Modeling Human Population Death Rates: A Bi-Dimensional Stochastic Gompertz Model with Correlated Wiener Processes

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Abstract

This study presents an innovative approach to human mortality data analysis, namely a transversal analysis across time using stochastic differential equation models, as a form of considering random environmental oscillations on the death rates. For each age between 0 and 99, we use a bi-dimensional stochastic Gompertz model with correlated Wiener processes to model the dynamics of female (first component of the stochastic process) and male (second component) crude death rates of the Portuguese population over the period 1940–2009. We test the complete model, with correlation between the unidimensional Wiener processes associated with males and with females, against the model without correlation effects. Results show significant correlations for most ages, particularly on ages below 5 and above 50.

1 Introduction

Population aging is becoming a very pertinent issue. In several and different contexts, like continuous health care and retirement funds, longevity is becoming a challenging issue. In the last decades, mortality has been exhaustively studied through both deterministic and stochastic models [1, 3, 8]. The most used of all, currently with many variations, is the Lee–Carter model [2, 5].

So why doing a transversal analysis of mortality data and why modeling death rates with stochastic differential equations (SDE) models? When analyzing simultaneously death rates of distinct ages across time, instead of doing a cohort (or longitudinal) data analysis, there is evidence that mortality at all ages is,

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