand function is discontinuous (like when $t = 0$) but now ULD 1 has an advantage because consumers have to incur transportation costs if they buy a house from ULD 2. Therefore we obtain equilibria where only ULD 1 operates, with full coverage (M1FC) for lower qualities and with partial coverage (M1PC) for higher qualities.

When $k_2 = 0$ or $k_2 = 6$ and $0 < k_1 < 6$, for low values of k_1 we obtain an equilibrium where ULD 1 operates with full coverage(M1FC) whereas for high values of k_1 only ULD 1 operates but with partial coverage (M1PC). Symmetrically when $k_1 = 0$ or $k_1 = 6$ and $0 < k_2 < 6$, we obtain an equilibrium where ULD 2 operates with partial coverage (M2PC) except for $k_2 = 1$ where also only ULD 2 operates but with full coverage.

When $k_2 > k_1 > 0$, we obtain four types of equilibria: both ULD operate with full coverage (DFC); or both ULD operate with full coverage but, $\hat{\theta} = \underline{\theta}$, in a kink case (DFCK); or for high values of k_2 and small values of k_1 only ULD 1 operates with full coverage (M1FC); for high values of k_2 and k_1 we obtain a equilibria where only ULD 1 operates but with partial coverage (M1PC).

When $k_1 > k_2 > 0$, we obtain the following types of equilibrium: for low values of k_1 we obtain an equilibrium where only ULD 1 operates with full coverage (M1FC); for intermediate values of k_1, we obtain an equilibrium where both ULD operate with full coverage (DFC) or the same but in a kink case (DFCK); or both operate with partial coverage (DPC); or when k_1 is very high we obtain the equilibrium where ULD 2 operate with partial coverage (M2PC).

Let us now analyze the equilibrium prices and profits for each ULD, for different quality levels. Figure 3.17 shows p_1^* as a function of k_1(on the left) and p_2^* as a function of k_2(on the right) for given values of k_2 and k_1, respectively.

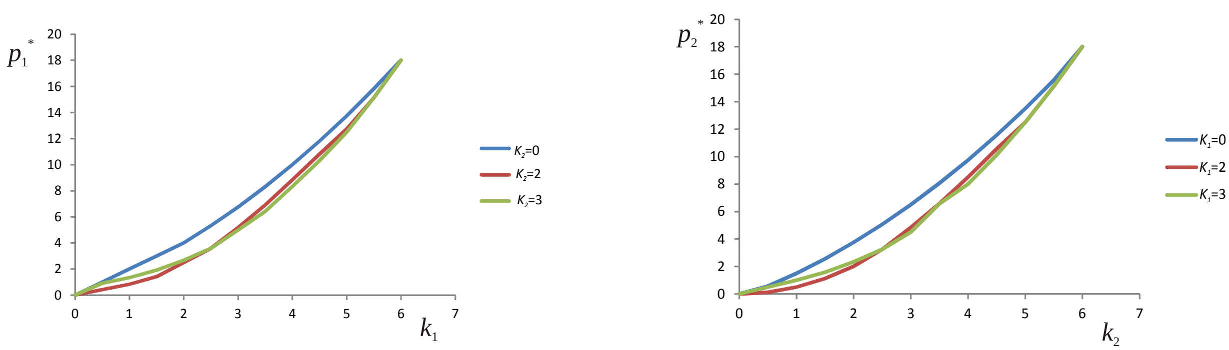

Figure 3.17: Equilibrium price of ULD 1 and ULD 2 as a function of their quality levels

The behavior of the equilibrium prices as a function of the ULD's quality is very similar to the one observed with nil transportation costs. For $k_2 = 0$, $k_2 = 2$ and $k_2 = 3$, as we can see, p_1 is an increasing and convex function of k_1. Similarly, for given values of k_1, p_2 is an increasing and convex function of k_2.

Result 3.20 *With positive unit transportation costs, for given the values of k_j, the equilibrium price of ULD i, p_i^*, is an increasing and convex function of its own quality, k_i.*

The intuition for this result is the same than in the case where $t = 0$. An increase in k_i affects positively the demand of ULD i and negatively the demand of ULD j, for given prices. Moreover increasing k_i leads to an increase in ULD i marginal costs. The positive impact on D_i and on the marginal cost lead to an higher p_i^* whereas the negative impact on D_j tends to decrease p_i^*, but the first two effects outweigh the last one. The convex shape of p_i^* can be explained by the convex shape of the marginal costs, which are a quadratic function of quality.

Figure 3.18 represents, on the left side, the values p_1^* as a function of k_2 (considering $k_1 = 2$; $k_1 = 4$ and $k_1 = 5$); on the right side, it shows the values of p_2^* as a function of k_1 (considering $k_2 = 2$; $k_2 = 4$ and $k_2 = 5$). Note that the equilibrium price of ULD 2 are slightly lower than the equilibrium prices of ULD 1. However, the behavior of the equilibrium prices as a function of the quality level of the rival ULD is very similar to the one observed when $t = 0$.

Result 3.21 *With positive unit transportation cost, for intermediate values of k_i, the equilibrium price of ULD i, p_i^*, is a non-monotonic function of k_j. For small values of k_j, p_i^* is decreasing with k_j but, after a certain point, p_i^* becomes an increasing function of k_j. Finally, for very high values of k_j, p_i^* is constant. In other words, for intermediate values of k_i and k_j, the equilibrium price of ULD i follows a U relationship with k_j.*

The explanation for this result is the same than when $t = 0$. When k_j increases, the demand of ULD i decreases, leading to lower optimal price for ULD i (the best response function of ULD i shifts to the left). However, when k_j increases, the demand and the marginal costs of ULD j increase, leading to an upward shift in the best response of ULD j. Since ULD j increases its price, and prices are strategic complements, in equilibrium ULD i also increases its price. Hence the two effects have opposite signs: the direct effect is negative whereas the indirect effect is positive. For small values of k_j the first effect dominates the second one (thus p_i^* decreases with k_j) while, for higher values of k_j, the second effect dominates (hence p_i^* is increasing with k_j).

As it can be seen in Figure 3.18, in the three curves of the two graphics, when the quality of the rival ULD is nil or when it is too high, ULD i is a monopolist and its optimal price does not depend on k_j. For intermediate values of k_j there is a duopoly and the price is below the monopoly price (due to the competition effect).

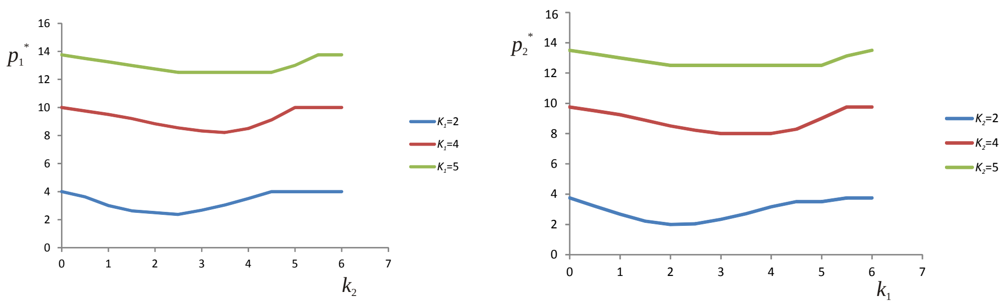

Figure 3.18: Equilibrium price of ULD 1 and ULD 2 as a function of the housing quality of ULD 2 and ULD 1, respectively.

The previous results show that, the existence of positive unit transportation costs does not change the overall pattern of the equilibrium prices. Nevertheless, it should be highlighted that the equilibrium prices are not symmetric when $t > 0$. However the biggest difference in terms of results is the one regarding the equilibrium profits. When the quality is too low or too high, we have a monopoly, and the shape of the profit is concave like the case of nil cost of transportation, as we can see in Figure 3.19. But when we have a duopoly, the result is very different, has we can see in the Figure 3.19 that represents the equilibrium profit of ULD 1, Π_1^*, as a function of k_1, considering $k_2 = 1$, $k_2 = 2.5$ and $k_2 = 4$ (on the left). In the same Figure, on the right, we have the equilibrium profit of ULD 2, Π_2^*, as a function of k_2. Those figures allows us to conclude the following:

Result 3.22 *With positive unit transportation cost, when k_j is nil or when k_j is too high, ULD i is a monopoly and the corresponding profit is a concave function of its own quality level, k_i. For low values of k_j, the profit of ULD i quasi-concave function of its own quality and the optimal profit occurs for $k_i > k_j$. For high values of k_j, the profit of ULD i quasi-concave function of its own quality and the optimal profit occurs for $k_i < k_j$. For* *intermediate values of k_2, the equilibrium profit of ULD 1 has a local maximum when $k_1 = k_2$. On the other hand, for intermediate values of k_1, the equilibrium profit of ULD 2 has two local maxima: one with $k_2 < k_1$ and the other one with $k_2 > k_1$.*

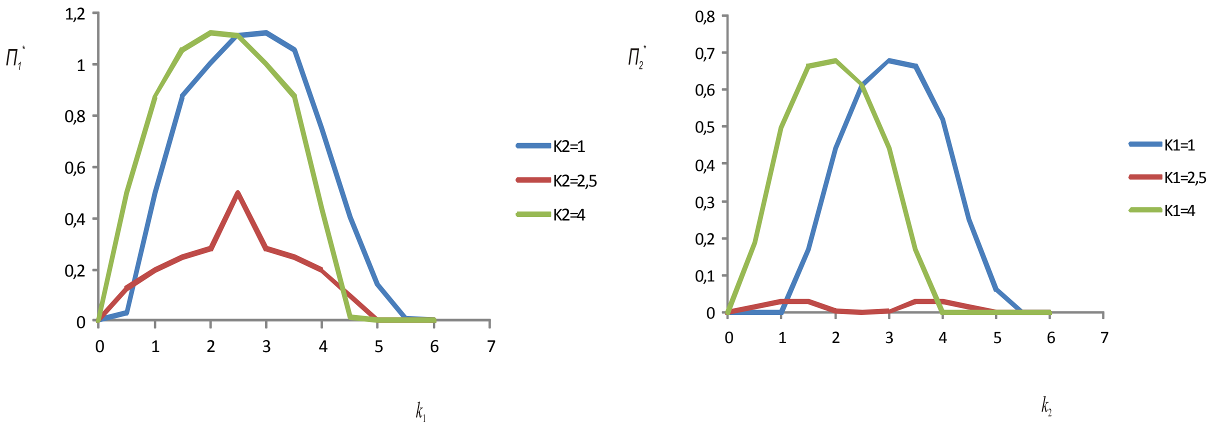

Figure 3.19: Equilibrium profit of ULD_1 and ULD_2 , Π_1^* and Π_2^*, as a function of its quality, $k_1$and k_2 , when the quality of the other ULD is equal to $1, 2.5$ and 4.

The equilibrium profit when only one of the ULD operates (because the other ULD has a nil quality or a too high quality) has a similar shape to the one observed when $t = 0$ (but the monopoly profit of ULD 2 is lower due to the transportation cost). However, for intermediate values of the rival ULD's quality, the behavior of the equilibrium profit is very different for ULD 1 and ULD 2. While ULD 2 gets an higher equilibrium profit when its quality is different from k_1, ULD 1 equilibrium profit may be higher when its quality is precisely the same than the quality of ULD 2. The explanation is that when the unit cost of transportation is different from zero, ULD 1 has an advantage when qualities are the same and is able to capture all the demand by charging $p_1 \leq c_2 + t$ (ULD 1 is a constrained monopolist). This result is important for the next chapter as it suggests that we may have a problem in finding an equilibrium in the first stage of the game.

3.7 Conclusion

In this chapter we analyzed the second stage of a two stage game between two urban land developers, in which one of the ULD is located at the CBD while the other is located in the periphery. In the first stage the two ULD simultaneously choose the quality of housing and, in the second stage, they simultaneously choose prices. This chapter solved the second stage price competition game, considering the qualities chosen in the first stage as given.

The chapter started with a literature review on vertical differentiation models, that allowed us to recognize the basic features of those models and to construct our model according the specificities of the urban land developers. Since the location of the house is an important characteristic, our model incorporates unit cost of transportation when a house is bought from the ULD located in the periphery. In addition we consider fixed and variable costs of quality improvement.

To solve the model, we started by deriving the demand functions of each ULD and by imposing conditions on the quality levels of the two ULDs so that their demand is positive. In addition we defined some cut-off valuations which enabled us to simplify the exposition. Next, we found analytically the Nash equilibrium for different quality vectors. Our analysis is very complete as we explore all the possible cases both in terms of who operates in the market (in some cases both operate while in others only one of the urban land developers operates) as well as in terms of the market coverage (in some cases the whole market is covered while in others there is only partial market coverage). For this reason, this chapter is an important contribution to the quality differentiation literature. Furthermore the chapter characterizes the Nash equilibrium (equilibrium type, equilibrium prices and equilibrium profits) for the different quality levels.

The results show that with nil transportation costs, the equilibrium price of a urban land developer is increasing with its housing quality, for given values of the quality of the other urban land developer. This result is also valid with positive unit cost of transportation. On the other hand, the equilibrium price of a urban land developer is a non-monotonic function of the quality of the rival ULD. In particular, for intermediate values of the other ULD quality, there is a U shaped relationship between the equilibrium price of a ULD and the housing quality of the other ULD. This result is also valid with positive transportation costs.

When the quality of an ULD is nil or very high, this ULD has zero demand and the other ULD is a monopolist. In this case the monopolist ULD optimal profit is a concave function of its housing quality: the equilibrium profit first grows with quality, up to a maximum, and then falls and becomes equal to zero. This result is also valid with positive unit cost of transportation. However when the two ULD have intermediate levels of quality and the unit transportation cost is nil, the equilibrium profit functions have two local maxima (one where the ULD chooses a quality lower than the rival, the other one where the ULD chooses a quality higher than the rival). Therefore, the equilibrium profit functions show the benefits of differentiating the quality. Moreover the result shows that, when the other ULD has a low quality it is better to differentiate by choosing a higher quality, whereas when the rival has a high quality it is better to differentiate by choosing a lower quality.

On the other hand, with positive unit transportation costs and intermediate quality levels, the ULD located at the periphery prefers to differentiate but the ULD located at the CBD may be better off by choosing a quality level equal to the rival's one and «exploiting» its locational advantage.

Appendix A – Demand functions

Case where $k_1 = k_2$

We start by analyzing the case when $k_1 = k_2 = k$. In this case the two urban land developers are offering precisely the same quality, thus there is no differentiation and the consumers always prefer the ULD that has a lower total price. For instance, the consumers strictly prefer ULD 1 if:

$$\theta k - p_1 > \theta k - t - p_2 \quad \Leftrightarrow \quad p_1 < p_2 + t$$

Similarly, if $p_2 < p_1 - t$ all consumers prefer ULD 2. Finally, when $p_1 = p_2 + t$ the consumers will be indifferent between buying from ULD 1 or ULD 2. In this case we assume that demand is equally divided among the two ULD. Consequently, demand is discontinuous. If $p_1 \leq \underline{\theta} k$ all consumers prefer to buy from ULD 1 than not to buy, hence the market is fully covered and the demand of ULD 1 is:

$$D_1(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + t \\ \frac{1}{2} & \text{if } p_1 = p_2 + t \\ 1 & \text{if } p_1 < p_2 + t \end{cases}$$

If $\underline{\theta} k < p_1 \leq (\underline{\theta} + 1) k$ the market is only partially covered when ULD 1 operates and demand of ULD 1 is:

$$D_1(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + t \\ \frac{1}{2}\left(\underline{\theta} + 1 - \frac{p_1}{k}\right) & \text{if } p_1 = p_2 + t \\ \underline{\theta} + 1 - \frac{p_1}{k} & \text{if } p_1 < p_2 + t \end{cases}$$

The demand function of firm 2 can be derived in a similar manner. If $p_2 \leq \underline{\theta}k - t$

$$D_2(p_1,p_2) = \begin{cases} 0 & \text{if } p_2 > p_1 - t \\ \frac{1}{2} & \text{if } p_2 = p_1 - t \\ 1 & \text{if } p_2 < p_1 - t \end{cases}$$

If $\underline{\theta}k - t < p_2 \leq (\underline{\theta}+1)\,k - t$ the demand of ULD 2 is:

$$D_2(p_1,p_2) = \begin{cases} 0 & \text{if } p_2 > p_1 - t \\ \frac{1}{2}\left(\underline{\theta}+1-\frac{p_2+t}{k}\right) & \text{if } p_2 = p_1 - t \\ \underline{\theta}+1-\frac{p_2+t}{k} & \text{if } p_2 < p_1 - t \end{cases}$$

Case where $k_2 > k_1$

If ULD 2 offers a higher quality house than ULD 1, the demand functions depend on the price differential, $p_2 - p_1$. If $p_2 - p_1 > (\underline{\theta}+1)(k_2 - k_1) - t$ all consumers prefer to buy from ULD 1 than from ULD 2. Depending on its price the ULD 1 may get the whole demand (if $p_1 \leq \underline{\theta}k_1$) or cover the market only partially (if $\underline{\theta}k_1 < p_1 \leq (\underline{\theta}+1)\,k_1$). Figure 3.20 illustrates these two cases.

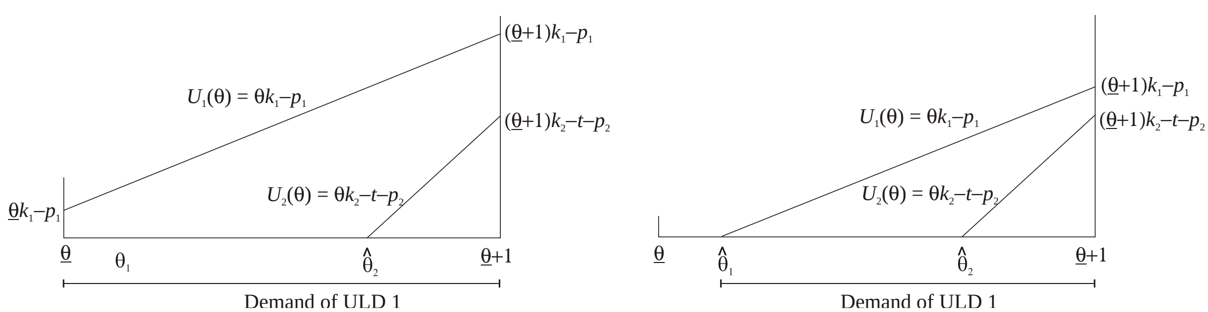

Figure 3.20: If the price differential $p_2 - p_1$ is high, ULD 2 gets no demand. Depending on p_1, ULD 1 may cover the whole market (left) or not (right).

Thus when $p_2 - p_1 > (\underline{\theta} + 1)(k_2 - k_1) - t$ the demand functions are:

$$D_1(p_1,p_2) = \begin{cases} 0 & \text{if } p_1 > (\underline{\theta}+1)\,k_1 \\ \underline{\theta} + 1 - \frac{p_1}{k_1} & \text{if } \underline{\theta}k_1 < p_1 \leq (\underline{\theta}+1)\,k_1 \\ 1 & \text{if } p_1 < \underline{\theta}k_1 \end{cases} \quad \text{and} \quad D_2(p_1,p_2) = 0$$

The opposite happens when the price differential is very low. If $p_2 - p_1 < \max\left(\underline{\theta}, \widehat{\theta}_1\right)(k_2 - k_1) - t$, ULD 1 has zero demand whereas ULD 2 covers the whole market (if $p_2 \leq \underline{\theta}k_2 - t$) or covers the market partially (if $\underline{\theta}k_2 - t < p_2 \leq (\underline{\theta}+1)\,k_2 - t$). Figure 3.21 illustrates these two cases.

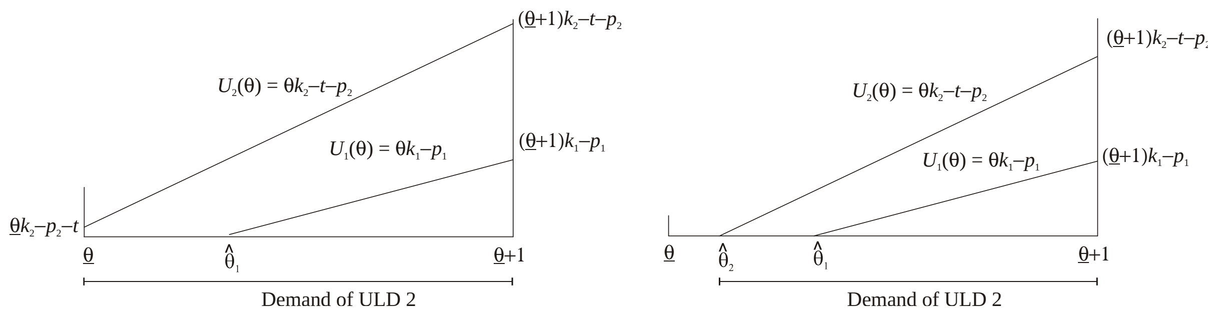

Figure 3.21: If the price differential $p_2 - p_1$ is low, ULD 1 gets no demand. Depending on p_2, ULD 2 may cover the whole market (left) or not (right).

Thus when $p_2 - p_1 < \max\left(\underline{\theta}, \widehat{\theta}_1\right)(k_2 - k_1) - t$ the demand functions are:

$$D_1(p_1,p_2) = 0 \quad \text{and} \quad D_2(p_1,p_2) = \begin{cases} 0 & \text{if } p_2 > (\underline{\theta}+1)\,k_2 - t \\ \underline{\theta} + 1 - \frac{p_2+t}{k_2} & \text{if } \underline{\theta}k_2 - t < p_2 \leq (\underline{\theta}+1)\,k_2 - t \\ 1 & \text{if } p_2 < \underline{\theta}k_2 - t \end{cases}$$

For price differentials between the two previous limits, both urban land developers operate. Figure 3.22 illustrates this case when there is full coverage (left) and when there is partial coverage (right). Note that the higher quality urban land developer, ULD 2, covers the higher valuation consumers while ULD 1 covers the lower valuation consumers. If p_1 is relatively high, the lower valuation consumers prefer not to buy. Thus when

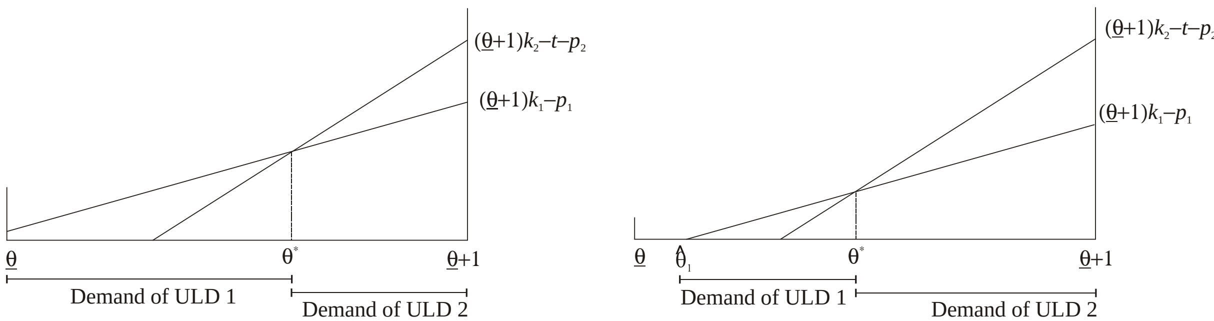

Figure 3.22: For intermediate price differentials, both firms operate either with full coverage (left) or with partial coverage (right). ULD 2 covers higher valuation consumers while ULD 1 covers lower valuation consumers.

$\max\left(\underline{\theta}, \widehat{\theta}_1\right)(k_2 - k_1) - t \; < p_2 - p_1 \leq (\underline{\theta} + 1)(k_2 - k_1) - t$ the demand function are:

$$D_1(p_1, p_2) = \begin{cases} 0 & \text{if } \; p_1 > (\underline{\theta} + 1)\, k_1 \\ \frac{p_2 - p_1 + t}{k_2 - k_1} - \frac{p_1}{k_1} & \text{if } \underline{\theta} k_1 < p_1 \leq (\underline{\theta} + 1)\, k_1 \\ \frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta} & \text{if } p_1 < \underline{\theta} k_1 \end{cases}$$

and

$$D_2(p_1, p_2) = \begin{cases} 0 & \text{if } \; p_2 > (\underline{\theta} + 1)\, k_2 - t \\ \underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1} & \text{if } p_2 \leq (\underline{\theta} + 1)\, k_2 - t \end{cases}$$

Case where $k_1 > k_2$

Let us now consider the case where ULD 1 offers a house of higher quality than ULD 2. If $p_1 - p_2 > (\underline{\theta} + 1)(k_1 - k_2) + t$ all consumers prefer to buy from ULD 2 than from ULD 1. Depending on its price ULD 2 may get the whole demand (if $p_2 \leq \underline{\theta} k_2 - t$) or cover the market only partially (if $\underline{\theta} k_2 - t < p_2 \leq (\underline{\theta} + 1)\, k_2 - t$). Figure 3.23 illustrates these two cases.

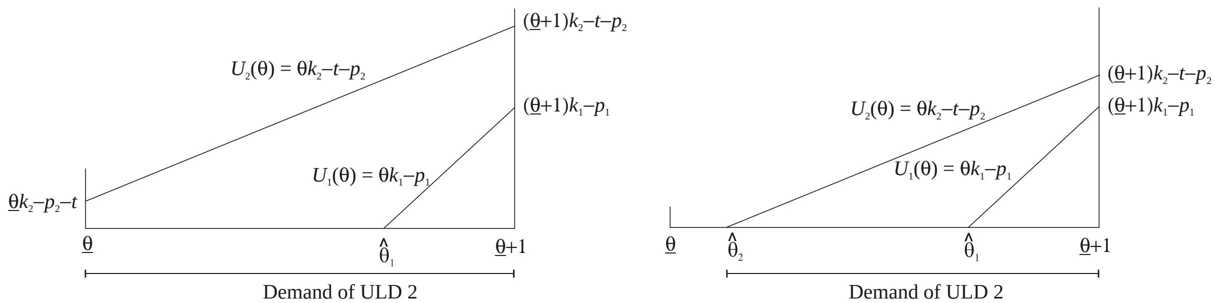

Figure 3.23: If the price differential $p_1 - p_2$ is high, ULD 1 gets no demand. Depending on p_2, ULD 2 may cover the whole market (left) or not (right).

Thus when $p_1 - p_2 > (\underline{\theta} + 1)(k_1 - k_2) + t$ the demand functions are:

$$D_1(p_1, p_2) = 0 \qquad \text{and} \qquad D_2(p_1, p_2) = \begin{cases} 0 & \text{if } p_2 > (\underline{\theta} + 1)\, k_2 - t \\ \underline{\theta} + 1 - \frac{p_2 + t}{k_2} & \text{if } \underline{\theta} k_2 - t < p_2 \leq (\underline{\theta} + 1)\, k_2 - t \\ 1 & \text{if } p_2 < \underline{\theta} k_2 - t \end{cases}$$

The opposite happens when the price differential is very low. If $p_1 - p_2 < \max\left(\underline{\theta}, \widehat{\theta}_2\right)(k_1 - k_2) + t$, ULD 2 has zero demand whereas ULD 1 covers the whole market (if $p_1 \leq \underline{\theta} k_1$) or covers the market partially (if $\underline{\theta} k_1 < p_1 \leq (\underline{\theta} + 1)\, k_1$. Figure 3.24 illustrates these two cases.

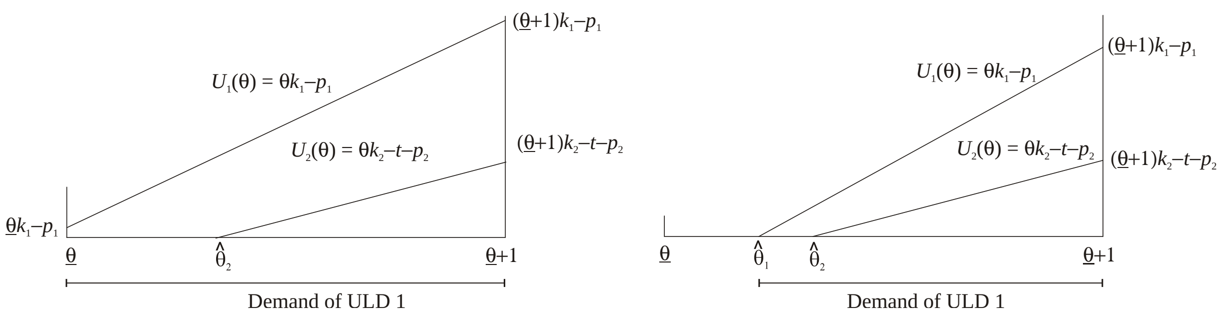

Figure 3.24: If the price differential $p_1 - p_2$ is low, ULD 2 gets no demand. Depending on p_1, ULD 1 may cover the whole market (left) or not (right).

Thus when $p_1 - p_2 < \max\left(\underline{\theta}, \widehat{\theta}_2\right)(k_1 - k_2) + t$ the demand functions are:

$$D_1(p_1,p_2) = \begin{cases} 0 & \text{if } \; p_1 > (\underline{\theta}+1)\,k_1 \\ \underline{\theta}+1-\frac{p_1}{k_1} & \text{if } \underline{\theta}k_1 < p_1 \leq (\underline{\theta}+1)\,k_1 \\ 1 & \text{if } p_1 < \underline{\theta}k_1 \end{cases} \quad \text{and} \quad D_2(p_1,p_2) = 0$$

For price differentials between the two previous limits, both urban land developers operate. Figure 3.24 illustrates this case, both when the market is whole covered (left) and when the market is not fully covered (right). Note that in both cases, ULD 1 serves the consumers who value most quality.

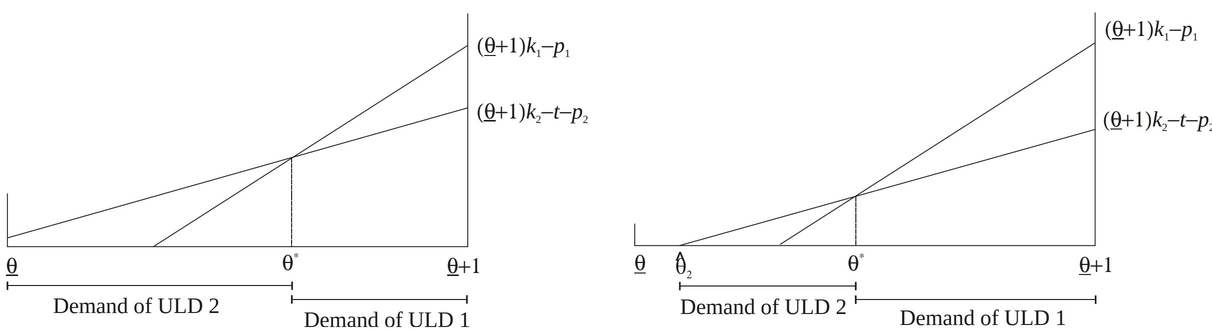

Figure 3.25: For intermediate price differentials, both firms operate either with full coverage (left) or with partial coverage (right). ULD 1 covers higher valuation consumers while ULD 2 covers lower valuation consumers.

Thus when $\max\left(\underline{\theta}, \widehat{\theta}_2\right)(k_1 - k_2) + t < p_1 - p_2 \leq (\underline{\theta}+1)(k_1 - k_2) + t$ the demand function are:

$$D_1(p_1,p_2) = \begin{cases} 0 & \text{if } \; p_1 > (\underline{\theta}+1)\,k_1 \\ \underline{\theta}+1-\frac{p_1-p_2-t}{k_1-k_2} & \text{if } p_1 \leq (\underline{\theta}+1)\,k_1 \end{cases}$$

and

$$D_2(p_1,p_2) = \begin{cases} 0 & \text{if } \; p_2 > (\underline{\theta}+1)\,k_2 - t \\ \frac{p_1-p_2-t}{k_1-k_2} - \frac{p_2+t}{k_2} & \text{if } \underline{\theta}k_2 - t < p_2 \leq (\underline{\theta}+1)\,k_2 - t \\ \frac{p_1-p_2-t}{k_1-k_2} - \underline{\theta} & \text{if } p_2 < \underline{\theta}k_2 - t \end{cases}$$

Appendix B – Gauss Program

```
/*****************************************************************/
/* This program computes the NE of the quality-price game for given quality levels */
/* (k1,k2). For each (k1,k2) the equilibrium prices, profit and type of equilibria is */
/* computed for each firm. */
/*****************************************************************/
/******* Parameters of the model **********************/
tetab=2;
t=0.5;
saltok=0.5; /* step size for the iterations on the quality levels*/
format /rdt 6,3;                    /* print number formatation */
tol=10^(-12);
/*****************************************************************/
/**Finding the second stage NE for various levels of (k1,k2) and saving ***********/
/******** the NE variables of each firm in a matrix ************/
/*****************************************************************/
k1min=0;
k2min=0;
k1max=2*(tetab+1);
k2max=2*(tetab+1);
niterk1=int((k1max-k1min)/saltok)+1; /* number of iterations for quality level of firm 1 */
niterk2=int((k2max-k2min)/saltok)+1; /* number of iterations for quality level of firm 2 */
pi1mat=ones(niterk1,niterk2); /* create matrix to save the NE profit of firm 1 */
pi1mat=pi1mat*(-5);
pi2mat=ones(niterk1,niterk2); /* create matrix to save the NE profit of firm 2 */
pi2mat=pi2mat*(-5);
typemat=zeros(niterk1,niterk2);
p1mat=ones(niterk1,niterk2); /* create matrix to save the NE prices of firm 1 */
p1mat=p1mat*(-5);
p2mat=ones(niterk1,niterk2); /* create matrix to save the NE prices of firm 1 */
p2mat=p2mat*(-5);
k2mat=zeros(1,niterk2);
k1mat=zeros(niterk1,1);
k1=k1min;
iterk1=1;
do while k1<= k1max;
c1=(k1^2)/2; /* marginal production costs as a function of k1 */
fcost1=0.001;
k2=k2min;
iterk2=1;
pi1eq=-5;
pi2eq=-5;
```

```
do while k2<= k2max;
c2=(k2^2)/2; /* marginal production costs as a function of k2 */
fcost2=0.001;
/******** Condition for a firn to have zero demand if price is ************/
/******** greater or equal than marginal cost ************/
k2dmin= (tetab+1)-sqrt((tetab+1)^2-2*t);
k2dmax=(tetab+1)+sqrt((tetab+1)^2-2*t);
k1dmax=2*(tetab+1);
k1dmin=0;
/**************************************************************/
/****** Cases where both firms have zero demand ***************************/
/**************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 le k1dmin or k1 ge k1dmax); /* both firms have
zero demand */
p1mat[iterk1,iterk2]=c1;
p2mat[iterk1,iterk2]=c2;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
if k2 eq 0;
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
typemat[iterk1,iterk2]=1;
goto nefoundc;
endif;
/**********************************************************************/
/****** Cases where only firm 2 has zero demand *********************/
/**********************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 gt k1dmin and k1 lt k1dmax); /* only firm 1
operates */
p2mat[iterk1,iterk2]=c2;
if k2 eq 0;
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
p1eq=(c1+k1*(tetab+1))/2|tetab*k1;
p1eq=maxc(p1eq);
tetahat1=p1eq/k1;
```

```
/*** check if firm 1 operates and has full coverage **/
if (p1eq eq(tetab*k1))and (p1eq ge c1);
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=2;
goto nefoundc;
endif;
/*** check if firm 1 operates and does not have full coverage **/
if (p1eq eq ((c1+k1*(tetab+1))/2)) and (tetahat1 le (tetab+1)) and (p1eq ge c1); /*check
if we are in this case */
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=3;
goto nefoundc;
endif;
endif;
/*********************************************************************/
/****** Cases where only firm 1 has zero demand **********************/
/*********************************************************************/
if (k1 le k1dmin or k1 ge k1dmax) and (k2 gt k2dmin and k2 lt k2dmax); /* only firm 2
operates */
p1mat[iterk1,iterk2]=c1;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
p2eq=(c2+k2*(tetab+1)-t)/2|k2*tetab-t;
p2eq=maxc(p2eq);
/*** check if firm 2 operates and has full coverage **/
if p2eq eq (k2*tetab-t) and p2eq ge c2;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=4;
goto nefoundc;
endif;
/*** check if firm 2 operates and does not have full coverage **/
tetahat2=(p2eq+t)/k2;
if p2eq eq (c2+k2*(tetab+1)-t)/2 and tetahat2 le (tetab+1) and p2eq ge c2; /*check if we
are in this case */
```

```
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=5;
goto nefoundc;
endif;
endif;
/*********************************************************************/
/****** Cases when k2 = k1 (but with positive demand) *****************/
/*********************************************************************/
if k2 eq k1;
if t eq 0;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
typemat[iterk1,iterk2]=6;
else;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
/**** check if there is partial coverage *******************/
p1eq=(c1+k1*(tetab+1))/2;
p1eqc=p1eq|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 gt tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi1eq=(tetab+1-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=7;
goto nefoundc;
endif;
p1eqc=(k1*tetab)|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 le tetab and p1eq ge c1;
pi1eq=(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=8;
goto nefoundc;
endif;
endif;
```

```
endif;
/*******************************************************************/
/****** Cases when k2 > k1 *****************************************/
/*******************************************************************/
if k2 gt k1;
/**** check if NE has full coverage and both firms operate ******/
p1eq=((1-tetab)*(k2-k1)+2*c1+c2+t)/3;
p2eq=((tetab+2)*(k2-k1)+2*c2+c1-t)/3;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
if tetahat1 le tetab and tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq
ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=9;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(k1*(tetab+1)*(k2-k1)+2*c1*k2+c2*k1+t*k1)/(4*k2-k1);
p2eq=(2*k2*(tetab+1)*(k2-k1)+c1*k2+2*c2*k2-t*(2*k2-k1))/(4*k2-k1);
tetahat1=p1eq/k1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetahat1 gt tetab and tetahat1 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and
p2eq ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-(p1eq/k1))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=10;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/**** check if NE has full coverage and both firms operate but in a kink case******/
p1eq=tetab*k1;
p2eq=((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
```

```
if tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=101;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((k2*c1/k1)-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetastar le tetahat2 and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=11;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
p2eq=p2eq|((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
p2eq=maxc(p2eq);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
if tetastar le tetab and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in this case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
```

```
typemat[iterk1,iterk2]=12;
goto nefoundc;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in this
case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=121;
goto nefoundc;
endif;
/**** check if NE only firm 2 operates and has full coverage but in a kink case******/
p1eq=c1;
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
tetahat2=(p2eq+t)/k2;
if tetastar eq tetab and tetahat2 le tetab and p2eq ge c2; /*check if, given prices, we are in
this case */
pi1eq=-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=122;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
p1eqc=((c1+k1*(tetab+1))/2)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
```

```
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=13;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=131;
goto nefoundc;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat1-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat1 le tetab and p1eq ge c1; /*check if, given prices, we
are in this case */
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=14;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=141;
goto nefoundc;
endif;
```

```
if tetadif1 le tol and tetastar ge (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=142;
goto nefoundc;
endif;
endif;
/**********************************************************************/
/****** Cases when k1 > k2 ********************************************/
/**********************************************************************/
/**** check if NE has full coverage and both firms operate ******/
if k1 gt k2;
p1eq=((tetab+2)*(k1-k2)+2*c1+c2+t)/3;
p2eq=((1-tetab)*(k1-k2)+2*c2+c1-t)/3;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetahat2 le tetab and tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq ge c2; /*check if, given prices, we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=15;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(2*k1*(tetab+1)*(k1-k2)+k1*(2*c1+c2+t))/(4*k1-k2);
p2eq=(k2*(tetab+1)*(k1-k2)+2*k1*c2+k2*c1-t*(2*k1-k2))/(4*k1-k2);
tetahat2=(p2eq+t)/k2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
if tetahat2 ge tetab and tetahat2 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if, given prices, we are in this case */
pi2eq=(tetastar-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-tetastar)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
```

```
typemat[iterk1,iterk2]=16;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/**** check if NE has full coverage and both firms operate but we are in a kink case ******/
p2eq=tetab*k2-t;
p1eq=((tetab+1)*(k1-k2)+c1+p2eq+t)/2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq ge c2; /*check if,
given prices, we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=161;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
p1eqc=((c1+k1*(tetab+1))/2)|(k1*(c2+t)/k2);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetahat1;
tetadif=abs(tetadif);
if tetastar le tetahat1 and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=17;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=171;
```

```
goto nefoundc;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|(tetab*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
if tetastar le tetab and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=18;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=181;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=19;
```

```
goto nefoundc;
endif;
if tetadif le tol and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=191;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat2-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=20;
goto nefoundc;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=201;
goto nefoundc;
endif;
if tetadif1 le tol and tetastar ge (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
```

```
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=202;
goto nefoundc;
endif;
endif;
nefoundc:
k2mat[1,iterk2]=k2;
k2=k2+saltok;
iterk2=iterk2+1;
endo;
k1mat[iterk1,1]=k1;
k1=k1+saltok;
iterk1=iterk1+1;
endo;
pidif=pi1mat-pi2mat';
pdif=p1mat-p2mat';
print "Type of NE";
print typemat;
print "Equilibrium prices for firm 1";
print p1mat;
print "Equilibrium prices for firm 2";
print p2mat;
print "Equilibrium profit for firm 1";
print pi1mat;
print "Equilibrium profit for firm 2";
print pi2mat;
print "profits matrizes difference";
print pidif;
print "prices matrizes difference";
print pdif;
```

Chapter 4

Housing quality choice in a price competition duopoly model

4.1 Introduction

In the previous chapter and in this one we consider a two stage model among two urban land developers,where one of the producers stays at the CBD while the other one has a more decentralized location. In the first stage of the game the two ULDs take simultaneously their quality decisions and in the second stage of the game they compete in prices.

The emphasis in the last chapter was in the price competition game among the two ULDs, for given quality levels. We started by deriving analytically the Nash equilibrium of the second stage of the game. Next, using numerical simulations we studied how the equilibrium changes with the qualities chosen in the first stage. In particular we looked at the equilibrium prices, at the equilibrium profits and at the type of equilibrium that occurs for each vector of qualities.

In this chapter we complete the analysis of the game by looking at the choice of the quality levels in the first stage of the game. Thus we determine the Subgame Perfect Nash Equilibrium (SPNE) of the quality-price game. Since it is impossible to get an analytical solution for the equilibrium qualities, we determine the Subgame Perfect Nash Equilibrium (SPNE) numerically. We used the Gauss software to create a program that computes the SPNE of the quality-price game. The program determines first the Nash equilibrium of the second stage game, for given quality levels. That is, for each (k_1, k_2) we obtain the equilibrium prices and profits for each ULD. This procedure is repeated for many (k_1, k_2), and the equilibrium values for profit is saved in two matrices: the profit matrix of ULD 1 and the profit matrix of ULD 2. These two matrices are then used to determine the best response functions of each ULD. To determine the best response function of ULD 1, for each k_2 we find the maximum value of ULD 1 profit in the column that corresponds to k_2 in the ULD 1 profit matrix and identify the value of k_1 that corresponds to it. Similarly, to identify the best response function of ULD 2, for each k_1 we find the maximum value of ULD 2 profit in the row that corresponds to k_1 in the ULD 2 profit matrix and identify the value of k_2 that corresponds to it. Next the Nash equilibrium of the first stage game is determined, by identifying the pairs of $(k_1,\ k_2)$ such that both ULDs are simultaneously in their best responses. After identifying the equilibrium values for (k_1, k_2), the corresponding NE of the second stage is determined. This procedure is repeated for many values of the unit transportation cost, t, and the lowest quality valuation, $\underline{\theta}$, to analyze how the equilibrium changes with these two parameters. The program computes the equilibrium qualities and the equilibrium profits for each ULD as well as the types of equilibria.

In this chapter we present the results of our numerical analysis of the SPNE. The chapter starts by looking at the best response functions of both ULDs, we analyze the best response functions for nil unit transportation costs as well as for positive unit transportation costs. Next we discuss how the equilibria change as we vary the unit transportation costs and the lowest quality valuation. In particular, we analyze the type of equilibria that occur, the changes in the equilibrium qualities and the changes in the equilibrium profits.

4.2 Best Response Functions

In this section we present graphically some of the results obtained in our numerical analysis. In order to give an idea of the shape of the best response functions and how they change as we vary the unit transportation cost, t, we present the best response functions with nil transportation case (in which case there is symmetry between the two ULDs) and the best response functions when t is positive (in which case there is no symmetry).

4.2.1 Best response for nil unit cost of transportation

First we analyze the best response function for nil cost of transportation ($t = 0$). In order do obtain the values for the best response function of both ULDs, we considered increments of 0.1 for k_1 and for k_2. We compute the best response functions for $\underline{\theta} = 0.5$, for $\underline{\theta} = 2$, and for $\underline{\theta} = 3.5$ (see in Figure 4.1). By looking at these best response functions we will get an idea of the impact of the quality valuation parameter on the best response functions.

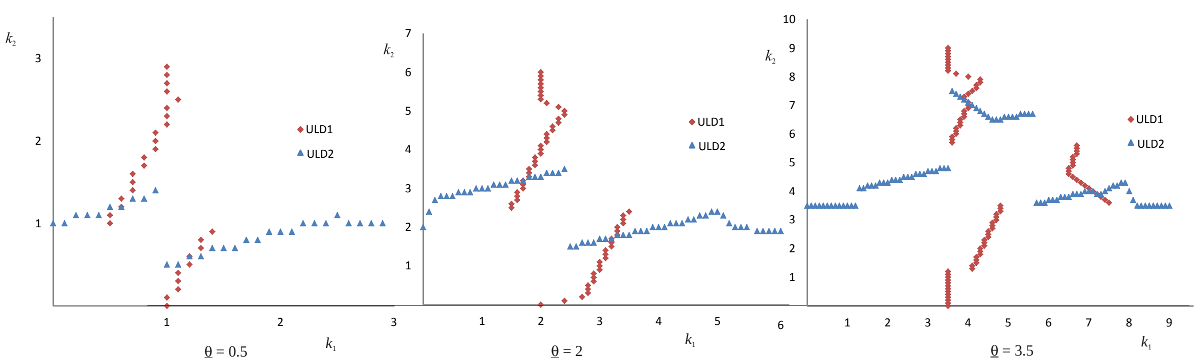

Figure 4.1: Best response functions for $t = 0$, when $\underline{\theta} = 0.5$, $\underline{\theta} = 2$ and $\underline{\theta} = 3.5$.

As we can see in Figure 4.1, for nil unit cost of transportation the best response functions for ULD 1 and ULD 2 are symmetric. Figure 4.1 shows that the best response functions when $\underline{\theta} = 0.5$ and when $\underline{\theta} = 2$ have a very similar shape. On the contrary, for $\underline{\theta} = 3.5$ the shape of the best response functions is quite different.

In the two first graphs we observe that when $k_2 = 0$, or when k_2 is very high, ULD 1 best response is to choose the monopoly quality level (for instance, when $\underline{\theta} = 2$ and $k_2 = 0$ or $k_2 > 5.6$, the best response of ULD 1 is $k_1^* = 2$). This explains the vertical segment of ULD 1 best response when k_2 is very high (similarly, the best response function of ULD 2 has an horizontal segment for very high values of k_1). In addition, for low values of k_2 it is optimal for ULD 1 to differentiate offering higher quality, whereas for higher values of k_2, it is optimal for ULD 1 to differentiate by offering lower quality. The best response of ULD 2 is similar. Therefore, when both ULDs operate, the best response functions show that it is optimal to choose a quality different from the quality of the rival ULD. When the other ULD offers a low quality, it is optimal to differentiate by offering an higher quality. When the other ULD offers a high quality, it is optimal to differentiate by offering a lower quality. When $\underline{\theta} = 0.5$ and when $\underline{\theta} = 2$, there are two SPNE that involve quality differentiation (in one ULD 1 has higher quality, in the other one the reverse happens). The following result summarizes these conclusions:

Result 4.1 *For low and intermediate values of the lowest quality valuation parameter, $\underline{\theta}$, and nil transportation cost, for low values of k_j it is optimal for ULD_i to differentiate by offering an higher quality level, $k_i^* > k_j$. On the other hand, for high values of k_j it is optimal for ULD_i to differentiate by offering a lower quality, $k_i^* < k_j$. When $k_j = 0$ or for very high values of k_j, ULD_i is a monopoly and it is optimal to offer the monopoly quality level. Furthermore, the best response functions are positively sloped, except when the ULD is about to become an unconstrained monopolist. There are two SPNE that involve quality differentiation.*

In Figure 4.1, the graphic on the right side, for $\underline{\theta} = 3.5$, shows a much more complex behavior and more discontinuities in the best response functions. We can see that for low values of k_2 (ranging from 0 to 1.2) ULD 1 offers the monopoly quality level $k_1 = 3.5$. This also happens for very high values of k_2 (ranging from 8.2 to 9). In this case, since the consumers value a lot quality, if ULD2 offers a very low quality it will have no demand and ULD 1 can behave as a monopolist. On the other hand, if ULD 2 offers a too high quality it will also have no demand (unless it charges a price below marginal cost) and again ULD 1 can behave as a monopolist. For values of k_2 between 1.3 and 3.5, ULD 1 differentiates offering higher qualities. For values of k_2 between 3.6 and 4.5, ULD 1 continues to differentiate offering higher qualities, but much higher qualities. In addition, in this region the best response function of ULD 1 is negatively sloped (that is, ULD 1 reduces its quality when k_2 increases). For k_2 ranging from 4.6 to 6.5, ULD 1 still differentiates offering higher qualities however its quality is again increasing with k_2. For k_2 ranging from 5.7 to 7.9, ULD 1 differentiates offering lower quality and its quality increases with k_2. For k_2 ranging from 8 to 8.1, ULD 1 still differentiates offering lower quality but its quality is decreasing with k_2.

In terms of equilibria, in our numerical simulations we got 8 equilibria when $t = 0$ and $\underline{\theta} = 3.5$. However we believe that this is due to the fact that numerically we have to define the increments in k_1 and k_2 and we may not have considered a grid fine enough to obtain only two equilibria.

4.2.2 Best response for positive unit cost of transportation

Let us now analyze the best response function for positive unit cost of transportation. In this case the best response functions of the two ULDs are no longer symmetric. Since we want to analyze the impact of changes in the unit cost of transportation, we set $\underline{\theta} = 1$, and obtain the best response functions for both ULDs, for $t = 0.2$, $t = 0.5$, and $t = 0.8$ (see Figure 4.2).

When $t = 0.2$ and $t = 0.5$ the shape of the best response function of ULD 2 is very similar to the one observed when $t = 0$. That is, for small values of k_1, ULD 2 wants to differentiate with higher qualities whereas for high values of k_1 ULD 2 wants to differentiate by offering lower qualities. For very high k_1, ULD 2 offers the monopoly quality level and the best response function is horizontal. On the other hand, when $t = 0.8$, the best response function of ULD 2 has a different behavior, since for intermediate values of k_1 the best response of ULD 2 is to choose $k_2 = 0$ (which implies that it will have

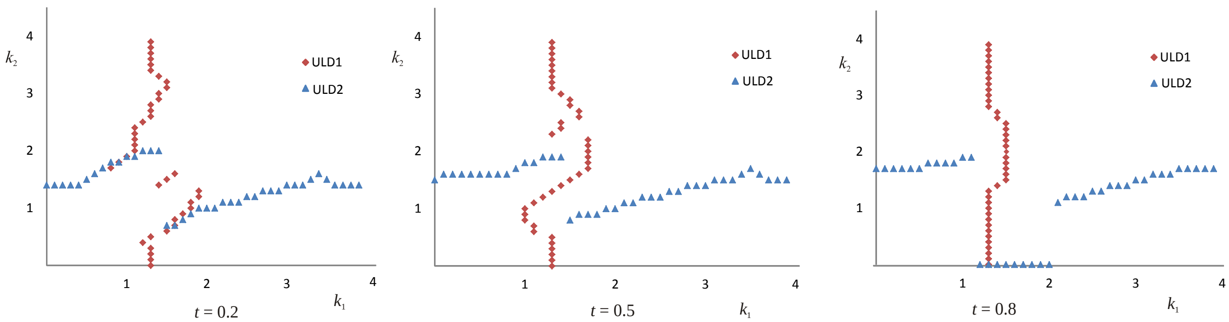

Figure 4.2: Best response functions for $\underline{\theta} = 1$ when $t = 0.2$, $t = 0.5$, and $t = 0.8$.

no demand). This means that if k_1 is intermediate, it is not profitable for ULD 2 to differentiate either by choosing a lower quality or by choosing an higher quality and thus ULD 2 ends up choosing a nil quality.

Regarding the shape of the best response of ULD 1, there are more differences relatively to the nil transportation cost case and as the value of t changes. For $t = 0.2$ we still observe that for relatively low value of k_2, ULD 1 wants to differentiate by offering higher qualities whereas for high values of k_2, ULD 1 wants to differentiate by offering lower qualities. However there are two novelties. The first is that offering the monopoly quality level is now a best response for some positive but very low values of k_2 (due to t being positive, if k_2 is below a certain level, ULD 2 has no demand). The other new feature is that there are some intermediate values of k_2 for which the best response of ULD 1 is to offer precisely the same quality, $k_1^* = k_2$). This last feature is even more visible when $t = 0.5$ where the best response function of ULD has a big segment that coincides with the 45^o line. As t increases the shape of the ULD 1 best response function starts having less clear discontinuities and becomes closer and closer to being a vertical line in the monopoly quality level. This means that as t increases, ULD 1 is able to have a behavior which does not differ much from the behavior of a monopolist. In particular for $t = 0.8$, ULD 1 either behaves as a monopolist (for low and for high values of k_2) or offers a quality which is slightly above the monopoly level.

In terms of equilibria when $\underline{\theta} = 1$, there are four SPNE when $t = 0.2$, there is no equilibrium when $t = 0.5$ since there is no intersection between the two best response functions (this can also be seen in Table 4.1), and there is a SPNE when $t = 0.8$ where ULD 1 is a monopolist since the interception of the two best response functions happens when $k_2 = 0$ and the best response function of ULD 1 is vertical (we can see in Table 4.1 that this is an equilibrium where ULD 1 has partial coverage).

4.3 Impact of parameters changes on the equilibria

In this section we analyze the changes on the equilibrium when t varies between 0 and 1 (with increments of 0.1), and $\underline{\theta}$ varies from 0 to 3.5 (with increments of 0.5). We first describe the type of equilibria that occurs for each combination of these two parameter values. Next we analyze how the equilibrium qualities and profits change as t increases and as $\underline{\theta}$ increases.

4.3.1 Type of equilibria

The type of equilibria that occurs depends on the values of parameters t and $\underline{\theta}$. Table 4.1 presents the type of equilibrium for several combinations of the parameters values (t varies between 0 and 1 and $\underline{\theta}$ varies between 0 and 3.5).

Table 4.1: Type of equilibria for several combinations of the parameters t (row) and $\underline{\theta}$ (column).

$t \mid \underline{\theta}$	0	0.5	1	1.5	2	2.5	3	3.5
0	DPC	DPC	DFCK	DFC	DFC	DFC	DFCK	DFCK
0.1	DPC	DPC	DFC	DFC	DFC	DFC	DFCK	DFCK
0.2	M1PC	DPC	DFC	DFC	DFC	DFC	DFC	DFCK
0.3	M1PC	NO	DFC	DFC	DFC	DFC	DFC	DFCK
0.4	M1PC	M1PC	NO	DFC	DFC	DFC	DFC	DFCK
0.5	M1PC	M1PC	NO	DFC	DFC	DFC	DFC	DFCK
0.6	M1PC	M1PC	NO	DFC	DFC	DFC	DFCK	DFCK
0.7	M1PC	M1PC	M1PC	DFC	NO	DFC	DFCK	DFCK
0.8	M1PC	M1PC	M1PC	NO	NO	DFCK	DFCK	DFCK
0.9	M1PC	M1PC	M1PC	NO	NO	DFC	DFCK	DFCK
1	M1PC	M1PC	M1PC	NO	NO	NO	DFCK	DFCK

For the parameters values considered, there are four types of equilibria. The equilibria where ULD 1 operates alone in the market with partial coverage are denoted by M1PC. The equilibria where both ULDs operate and there is partial coverage of the market, since the lower valuation consumers are not served, are denoted by DPC. The equilibria where both ULDs operate in the market but with full coverage are denote by DFC. Finally, there are equilibria where both ULDs operate in the market with full coverage but we are in a kink case (DFCK) where even a very small increase in prices would lead to partial coverage. We describe the cases where no equilibria occurs as NO, the explanation for these cases is on the discontinuities of the qualities best response functions combined with the fact that the best response functions are asymmetric when t is positive.

The results are very clear. For low values of $\underline{\theta}$ there is partial coverage whereas for higher values of values of $\underline{\theta}$ there is full coverage of the market. For low values of the quality valuation parameter, when t is small (ranging from 0 to 0.2), we obtain the type of equilibria DPC, where both ULDs operate but with partial coverage. As the values of t rise from 0.2 to 1, and still with small values of $\underline{\theta}$ (ranging from 0 to 1) we may either have the equilibrium where ULD 1 operates as a monopoly with partial coverage (M1PC) or have no equilibrium. The explanation for the existence of equilibria with monopoly and partial coverage, is that ULD1 as an advantage with respect to its rival when the unit cost of transportation is high, since ULD 1 is located at the CBD while the rival is not. It also interesting to note that the cases of no equilibria are a sort of transition case between the case of duopoly equilibria with partial coverage and the case of monopoly with partial coverage.

For high values of the taste parameter $\underline{\theta}$, namely when the lowest valuation of housing quality is between 1.5 and 3.5, we only get equilibria where the two ULDs operate with full coverage (DFC) or the two ULDs operate with full coverage but in a kink case (DFCK) or, for higher values of t, we may get no equilibrium.

4.3.2 Equilibrium qualities and profits

Having the matrices for the SPNE qualities k_1^* and k_2^*, and the matrices for the corresponding equilibrium profits, Π_1^* and Π_2^*, we now analyze how the equilibrium qualities and profits changes with t and $\underline{\theta}$. Figure 4.3 shows how the equilibrium qualities, k_1^* and k_2^*, change with the unit cost of transportation t, considering the cases when $\underline{\theta} = 1$ and when $\underline{\theta} = 3$. The figure illustrates what happens to the equilibria where, under duopoly, ULD 1 has the lowest quality ($k_1^* < k_2^*$).

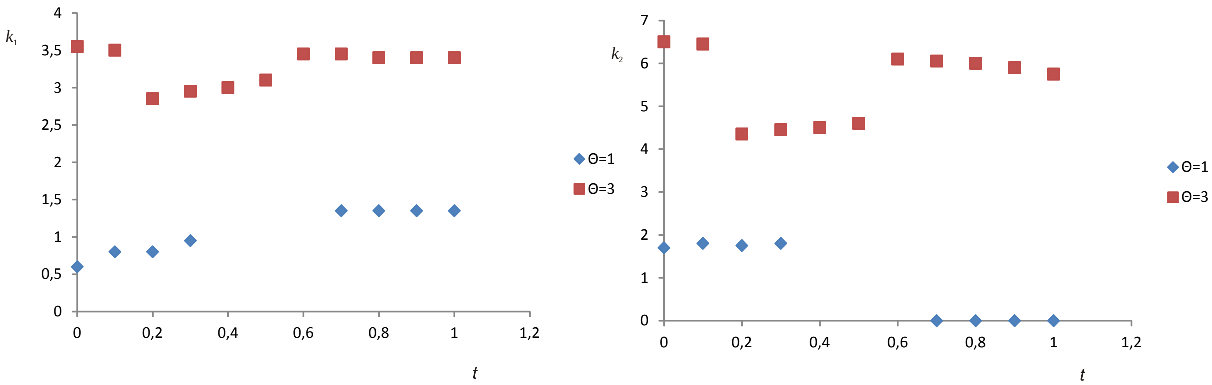

Figure 4.3: Equilibrium qualities of ULD 1 and ULD 2 as a function of t, when $\underline{\theta} = 1$ and when $\underline{\theta} = 3$.

In Figure 4.3 , on the left we can see k_1^* as a function of t. When $\underline{\theta} = 1$, the equilibrium quality of ULD 1 starts by being increasing with the unit cost of transportation, then for values of t greater than 0.7 the quality is constant with t. The fact that k_1^* does not change with t for $t \geq 0.7$ is explained by the fact that ULD 1 is a monopolist for these very high values of the unit transportation costs. When $\underline{\theta} = 3$, we can observe that for low values of t (ranging from 0 to 0.2) the housing quality of ULD 1 decreases with t, but for t between $t = 0.3$ and $t = 0.6$ ULD 1 quality is increasing with t. Finally, for $t \geq 0.8$ the housing quality of ULD 1 is constant with t.

In the graphic on the right we can see the plot for the equilibrium quality of ULD 2, k_2^*, as a function of t. When $\underline{\theta} = 1$, we can observe that k_2^* increases with t, from 0 to 0.1, but then it decreases from 0.1 to 0.2, however the equilibrium quality maintains an almost constant value for low values of t (from $t = 0$ to $t = 0.3$). For greater values of t (ranging from 0.7 to 1) the equilibrium qualities of ULD 2 are nil and hence ULD 1 is a monopoly. When $\underline{\theta} = 3$, the behavior of the equilibrium quality of ULD2 is similar (although with bigger jumps) to the one observed for $ULD1$. In fact k_2^* starts by decreasing with t (from 0 to 0.2), jumps down and becomes increasing with t (from 0.3 to 0.6). Finally, for t ranging from 0.6 to 1 the ULD 2 quality decreases slightly with t.

To analyze how the equilibrium qualities, k_1^* and k_2^*, change with $\underline{\theta}$, we represent the cases where $t = 0.2$ and $t = 0.5$. In Figure 4.4, on the left we have the equilibrium quality of ULD 1, k_1^* as a function of $\underline{\theta}$. In this graphic we can observe that the equilibrium quality, k_1^*, is increasing with the valuation of the housing quality, and that the equilibrium quality is higher for the series that represents the larger unit cost of transportation, $t = 0.5$.

On the right side of Figure 4.3 we have a graphic for the equilibrium quality of ULD 2, k_2^* as a function of $\underline{\theta}$. When $t = 0.2$, the equilibrium quality of ULD 2 is increasing with the valuation of housing quality. When $t = 0.5$, the quality also grows with the valuation of the housing quality, however the equilibrium quality of ULD 2 is nil for $\underline{\theta} = 0$ and $\underline{\theta} = 0.5$, since for those values of $\underline{\theta}$ we have a monopoly of ULD 1. The explanation for the increasing quality with the valuation of housing quality is obvious. If the consumers value more the housing quality then the ULDs have an interest in offering higher quality of housing.

Result 4.2 *For given values of the unit cost of transportation, t, the equilibrium qualities of ULD 1 and ULD 2, k_1^* and k_2^*, are increasing with the lowest valuation of housing quality, $\underline{\theta}$.*

We now examine the behavior of the equilibrium profit of each ULD as a function of t and as a function of $\underline{\theta}$. Figure 4.5 shows the behavior of the equilibrium profit of both ULDs as a function of t, considering the case where $\underline{\theta} = 0.5$ and the case where $\underline{\theta} = 2.5$.

The left side of Figure 4.5 shows that, for ULD 1, when the consumers have a low quality valuation ($\underline{\theta} = 0.5$), the equilibrium profit starts to slightly rise with the unit cost of transportation, but then becomes constant at a very low value. On the contrary, when

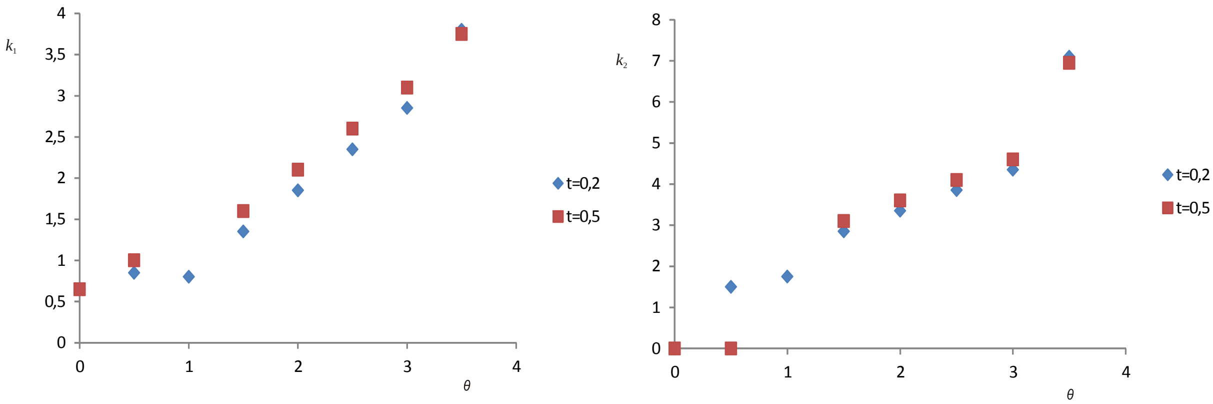

Figure 4.4: Equilibrium qualities of ULD 1 and ULD 2 as a function of $\underline{\theta}$, when $t = 0.2$ and when $t = 0.5$.

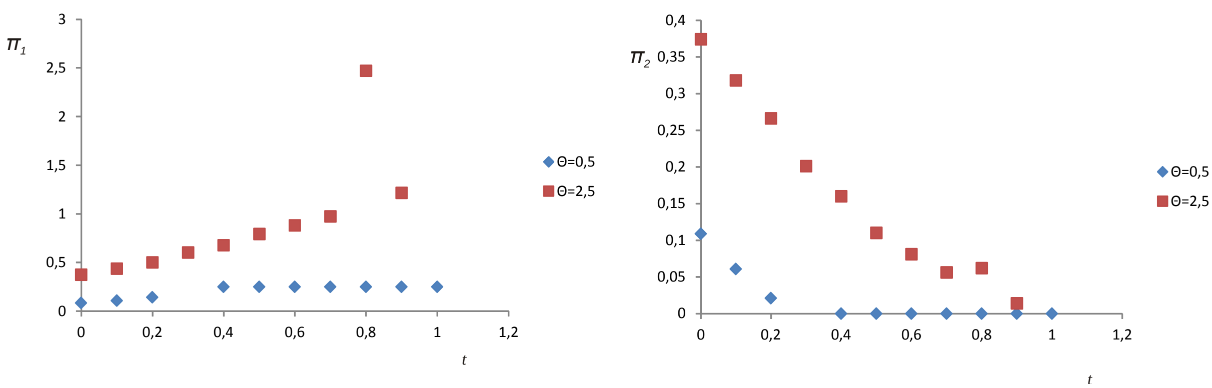

Figure 4.5: Equilibrium profits of ULD 1 and ULD 2 as a function of t, when $\underline{\theta} = 0.5$ and when $\underline{\theta} = 2.5$.

the quality valuation of the consumer is high ($\underline{\theta} = 2.5$), the equilibrium profit increases more steeply with the unit cost of transportation. For $t = 0.8$ we have a very high profit value (remember that this is an equilibrium where we have a kink case). This graph shows that ULD 1 benefits from the existence of the transportation costs since it has a strong advantage due to its location at the CBD. This benefit is even more effective when we have a high valuation of housing quality by the consumers.

Result 4.3 *In general, the equilibrium profits of ULD 1 are non-decreasing with the unit cost of transportation.*

On the other hand, the right side of Figure 4.5 reveals that, for the ULD 2, in both cases, $\underline{\theta} = 0.5$ and $\underline{\theta} = 2.5$ (for low and high valuation of housing quality), the equilibrium profit declines strongly as the unit cost of transportation increases. Like we explained above this is due to the fact that ULD 2 has a peripheral location, so the unit cost of transportation is a disadvantage for ULD 2, and therefore the equilibrium profits decline with the increase of the unit cost of transportation. Again for $t = 0.8$, we have an increase of the equilibrium profit, we think that the explanation is because we are in a kink case.

Result 4.4 *In general, the equilibrium profits of ULD 2 are non-increasing with the unit cost of transportation.*

Figure 4.6 shows the equilibrium profits as a function of $\underline{\theta}$, when $t = 0$ and when $t = 0.5$. As we can see on the left graph, the equilibrium profit of ULD 1, when $t = 0.5$, there is an increase of the equilibrium profit with $\underline{\theta}$ for smaller values of $\underline{\theta}$ (ranging from 0 to 1.5). For values of $\underline{\theta}$ between 1.5 and 3 the equilibrium profit of ULD 1 is constant with $\underline{\theta}$. Finally, for $\underline{\theta} = 3.5$ we have a very high equilibrium profit value. Observing the case when $t = 0$, we can conclude that we have the same pattern of the case with $t = 0.5$. We conclude that the equilibrium profit of ULD 1 grows sharply for very high valuations of house quality.

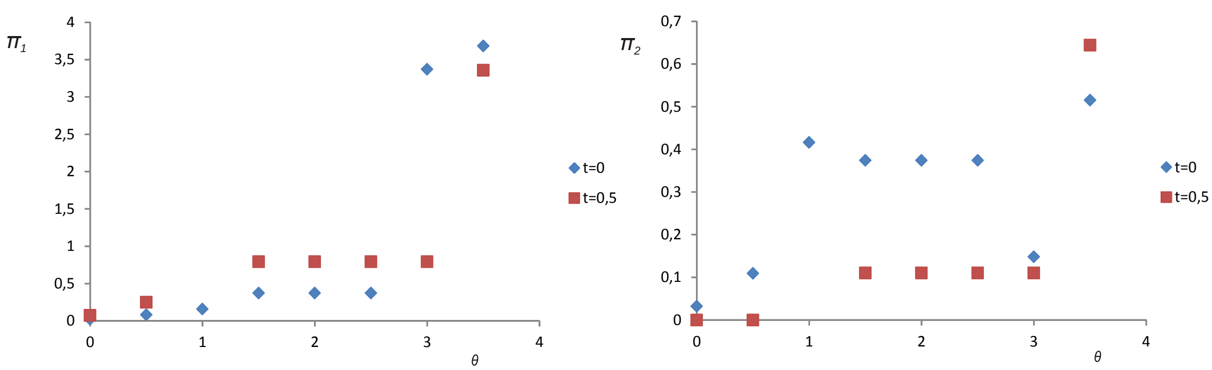

Figure 4.6: Equilibrium profits of ULD 1 and ULD 2 as function of $\underline{\theta}$, when $t = 0$ and when $t = 0.5$

Analyzing the equilibrium profit of ULD 2 as a function of $\underline{\theta}$ (the graphic on the right in Figure 4.6), we observe that for $t = 0.5$, for $\underline{\theta} = 0$ and $\underline{\theta} = 0.5$ the equilibrium profit of ULD 2 is nil. From $\underline{\theta} = 1.5$ to $\underline{\theta} = 3$ and $t = 0.5$, ULD 2 has a constant equilibrium profit. Finally, for $\underline{\theta} = 3.5$ the profit of ULD 2 grows sharply. We conclude that for the two ULDs the equilibrium profit grows for high valuations of housing quality.

Result 4.5 *For $t = 0$ and $t = 0.5$, the equilibrium profit of ULD 1 and ULD 2 grows sharply for high valuations of housing quality.*

4.4 Conclusion

In this chapter we solved numerically the first stage of the quality-price game, by using a Gauss program. The chapter started by looking at the best response functions for both ULDs. Next we studied the impact of changes in the unit transportation cost and in the lowest valuation of housing quality parameter on the equilibria. In particular, we investigated the type of equilibria that happen, the impact on the equilibrium qualities and the impact on the equilibrium profits.

Regarding the best response functions, when the unit transportation cost are null, we can conclude that if the rival ULD offer low quality, it is optimal to differentiate by offering a higher quality level. On the other hand, if the rival ULD offers a high quality, it is optimal to differentiate by offering a lower quality. Finally, when the rival ULD offers a very high quality, the ULD is a monopoly and offers the optimal monopoly quality. The best response functions are discontinuous and, typically, there are two subgame perfect Nash equilibria, involving quality differentiation. In one equilibrium ULD offers a lower quality that ULD 2, in the other equilibrium the reverse happens. For higher housing quality valuations the best response functions present even more discontinuities and there are intermediate values of the rival quality valuation where the best response functions are negatively sloped. However the best response functions still show the «desire to differentiate» from the rival.

With positive transportation costs, the best response of ULD 2 is similar to the one observed without transportation costs. However, when the unit transportation cost is high and ULD 1 offers an intermediate quality, it may not be profitable for ULD 2 to differentiate and ULD 2 may be better off by offering a nil quality. On the other hand, the best response function of ULD 1 shows some interesting features, including the fact that, for intermediate values of the quality offered by the rival, ULD 1 best response may be to offer precisely the same quality. Another feature is that there are more values of the quality offered by ULD 2, where ULD 1 is a monopolist. Furthermore, as the transportation costs increase, the best response function of ULD 1 becomes closer and closer to the quality offered by a monopolist (ULD 1 is a sort of constrained monopolist). With positive transportation costs, we may still have cases where there are two SPNE, but we may have cases with no equilibrium and we may also have cases where there is a unique equilibrium where only ULD 1 operates.

In the second part of this chapter we analyzed the impact of changes in the parameter values on the equilibria. Our first conclusion is that the type of equilibria that occur, depends on the combinations of the parameters values. When the lowest housing quality valuation is small and for low values of the unit cost of transportation, in equilibrium both urban land developers operate but with partial coverage of the market. For higher values of the unit transportation cost, we have an equilibrium where the ULD 1 is a monopoly with partial coverage, this is due to the advantage that ULD 1 has as consequence of the unit cost of transportation. Moreover we may also have cases where no equilibria exists.

For higher values of the lowest housing quality valuation parameter, we have two similar types of equilibria: in the first the two ULDs operate with full coverage and, in the second, the two ULDs operate with full coverage, but in a kink case where even a small increase in prices would imply partial coverage. The fact that the market is fully covered when the consumers value a lot housing quality is intuitive, because in this case the two urban land developers have interest in serving all the consumers.

Besides looking at the type of equilibria that arise for each combination of the parameters values, we also studied how the equilibrium values of qualities and profits change with the transportation costs and with the lowest quality valuation parameter. Our numerical results show that, for given values of the unit cost of transportation, the qualities of both ULDs are increasing with the valuation of housing quality. Similarly, the equilibrium profits of both ULDs are increasing with the valuation of housing quality.

Moreover, the equilibrium profits of the urban land developer located at the CBD (ULD 1), when we have a high value of the lowest valuation of housing quality, are increasing with the unit transportation cost. And for ULD 2 (the ULD located far away from the CBD), the equilibrium profits decreases with the unit cost of transportation, this reflects the disadvantage of this ULD with the unit cost of transportation.

We could not observe a conclusive pattern of how the quality that each ULD offers varies with the unit cost of transportation. But for high values of the unit cost of transportation, and small values of the housing quality valuation, the urban land developer located at the periphery (ULD 2) does not operate in the market, due to its disadvantage with the unit cost of transportation.

Appendix - Gauss Program

```
/***************************************************************/
/* This program computes the SPNE of the quality-price game. */
/* The program determines first the NE of the second stage game, for */
/*given quality levels (k1,k2) and for each (k1,k2) the equilibrium profit */
/* of each firm, (Pi1,Pi2), is computed. This is repeated for many (k1,k2) */
/* of each firm, (Pi1,Pi2), is computed and saved in two matrices. */
/* Next the NE of the first stage game is determined (k1eq,k2eq) and the */
/* corresponding NE of the second stage game is determined. */
/* This procedure is repeated for many values of the parameter values*/
/* so as to analyze how the equilibrium changes with changes in the parameters*/
/***************************************************************/
/****** Inicial parameters of the model **********************/
tol=10^(-12);
tetabmin=0;
tetabmax=3,5;
tmin=0; /* minimum value of transportation cost */
tmax=1;
saltetab=0.5;
saltot=0.1;
saltok=0.05; /* step size for the iterations on the quality levels*/
nitert=int((tmax-tmin)/saltot)+1; /* number of iterations for transportation cost*/
nitertb=int((tetabmax-tetabmin)/saltetab)+1; /* number of iterations for quality level of firm 1*/
```

```
/**** Create matrices to keep the SPNE values of qualities, prices, profits and welfare ****/
k1eqmat1=zeros(nitert,nitertb);
k2eqmat1=zeros(nitert,nitertb);
k1eqmat2=zeros(nitert,nitertb);
k2eqmat2=zeros(nitert,nitertb);
typemat1=zeros(nitert,nitertb);
typemat2=zeros(nitert,nitertb);
p1eqmat1=zeros(nitert,nitertb);
p2eqmat1=zeros(nitert,nitertb);
p1eqmat2=zeros(nitert,nitertb);
p2eqmat2=zeros(nitert,nitertb);
pi1eqmat1=zeros(nitert,nitertb);
pi2eqmat1=zeros(nitert,nitertb);
pi1eqmat2=zeros(nitert,nitertb);
pi2eqmat2=zeros(nitert,nitertb);
tmat=zeros(nitert,1);
tetabmat=zeros(1,nitertb);
nspnemat=zeros(nitert,nitertb);
/***** Start iterations of level the of transportation costs and lowest valuation ********/
t=tmin;
itert=1;
do while t<= tmax;
tetab=tetabmin;
itertb=1;
do while tetab<= tetabmax;
/****************************************************************/
/******** Finding the second stage NE for various levels of (k1,k2) and saving ***/
/******** the NE variables of each firm in a matrix ************/
/****************************************************************/
k1min=0;
k2min=0;
k1max=2*(tetab+1);
k2max=2*(tetab+1);
niterk1=int((k1max-k1min)/saltok)+1; /* number of iterations for quality level of firm 1 */
niterk2=int((k2max-k2min)/saltok)+1; /* number of iterations for quality level of firm 2 */
pi1mat=ones(niterk1,niterk2); /* create matrix to save the NE profit of firm 1 */
pi1mat=pi1mat*(-5);
pi2mat=ones(niterk1,niterk2); /* create matrix to save the NE profit of firm 2 */
pi2mat=pi2mat*(-5);
typemat=zeros(niterk1,niterk2);
p1mat=ones(niterk1,niterk2); /* create matrix to save the NE prices of firm 1 */
p1mat=p1mat*(-5);
p2mat=ones(niterk1,niterk2); /* create matrix to save the NE prices of firm 1 */
```

```
p2mat=p2mat*(-5);
k2mat=zeros(1,niterk2);
k1mat=zeros(niterk1,1);
k1=k1min;
iterk1=1;
do while k1<= k1max;
c1=(k1^2)/2; /* marginal production costs as a function of k1 */
fcost1=0.001;
k2=k2min;
iterk2=1;
pi1eq=-5;
pi2eq=-5;
do while k2<= k2max;
c2=(k2^2)/2; /* marginal production costs as a function of k2 */
fcost2=0.001;
/******** Condition for a firn to have zero demand if price is ************/
/******** greater or equal than marginal cost ************/
k2dmin= (tetab+1)-sqrt((tetab+1)^2-2*t);
k2dmax=(tetab+1)+sqrt((tetab+1)^2-2*t);
k1dmax=2*(tetab+1);
k1dmin=0;
/******************************************************************/
/****** Cases where both firms have zero demand ****************************/
/******************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 le k1dmin or k1 ge k1dmax); /* both firms have
zero demand */
p1mat[iterk1,iterk2]=c1;
p2mat[iterk1,iterk2]=c2;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
if k2 eq 0;
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
typemat[iterk1,iterk2]=1;
goto nefoundc;
endif;
/************************************************************************/
/****** Cases where only firm 2 has zero demand ***********************/
```

```
/**********************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 gt k1dmin and k1 lt k1dmax); /* only firm 1
operates */
p2mat[iterk1,iterk2]=c2;
if k2 eq 0;
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
p1eq=(c1+k1*(tetab+1))/2|tetab*k1;
p1eq=maxc(p1eq);
tetahat1=p1eq/k1;
/*** check if firm 1 operates and has full coverage **/
if (p1eq eq(tetab*k1))and (p1eq ge c1);
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=2;
goto nefoundc;
endif;
/*** check if firm 1 operates and does not have full coverage **/
if (p1eq eq ((c1+k1*(tetab+1))/2)) and (tetahat1 le (tetab+1)) and (p1eq ge c1); /*check
if we are in this case */
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=3;
goto nefoundc;
endif;
endif;
/**********************************************************************/
/****** Cases where only firm 1 has zero demand **********************/
/**********************************************************************/
if (k1 le k1dmin or k1 ge k1dmax) and (k2 gt k2dmin and k2 lt k2dmax); /* only firm 2
operates */
p1mat[iterk1,iterk2]=c1;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
p2eq=(c2+k2*(tetab+1)-t)/2|k2*tetab-t;
p2eq=maxc(p2eq);
```

```
/*** check if firm 2 operates and has full coverage **/
if p2eq eq (k2*tetab-t) and p2eq ge c2;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=4;
goto nefoundc;
endif;
/*** check if firm 2 operates and does not have full coverage **/
tetahat2=(p2eq+t)/k2;
if p2eq eq (c2+k2*(tetab+1)-t)/2 and tetahat2 le (tetab+1) and p2eq ge c2; /*check if we
are in this case */
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=5;
goto nefoundc;
endif;
endif;
/*********************************************************************/
/****** Cases when k2 = k1 (but with positive demand) *****************/
/*********************************************************************/
if k2 eq k1;
if t eq 0;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
typemat[iterk1,iterk2]=6;
else;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
/**** check if there is partial coverage ******************/
p1eq=(c1+k1*(tetab+1))/2;
p1eqc=p1eq|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 gt tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi1eq=(tetab+1-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=7;
goto nefoundc;
```

```
endif;
p1eqc=(k1*tetab)|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 le tetab and p1eq ge c1;
pi1eq=(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=8;
goto nefoundc;
endif;
endif;
endif;
/************************************************************************/
/****** Cases when k2 > k1 *********************************************/
/************************************************************************/
if k2 gt k1;
/**** check if NE has full coverage and both firms operate ******/
p1eq=((1-tetab)*(k2-k1)+2*c1+c2+t)/3;
p2eq=((tetab+2)*(k2-k1)+2*c2+c1-t)/3;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
if tetahat1 le tetab and tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=9;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(k1*(tetab+1)*(k2-k1)+2*c1*k2+c2*k1+t*k1)/(4*k2-k1);
p2eq=(2*k2*(tetab+1)*(k2-k1)+c1*k2+2*c2*k2-t*(2*k2-k1))/(4*k2-k1);
tetahat1=p1eq/k1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetahat1 gt tetab and tetahat1 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-(p1eq/k1))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
```

```
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=10;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/**** check if NE has full coverage and both firms operate but in a kink case******/
p1eq=tetab*k1;
p2eq=((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
if tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we
are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=101;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((k2*c1/k1)-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetastar le tetahat2 and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=11;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
p2eq=p2eq|((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
p2eq=maxc(p2eq);
tetahat2=(p2eq+t)/k2;
```

```
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
if tetastar le tetab and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in
this case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=12;
goto nefoundc;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in this
case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=121;
goto nefoundc;
endif;
/**** check if NE only firm 2 operates and has full coverage but in a kink case******/
p1eq=c1;
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
tetahat2=(p2eq+t)/k2;
if tetastar eq tetab and tetahat2 le tetab and p2eq ge c2; /*check if, given prices, we are in
this case */
pi1eq=-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=122;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
```

```
p1eqc=((c1+k1*(tetab+1))/2)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=13;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=131;
goto nefoundc;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat1-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat1 le tetab and p1eq ge c1; /*check if, given prices, we
are in this case */
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=14;
```

```
goto nefoundc;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=141;
goto nefoundc;
endif;
if tetadif1 le tol and tetastar ge (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=142;
goto nefoundc;
endif;
endif;
/******************************************************************/
/****** Cases when k1 > k2 *******************************************/
/******************************************************************/
/**** check if NE has full coverage and both firms operate ******/
if k1 gt k2;
p1eq=((tetab+2)*(k1-k2)+2*c1+c2+t)/3;
p2eq=((1-tetab)*(k1-k2)+2*c2+c1-t)/3;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetahat2 le tetab and tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq
ge c2; /*check if we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=15;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(2*k1*(tetab+1)*(k1-k2)+k1*(2*c1+c2+t))/(4*k1-k2);
```

```
p2eq=(k2*(tetab+1)*(k1-k2)+2*k1*c2+k2*c1-t*(2*k1-k2))/(4*k1-k2);
tetahat2=(p2eq+t)/k2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
if tetahat2 ge tetab and tetahat2 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we are in this case */
pi2eq=(tetastar-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-tetastar)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=16;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/**** check if NE has full coverage and both firms operate but we are in a kink case ******/
p2eq=tetab*k2-t;
p1eq=((tetab+1)*(k1-k2)+c1+p2eq+t)/2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq ge c2; /*check if, given prices, we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=161;
goto nefoundc; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
p1eqc=((c1+k1*(tetab+1))/2)|(k1*(c2+t)/k2);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetahat1;
tetadif=abs(tetadif);
if tetastar le tetahat1 and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
```

```
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=17;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=171;
goto nefoundc;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|(tetab*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
if tetastar le tetab and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=18;
goto nefoundc;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typemat[iterk1,iterk2]=181;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
```

```
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=19;
goto nefoundc;
endif;
if tetadif le tol and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=191;
goto nefoundc;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat2-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=20;
goto nefoundc;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
```

```
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=201;
goto nefoundc;
endif;
if tetadif1 le tol and tetastar ge (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typemat[iterk1,iterk2]=202;
goto nefoundc;
endif;
endif;
nefoundc:
k2mat[1,iterk2]=k2;
k2=k2+saltok;
iterk2=iterk2+1;
endo;
k1mat[iterk1,1]=k1;
k1=k1+saltok;
iterk1=iterk1+1;
endo;
/**************************************************************/
/************* Find the SPNE levels of k1 and k2 **************************/
/**************************************************************/
iterk1=1;
nspne=0;
k1eq=-1;
k2eq=-1;
do while iterk1 <= niterk1;
iterk2=1;
do while iterk2 <= niterk2;
pi1col=pi1mat[.,iterk2];
pi2row=pi2mat[iterk1,.];
pi2col=pi2row';
if pi1mat[iterk1,iterk2]==maxc(pi1col) and pi2mat[iterk1,iterk2]==maxc(pi2col); /* this checks if a given (k1,k2) is a NE */
/* if we are in the region where no NE of 2nd stage game was found, jump to line with level notane */
```

```
if pi1mat[iterk1,iterk2]==(-500) or pi2mat[iterk1,iterk2]==(-500);
goto notane;
else;
k1eq=k1min+saltok*(iterk1-1); /* if NE is in feasible region, this gives us SPNE value of k1 */
k2eq=k2min+saltok*(iterk2-1); /* if NE is in feasible region, this gives us SPNE value of k2 */
nspne=nspne+1;
typeeq=0;
/* Identify the type of equilibrium and compute SPNE prices and profits */
k1=k1eq;
k2=k2eq;
c1=(k1^2)/2;
c2=(k2^2)/2;
fcost1=0.001;
fcost2=0.001;
/**************************************************************************/
/****** Cases where both firms have zero demand ***************************/
/**************************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 le k1dmin or k1 ge k1dmax); /* in this case both firms have zero demand */
p1mat[iterk1,iterk2]=c1;
p2mat[iterk1,iterk2]=c2;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
if k2 eq 0;
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
typeeq=1;
goto nefound;
endif;
/*********************************************************************/
/****** Cases where only firm 2 has zero demand **********************/
/*********************************************************************/
if (k2 le k2dmin or k2 ge k2dmax) and (k1 gt k1dmin and k1 lt k1dmax); /* in this case only firm 1 operates */
p2mat[iterk1,iterk2]=c2;
if k2 eq 0;
```

```
pi2mat[iterk1,iterk2]=0;
else;
pi2mat[iterk1,iterk2]=-fcost2;
endif;
p1eq=(c1+k1*(tetab+1))/2|tetab*k1;
p1eq=maxc(p1eq);
tetahat1=p1eq/k1;
/*** check if firm 1 operates and has full coverage **/
if (p1eq eq(tetab*k1))and (p1eq ge c1);
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=2;
goto nefound;
endif;
/*** check if firm 1 operates and does not have full coverage **/
if (p1eq eq ((c1+k1*(tetab+1))/2)) and (tetahat1 le (tetab+1)) and (p1eq ge c1); /*check
if we are in this case */
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=3;
goto nefound;
endif;
endif;
/*********************************************************************/
/****** Cases where only firm 1 has zero demand **********************/
/*********************************************************************/
if (k1 le k1dmin or k1 ge k1dmax) and (k2 gt k2dmin and k2 lt k2dmax); /* in this case
only firm 2 operates */
p1mat[iterk1,iterk2]=c1;
if k1 eq 0;
pi1mat[iterk1,iterk2]=0;
else;
pi1mat[iterk1,iterk2]=-fcost1;
endif;
p2eq=(c2+k2*(tetab+1)-t)/2|k2*tetab-t;
p2eq=maxc(p2eq);
/*** check if firm 2 operates and has full coverage **/
if p2eq eq (k2*tetab-t) and p2eq ge c2;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
```

```
typeeq=4;
goto nefound;
endif;
/*** check if firm 2 operates and does not have full coverage **/
tetahat2=(p2eq+t)/k2;
if p2eq eq (c2+k2*(tetab+1)-t)/2 and tetahat2 le (tetab+1) and p2eq ge c2; /*check if we
are in this case */
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=5;
goto nefound;
endif;
endif;
/*********************************************************************/
/****** Cases when k2 = k1 (but with positive demand) *****************/
/*********************************************************************/
if k2 eq k1;
if t eq 0;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
typeeq=6;
else;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
/**** check if there is partial coverage ******************/
p1eq=(c1+k1*(tetab+1))/2;
p1eqc=p1eq|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 gt tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi1eq=(tetab+1-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=7;
goto nefound;
endif;
p1eqc=(k1*tetab)|(c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
if tetahat1 le tetab and p1eq ge c1;
```

```
pi1eq=(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=8;
goto nefound;
endif;
endif;
endif;
/*****************************************************************/
/****** Cases when k2 > k1 ******************************************/
/*****************************************************************/
if k2 gt k1;
/**** check if NE has full coverage and both firms operate ******/
p1eq=((1-tetab)*(k2-k1)+2*c1+c2+t)/3;
p2eq=((tetab+2)*(k2-k1)+2*c2+c1-t)/3;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
if tetahat1 le tetab and tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq
ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=9;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(k1*(tetab+1)*(k2-k1)+2*c1*k2+c2*k1+t*k1)/(4*k2-k1);
p2eq=(2*k2*(tetab+1)*(k2-k1)+c1*k2+2*c2*k2-t*(2*k2-k1))/(4*k2-k1);
tetahat1=p1eq/k1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetahat1 gt tetab and tetahat1 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and
p2eq ge c2; /*check if we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-(p1eq/k1))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=10;
goto nefound; /* NE was found so we can jump to the end of the if loop */
```

```
endif;
/**** check if NE has full coverage and both firms operate but in a kink case******/
p1eq=tetab*k1;
p2eq=((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
if tetastar ge tetab and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if, given
prices, we are in this case */
pi1eq=((p2eq-p1eq+t)/(k2-k1)-tetab)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=101;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((k2*c1/k1)-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
if tetastar le tetahat2 and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=11;
goto nefound;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
p2eq=p2eq|((tetab+1)*(k2-k1)+c2-t+p1eq)/2;
p2eq=maxc(p2eq);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
```

```
if tetastar le tetab and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in this case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=12;
goto nefound;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2; /* check if, given prices, we are in this case */
p1mat[iterk1,iterk2]=p1eq;
pi1mat[iterk1,iterk2]=-fcost1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=121;
goto nefound;
endif;
/**** check if NE only firm 2 operates and has full coverage but in a kink case******/
p1eq=c1;
p2eqc=(tetab*k2-t)|(tetab*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetahat1=p1eq/k1;
tetahat2=(p2eq+t)/k2;
if tetastar eq tetab and tetahat2 le tetab and p2eq ge c2; /*check if, given prices, we are in this case */
pi1eq=-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
pi2eq=((tetab+1)-(p2eq-p1eq+t)/(k2-k1))*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=121;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
p1eqc=((c1+k1*(tetab+1))/2)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
```

```
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=13;
goto nefound;
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=c2;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=131;
goto nefound;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|((tetab+1)*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
tetahat1=p1eq/k1;
p2eq=c2;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat1-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat1 le tetab and p1eq ge c1; /*check if, given prices, we
are in this case */
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=14;
goto nefound;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
```

```
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=141;
goto nefound;
endif;
if tetadif1 le tol and tetastar ge (tetab+1) and p1eq ge c1;
p2mat[iterk1,iterk2]=p2eq;
pi2mat[iterk1,iterk2]=-fcost2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=142;
goto nefound;
endif;
endif;
/*************************************************************************/
/****** Cases when k1 > k2 *********************************************/
/*************************************************************************/
/**** check if NE has full coverage and both firms operate ******/
if k1 gt k2;
p1eq=((tetab+2)*(k1-k2)+2*c1+c2+t)/3;
p2eq=((1-tetab)*(k1-k2)+2*c2+c1-t)/3;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetahat2 le tetab and tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq
ge c2; /*check if we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=15;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if NE does not have full coverage and both firms operate **/
p1eq=(2*k1*(tetab+1)*(k1-k2)+k1*(2*c1+c2+t))/(4*k1-k2);
p2eq=(k2*(tetab+1)*(k1-k2)+2*k1*c2+k2*c1-t*(2*k1-k2))/(4*k1-k2);
tetahat2=(p2eq+t)/k2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
```

```
if tetahat2 ge tetab and tetahat2 le tetastar and tetastar le (tetab+1)and p1eq ge c1 and p2eq ge c2; /*check if we are in this case */
pi2eq=(tetastar-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-tetastar)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=16;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/**** check if NE has full coverage and both firms operate but we are in a kink case ******/
p2eq=tetab*k2-t;
p1eq=((tetab+1)*(k1-k2)+c1+p2eq+t)/2;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetahat2=(p2eq+t)/k2;
if tetastar ge tetab and tetastar le (tetab+1) and p1eq ge c1 and p2eq ge c2; /*check if, given prices, we are in this case */
pi2eq=((p1eq-p2eq-t)/(k1-k2)-tetab)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
pi1eq=((tetab+1)-(p1eq-p2eq-t)/(k1-k2))*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=161;
goto nefound; /* NE was found so we can jump to the end of the if loop */
endif;
/*** check if in the NE only firm 1 operates and does not have full coverage **/
p1eqc=((c1+k1*(tetab+1))/2)|(k1*(c2+t)/k2);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetahat1;
tetadif=abs(tetadif);
if tetastar le tetahat1 and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=17;
goto nefound;
```

```
endif;
if tetadif le tol and tetahat1 ge tetab and tetahat1 le (tetab+1) and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=((tetab+1)-tetahat1)*(p1eq-c1)-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=171;
goto nefound;
endif;
/*** check if in the NE only firm 1 operates and has full coverage **/
p1eqc=(tetab*k1)|(tetab*(k1-k2)+c2+t);
p1eq=minc(p1eqc);
p2eq=c2;
tetahat1=p1eq/k1;
tetastar=(p1eq-p2eq-t)/(k1-k2);
tetadif=tetastar-tetab;
tetadif=abs(tetadif);
if tetastar le tetab and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=18;
goto nefound;
endif;
if tetadif le tol and tetahat1 le tetab and p1eq ge c1;
pi2mat[iterk1,iterk2]=-fcost2;
p2mat[iterk1,iterk2]=c2;
pi1eq=p1eq-c1-fcost1;
pi1mat[iterk1,iterk2]=pi1eq;
p1mat[iterk1,iterk2]=p1eq;
typeeq=181;
goto nefound;
endif;
/*** check if in the NE only firm 2 operates and does not have full coverage **/
p2eqc=((c2+k2*(tetab+1)-t)/2)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
```

```
tetadif=abs(tetadif);
if tetastar ge (tetab+1) and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=19;
goto nefound;
endif;
if tetadif le tol and tetahat2 ge tetab and tetahat2 le (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=((tetab+1)-tetahat2)*(p2eq-c2)-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=191;
goto nefound;
endif;
/*** check if in the NE only firm 2 operates and has full coverage **/
p2eqc=(tetab*k2-t)|((tetab+1)*(k2-k1)+c1-t);
p2eq=minc(p2eqc);
tetahat2=(p2eq+t)/k2;
p1eq=c1;
tetastar=(p2eq-p1eq+t)/(k2-k1);
tetadif=tetastar-(tetab+1);
tetadif=abs(tetadif);
tetadif1=tetahat2-tetab;
tetadif1=abs(tetadif1);
if tetastar ge (tetab+1) and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=20;
goto nefound;
endif;
if tetadif le tol and tetahat2 le tetab and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
```

```
p2mat[iterk1,iterk2]=p2eq;
typeeq=201;
goto nefound;
endif;
if tetadif1 le tol and tetastar ge (tetab+1) and p2eq ge c2;
pi1mat[iterk1,iterk2]=-fcost1;
p1mat[iterk1,iterk2]=c1;
pi2eq=p2eq-c2-fcost2;
pi2mat[iterk1,iterk2]=pi2eq;
p2mat[iterk1,iterk2]=p2eq;
typeeq=202;
goto nefound;
endif;
endif;
endif;
endif;
nefound:
if nspne eq 1;
k1eqmat1[itert,itertb]=k1eq; /*save the SPNE of k1 in a matrix */
k2eqmat1[itert,itertb]=k2eq;
/* save the NE profit of firm 1 in a matrix, where each row corresponds to a value of k1,
and each columnn to the value of k2 */
pi1eqmat1[itert,itertb]=pi1eq;
/* save the NE profit of firm 2 in a matrix, where each row corresponds to a value of k1,
and each columnn to the value of k2 */
pi2eqmat1[itert,itertb]=pi2eq;
typemat1[itert,itertb]=typeeq;
endif;
if nspne eq 2;
k1eqmat2[itert,itertb]=k1eq; /*save the SPNE of k1 in a matrix */
k2eqmat2[itert,itertb]=k2eq;
/* save the NE profit of firm 1 in a matrix, where each row corresponds to a value of k1,
and each columnn to the value of k2 */
pi1eqmat2[itert,itertb]=pi1eq;
/* save the NE profit of firm 2 in a matrix, where each row corresponds to a value of k1,
and each columnn to the value of k2 */
pi2eqmat2[itert,itertb]=pi2eq;
typemat2[itert,itertb]=typeeq;
endif;
notane:
iterk2=iterk2+1;
endo;
iterk1=iterk1+1;
```

```
endo;
nspnemat[itert,itertb]=nspne;
tetab=tetab+saltetab;
itertb=itertb+1;
endo;
t=t+saltot;
itert=itert+1;
endo;
print "nº de equilíbrios";
print nspnemat;
print "k1 equilíbrio - primeira matriz";
print k1eqmat1;
print "k2 equilíbrio - primeira matriz";
print k2eqmat1;
print "tipo de equilíbrio - primeira matriz";
print typemat1;
print "Lucro 1 de equilíbrio - primeira matriz";
print pi1eqmat1;
print "Lucro 2 de equilíbrio - primeira matriz";
print pi2eqmat1;
print "k1 equilíbrio - segunda matriz";
print k1eqmat2;
print "k2 equilíbrio - segunda matriz";
print k2eqmat2;
print "tipo de equilíbrio - segunda matriz";
print typemat2;
print "Lucro 1 de equilíbrio - segunda matriz";
print pi1eqmat2;
print "Lucro 2 de equilíbrio - segunda matriz";
print pi2eqmat2;
```

Conclusion

This thesis studied the supply side of the housing market taking into account the strategic interactions that occur between urban land developers. The thesis started by reviewing the literature on new housing supply, concluding that there are very few studies where strategic interactions are taken into account. Next we developed a vertical differentiation model with two urban land developers. The two producers first simultaneously decide the quality of housing and then compete in prices. We assumed that one of the producers stays at the CBD while the other has a more decentralized location. Moreover, our model assumed that a quality improvement has fixed costs but it also increases the marginal production cost. In this chapter we summarize the main conclusions of our study.

The literature review allowed us to conclude that housing supply is understudied comparing with the extensive literature on housing demand. In spite of this, various studies have been undertaken, mainly empirical studies but also some theoretical studies. Relatively to the empirical studies, there are some studies using cross section or panel data sets for metropolitan areas, but most of the studies use aggregated time series data. Curiously, in spite of the differences regarding the type of data and econometric estimation methods, the main results are quite consistent across studies. Excluding some earlier studies like Muth (1960) and Follain (1979), we can reject a perfectly elastic supply of housing. Most studies find an elastic housing supply but there are some studies that obtain below unit elasticities. The studies that distinguish between short run and long run elasticities reveal that price elasticity of housing supply is lower in the short run. Moreover, the studies that allow comparisons across countries or regions show that there are significant differences in supply elasticities between countries and regions. For instance, the values of the price elasticity of supply are higher in the USA than in the UK. Regarding the other determinants of housing supply, most empirical results are according to the theoretical predictions. For instance, financial costs, inflation and sales delay influence negatively the housing supply. However there are also some results which are unexpected, namely the inconclusive results with respect to the impact of construction costs. One possible explanation for this inconclusive results is the difficulty in measuring accurately the construction costs.

Our review on the articles that use game theory/ industrial organization models of housing supply showed that the strategic interaction between land developers or constructors is still understudied and hence there is a lot of potential in exploring this type of models. We believe that there is a need to increase our understanding of the behavior of constructors and land developers. This deeper understanding can come from the development of theoretical models predicting their decisions in a context where there exists strategic interactions between land developers and the estimation of empirical models based on micro data. Strategic interaction models of housing supply may allow us to understand how land developers make their decisions regarding the house location and house quality, may allow us to explore the market structure of the housing market and test if the market is competitive or if the land developers have some oligopolistic power. By using data where the unit of analysis is the land developer, we may be able to resolve some counter-intuitive results such as those obtained with respect to the impact of construction costs.

Most of the literature on housing markets assumes that the housing industry is perfectly competitive, but there is a growing support for imperfect competition models (Arnott and Igarashi, 2000; Baudewyns, 2000). The existence of differences in the housing quality, differences in housing accessibility, differences in households tastes, can be sources of market power and lead to strategic interactions between the urban land developers. Therefore, in this study we applied game theory and industrial organization tools to model housing supply. In chapters 3 and 4, we developed a dynamic duopoly game with two stages. Our model is naturally a vertical differentiation model. This type of models has been extensively studied in the literature in industrial economics, including models with endogenous quality choice. However most of the literature that we have reviewed consists of general theoretical models, that do not take into account the specificities of the housing market, such as the location of the house.

The literature on vertical product differentiation models, specifically with endogenous quality choice, can be divided according to the assumption that is made about the nature of the costs of quality improvement. Some authors like Lambertini (2012) assume that there are fixed costs of quality improvement while variable costs do not change with quality. This assumption is reasonable when producers improve quality by advertising or by research and development. Other authors like Aoki (1996), argue that higher quality requires more expensive inputs or a more specialized labour force. Motta (1993) compares the two assumptions about the nature of the costs of quality improvement however he does not incorporate simultaneously the two types of costs. The model developed in this thesis, assumes that a quality improvement has fixed costs and also variable costs. Thus a quality improvement has cost implications both for the price-stage game as well as for the quality-stage game. This is a contribution to the literature on vertical differentiation, since none of the existent studies incorporates simultaneously these two types of costs of increasing quality. Another contribution of our study is that we analyzed if there is full or partial coverage of the market in the subgame perfect equilibrium whereas most of the studies on vertical differential models consider either full coverage or partial coverage.

In the third and fourth chapter we analyzed a dynamic model with two ULDs, where producers first decide the housing quality and then compete in prices. We assume that one of the producers stays at the CBD while the other has a more decentralized location. Our model also considers, in the utility function a transport cost by unit of distance. In the third chapter we solved the price competition game, considering the qualities chosen in the first stage as given. In chapter 4 we solved the first stage of the game, finding the equilibrium qualities for different combinations of the unit transportation cost and the quality valuation parameter.

In chapter 3, we derived analytically the Nash equilibrium of the price-game, for given quality levels. Our analysis is very exhaustive as we explore all the possible cases both in terms of who operates in the market, in some cases only one ULD operates in the market and in others both operate, as well as in terms of the market coverage, in some cases we have full coverage of the market and in others we have partial coverage of the market. Because of this, the third chapter is an important contribution to the quality differentiation literature. We used numerical analysis, utilizing a Gauss program to obtain and characterize the Nash equilibrium of the price competition game, the equilibrium type, equilibrium prices and equilibrium profits for the different quality levels.

The results show that with nil transportation costs, the equilibrium price of a urban land developer is increasing with its housing quality, for given values of the quality of the other urban land developer. This result is also valid with positive unit cost of transportation. On the other hand, the equilibrium price of a urban land developer is a non-monotonic function of the quality of the rival ULD. In particular, for intermediate values of the other ULD quality, there is a U shaped relationship between the equilibrium price of a ULD and the housing quality of the other ULD. This result is also valid with positive transportation costs. This interesting result is due to the existence of direct and strategic effects. When the housing quality of the rival ULD increases, the demand of the ULD decreases, which tends to decrease price. However, an increase in the housing quality of the rival, increases the rival's demand and the rival's marginal cost, and hence the rival has an incentive to increase its price. Since prices are strategic complements this implies that the ULD increases its price. The non-monotonic relationship happens because for lower values of the rival ULD housing quality, the direct effect dominates, whereas for higher values of the rival's quality the strategic effect dominates.

When the quality of an ULD is nil or very high, this ULD has zero demand and the other ULD is a monopolist. In this case the monopolist ULD optimal profit is a concave function of its housing quality: the equilibrium profit first grows with quality, up to a maximum, and then falls and becomes equal to zero. This result is also valid with positive unit cost of transportation. However when the two ULD have intermediate levels of quality and the unit transportation cost is nil, the equilibrium profit functions may have two local maxima (one where the ULD chooses a quality lower than the rival, the other one where the ULD chooses a quality higher than the rival). Therefore, the equilibrium profit functions show the benefits of differentiating the housing quality. Moreover the result shows that, when the other ULD has a low quality it is better to differentiate by choosing a higher quality, whereas when the rival has a high quality it is better to differentiate by choosing a lower quality. On the other hand, with positive unit transportation costs and intermediate quality levels, the ULD located at the periphery prefers to differentiate but the ULD located at the CBD may be better off by choosing a quality level equal to the rival's one and «exploiting» its locational advantage.

In chapter 4 we solved numerically the first stage of the quality-price game, by using a Gauss program. We first analyzed the best response function for the two ULDs. For nil unit transportation cost, we can affirm that if the rival ULD offers low housing quality, it is optimal for the ULD to differentiate by offering higher quality. Conversely, if the rival offers a high quality, it is optimal for the ULD to differentiate by offering a lower quality. Finally, when the rival ULD offers a very high quality (so high that rival's marginal costs are so high that the rival has no demand unless its price is below marginal costs), the ULD offers the optimal monopoly quality. The best response functions are discontinuous, and there are two SPNE, that involves quality differentiation. If we have positive unit transportation costs, the best response function of ULD 2 is almost the same has the one with nil unit transportation costs, except when the unit transportation cost is high. In this case, if ULD 1 offers intermediate quality, ULD 2 may be better off by choosing nil quality. Furthermore, the best response function of ULD 1 has same interesting features. In particular, for intermediate values of the quality offered by the rival, ULD 1 best response may be to offer the same quality and there are more values of the quality offered by ULD 2, for which ULD 1 is a monopoly.

Based on the numerical analysis of chapter 4, we concluded that the type of equilibria that happens depends on the combinations of the parameters (unit transportation cost and the lowest consumer's quality valuation). When the housing quality valuation is small and for low values of the unit cost of transportation, in equilibrium both urban land developers operate but with partial coverage of the market. For higher values of the unit transportation cost, we have an equilibrium where the ULD 1 is a monopoly with partial coverage. Moreover we may also have cases where no equilibria exists. For higher values of the housing quality valuation parameter, we have two similar types of equilibria where the two ULDs operate with full coverage. These results are quite intuitive. The fact that the market is fully covered when the consumers value a lot housing quality is expected, because in this case the two urban land developers have interest in serving all the consumers. Similarly, when the valuation of the housing quality is lower and transportation costs are higher it is also natural that in equilibrium only ULD 1 operates with partial coverage, since this ULD has a locational advantage as the consumers do not incur in transportation costs if the buy a house from ULD 1.

In chapter 4, we also studied how the equilibrium values of qualities and profits change with the unit transportation cost and with the quality valuation parameter. Our numerical results showed that, for given values of the unit cost of transportation, the equilibrium qualities and the equilibrium profits of both ULDs are increasing with the valuation of housing quality. Moreover, the equilibrium profits of the urban land developer located at the CBD (ULD 1), when we have a high value of the lowest valuation of housing quality, are increasing with the unit transportation cost. And for ULD 2 (the ULD located far away from the CBD), the equilibrium profits decreases with the unit cost of transportation, this reflects the disadvantage of this ULD with the unit cost of transportation.

Regarding future research we have some suggestions that can improve the study of strategic interactions among urban land developers. The first one, is to consider the location as endogenous. This implies one more stage in the model. In this case the urban land developers first choose the location of their construction, in the second stage they choose the quality and in the last stage they compete in prices. In such a model it will be important to consider that different locations may have different land prices. A second improvement may be to study the social optimality of the market equilibrium outcomes, by including a welfare measure, that can be incorporated in the Gauss program. A final suggestion is to study the implications of changes in the marginal cost function. We assumed that marginal production costs are a convex function of the quality level, but it would be interesting to explore what would happen under other assumptions relatively to the way marginal production costs change with the quality.

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UNIVERSIDADE DE ÉVORA
INSTITUTO DE INVESTIGAÇÃO
E FORMAÇÃO AVANÇADA

Contactos:
Universidade de Évora
Instituto de Investigação e Formação Avançada - IIFA
Palácio do Vimioso | Largo Marquês de Marialva, Apart. 94
7002-554 Évora | Portugal
Tel: (+351) 266 706 581
Fax: (+351) 266 744 677
email: iifa@uevora.pt