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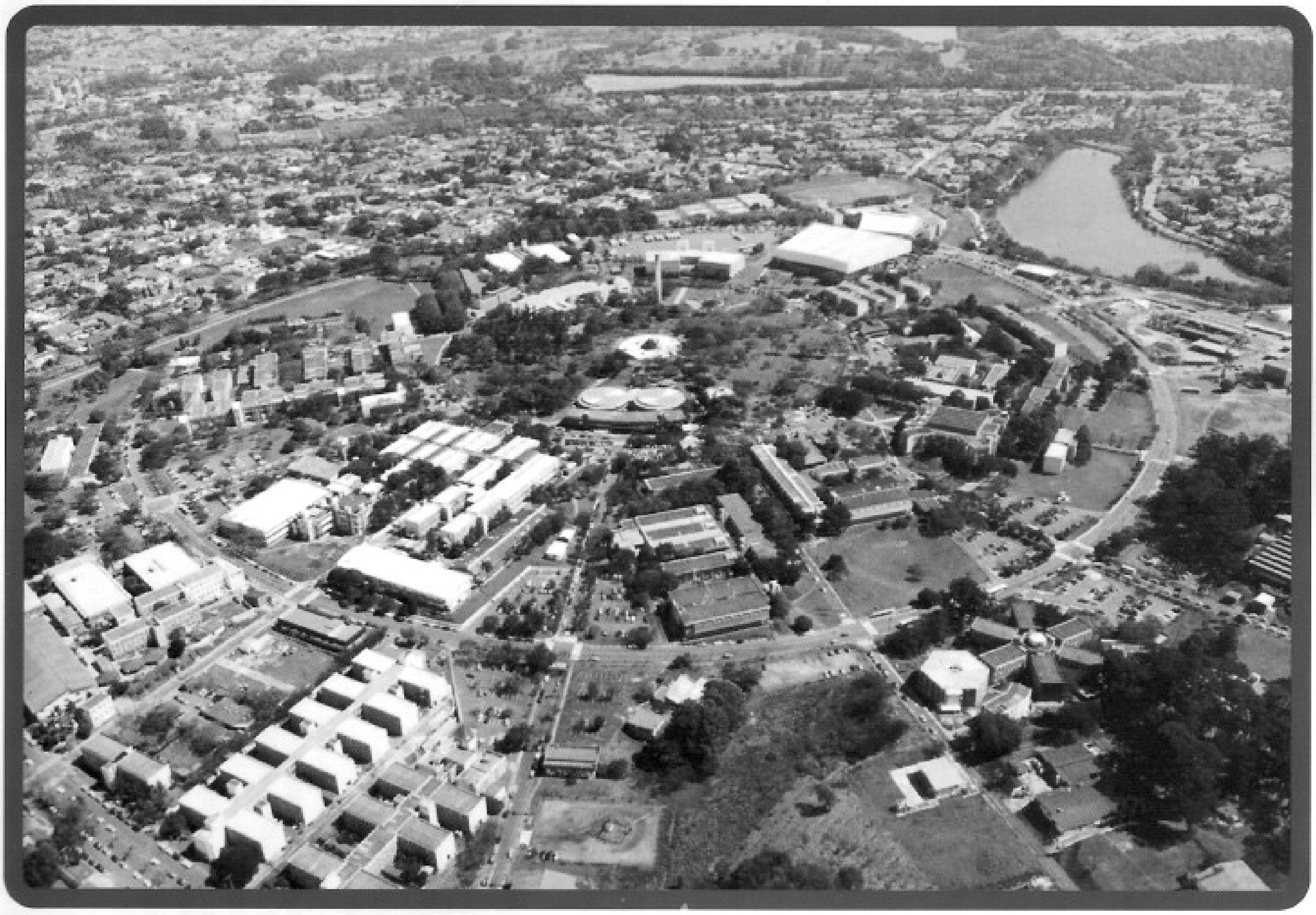
PROGRAM



ABSTRACT BOOK

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Animal growth in random environments

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The models used to describe the growth of animals in terms of their weight (or size) $X(t)$ at time t assume the form of a differential equation ([1],[2],[3]). One can see that the models commonly used, after a suitable change of variable $Y(t)=g(X(t))$ with g strictly increasing, can be written in the form

(1) $\quad dY(t)/dt = b(A-Y(t)), \quad Y(0)=y_0.$

The Bertalanffy-Richards model corresponds to the choice $g(x)=x^c$ with $c>0$ (typical choices are $c=1$, the Mitscherlich model, and $c=1/3$) and the Gompertz model corresponds to $g(x)=\ln x$.

Here, $y_0=g(x_0)$ and $A=g(a)$, where x_0 is the weight at birth and a is the asymptotic weight (weight at maturity); b is a rate of approach to maturity.

If the animal is growing in a randomly fluctuating environment, we can model growth ([4]) through a stochastic differential equation of the form

(2) $\quad dY(t) = b(A-Y(t))\,dt + \sigma\, dW(t),$

where σ measures the strength of environmental fluctuations and $W(t)$ is a Wiener process. This is equivalent to consider that the maturity value A in (1) is perturbed by white noise fluctuations.

The transient and stationary behaviours of (2) are well-known (see, for instance, [5]) and the consequences to individual growth and to the distribution of weights (or sizes) in a population of individuals are considered. We also study the properties of the time required for an individual to reach a given size a^* (if a^* is close to a, we may call it the time to maturity, which is relevant in the exploitation of farm animals and in the design of harvesting policies).

Realistically, however, one may expect different individuals to have different maturity values A. We therefore study the distribution of weights in a population when the distribution of A values among its individuals is Gaussian.

We also consider the problem of parameter estimation (by maximum likelihood and, for small samples, the use of bootstrap methods to obtain the distribution of the estimators). The problem of prediction of the size of individuals in future times is also considered. These statistical issues studied here are extremely important for the practical application of the models. We consider the case where we have data at several time instants coming either from a single individual or from several individuals.

The results and methods are illustrated using bovine growth data provided by Carlos Roquete (University of Évora).

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