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How does product market structure influence financial structure and bankruptcy risk?

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Abstract

The decision-making process is crucial to the success or failure of an organization. We can analyze financial decisions (particularly decisions regarding capital structure) and / or operational decisions (quantities and prices to be charged). These decisions are influenced by the dynamic economic and competitive environment in which firms live.

The dissertation aims, using a game theoretical framework, to examine how market structure influences financial and product market decisions and consequently the bankruptcy risk .It analyzes the impact of changes at the level of demand uncertainty, in the degree of product differentiation and the asymmetry in marginal production costs on the risk of bankruptcy. The analysis is performed assuming a duopoly market where there is uncertainty in demand and where firms compete in quantities.

Keywords:Product Market, Financial Structure, Bankruptcy Risk

Como é que a estrutura de mercado influencia a estrutura de capital e o risco de falência?

Resumo

O processo de tomada de decisão é crucial para o sucesso ou insucesso de uma organização. Pode-se falar em decisões financeiras (particularmente decisões quanto à estrutura de capital) e/ou em decisões operacionais (quantidades e preços ótimos a praticar). Estas decisões são influenciadas pelo contexto económico e concorrencial dinâmico em que as empresas vivem.

Esta dissertação pretende, utilizando o enquadramento da teoria dos jogos, analisar como é que a estrutura de mercado influencia as decisões financeiras e do mercado do produto e consequentemente o risco de falência. É analisado o impacto de mudanças no nível de incerteza na procura, no grau de diferenciação do produto e na assimetria nos custos de produção sobre o risco de falência. A análise será feita assumindo um mercado duopólio onde existe incerteza na procura e onde as empresas competem em quantidades.

Palavras-chave: Competição no mercado do produto, Estrutura de capital, risco de falência

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Chapter 1

Introduction

1.1 Background

The decision-making process is crucial to the success or failure of an organization. We can analyze financial decisions (particularly decisions regarding capital structure) and / or operational decisions (quantities and prices to be charged). These decisions are influenced by the dynamic economic and competitive environment where firms live. Wrong financial and operational decisions can lead the firm into a deteriorating financial situation and may even lead to bankruptcy.

Decisions on the capital structure and output market decisions influence the success or failure of an organization. Hence the way in which these type of decisions affect the bankruptcy probability is an important topic to be analyzed. The link between the capital structure decision and product market decisions has been addressed, both in the corporate finance literature and in the industrial organization literature. However the analysis of the influence of these two types of decisions on the probability of bankruptcy is almost nonexistent. In fact, the bankruptcy probability has been addressed separately without considering these strategic decisions. Considering the negative social and economic impacts of bankruptcy, many researchers have searched for the best model to predict and explain the risk of bankruptcy but there is a lack of theoretical models explaining the relationship between capital structure, output market competition and bankruptcy.

The study of how market structure influences financial decisions and output market decisions and their impact on the bankruptcy probability is a topic of high relevance for at least two reasons: the definition of an optimal capital structure that maximizes the value of a company, taking into account the possibility of default or bankruptcy resulting from various factors (uncertainty in demand, the degree of product differentiation or the level of asymmetry in marginal production costs) inherent in a competitive economy, allows for a better prediction of financially difficult situations, reducing the number of bankruptcies,which results not only in the decrease of unemployment, but also in a substantial decrease in the direct and indirect costs incurred by the firm; the awareness of this reality in advance allows businesses to firstly take preventive actions, and establish long-term relationships with other stakeholders, with less fear of contract default, increasing the viability of sustainable business.

We can conclude that the definition of a capital structure and a productive structure (quantity produced) that maximize the expected value of the firm (value to shareholders and debt holders) incorporating the probability of financial distress or even bankruptcy, presents an important contribution. The consideration of a model where there is strategic interaction among agents and where the impact of changes in the parameters are investigated, provides a quite realistic and complete scenario.

1.2 Problem and purposes

This study aims to examine analytically and numerically, how the market structure influences financial decisions and decisions in the product market (the optimal amount to produce) and, consequently, the bankruptcy risk and other important variables for the firm (interest rate, expected equity value, expected debt value and welfare). Our objective is to analyze the impact of changes in the level of uncertainty in demand, the degree of product differentiation and the asymmetry in marginal production costs on the risk of bankruptcy. We aim to analyze if an increase in the uncertainty and in the degree of product differentiation leads to an increase in debt and in the quantity to be produced; to understand if an increase in the uncertainty leads to an increase on the bankruptcy probability; to identify if an increase in the degree of product differentiation leads to an increase on the bankruptcy probability; to analyze if an unilateral increase in the marginal cost of production leads to an increase on the bankruptcy probability in both firms (efficient and inefficient firm), and finally to understand the impact of increased uncertainty in demand, the degree of product differentiation and the degree of asymmetry on interest rate, expected equity value, expected debt value and welfare.

1.3 Methodology

The work aims to do an analytical resolution of a game theory model: a duopoly model (there are only two firms in the market) where there is uncertainty in demand. Firms' decisions are made with the objective of maximizing their expected value (value for shareholders and creditors). The model incorporates two decisions stages (in the first stage firms decide on the financial structure and in the second stage firms decide how much to produce, that is, we assume Cournot competition).

As previously mentioned, the study aims to analyze a problem with strategic interaction in which the objective function is the maximization of the firm value. The model has two stages and it will be solved by backwards induction, i.e., first we will determine the equilibrium quantities, quantities that maximize shareholder value. After calculating the equilibrium quantity, we will determine the corresponding optimal financial structure. Part of the model will be deduced analytically, but due to its complexity and to infer the sensitivity to the parameters, we have to resort to a numerical analysis (we use program GAUSS to do this numerical analysis).

The study aims to examine, analytically and numerically, how the market structure influences financial decisions and output market decisions. First the model will be deducted analytically, particularly the first order conditions in the second stage of the game (conditions that maximize the expected value of the firm to shareholders). The rest of the model is solved numerically. We determine numerically the equilibrium quantities and later (backward induction) it, we determine the equilibrium debt levels and the equilibrium bankruptcy probabilities. Through numerical analysis it is possible to study the impact of changes on demand uncertainty, the degree of differentiation between products and the marginal costs' asymmetry on the equilibrium output, the equilibrium debt levels and the equilibrium probability of bankruptcy.

1.4 Structure of dissertation

In order to be able to analyze how the market structure influences financial decisions and decisions in the product market and consequently the bankruptcy risk, this dissertation is organized as follows.

After this brief introductory chapter, Chapter 2 reviews the literature that relates capital structure and output market competition. We also present the main developments in the bankruptcy probability literature. We end this chapter with the presentation of studies that relate the three areas: studies that relate capital structure, output market and bankruptcy probability.

Chapter 3 presents the research methodology. Specifically, we present the model and its resolution. Firstly we present the resolution of the second stage of the game and then, through the backwards induction process, we present the solution of the first stage.

Chapter 4 is devoted to expose the numerical results and their subsequent analysis. We begin to present the results in the symmetric case, i.e. when the firms have the same marginal production cost, and then we present the asymmetric case, assuming that one firm is more efficient than the other.

Chapter 5 concludes this dissertation summarizing the main results of this work, pointing out same of its limitations and outlining some directions for future research.

Chapter 2

Literature Review

2.1 Introduction

Bankruptcies have a social and economic impact which explains why many researchers are interested in finding the best form to predict the bankruptcy risk. Decisions regarding capital structure and output market influence the success or failure of a firm, so this kind of decision will influence the probability of bankruptcy.

According to Craven and Islam (2013) “the choice of capital structure is, therefore, important for influencing the value of the firm” and Wanzeried (2003) states that “...the choice of a firm’s capital structure is in fact closely related to its output market decisions...the choice of firms’ capital structure depends on specific output market characteristics such as substitutability between different varieties and volatility in demand”. The decision on the capital structure does not affect only shareholders and creditors, but it also affects “non-financial stakeholders” such as customers, suppliers and employees. This was addressed in the work of Titman (1984), Cornell and Shapiro (1987) and Menendez (2002).

This chapter aims to present the main references regarding the link between financing and output market decisions and their relationship with the bankruptcy probability. The analysis is divided into three parts: the first part reviews the existing studies that relate the decisions on capital structure and market output; the aim of the second part is to present the main references regarding to the probability of bankruptcy and, finally, in the third part we present the main studies that relate the two previous points, i.e., linking financing and output market decisions with the bankruptcy probability.

2.2 Studies relating capital structure and output market

The link between financing and output market decisions began to emerge with the pioneering work of Modigliani and Miller (1958). The authors argue (taking into account their propositions I and II, where they consider an economy without taxes) that in a perfect capital market, the capital structure is irrelevant in determining the firm value, the important thing is the value created by the assets. In their framework there is no relationship between financial structure and output market decisions. In 1963 Modigliani and Miller, restated the propositions I and II considering an economy with taxes, they claim that a firm reaches its maximum value when fully indebted as it is when it gets the maximum tax benefit. Theories of capital structure that followed (the trade-off theory and the pecking order theory) support the existence of an optimal capital structure, but they do not incorporate in their analysis the interdependence of financing decisions and output market decisions. According to Lee (2000, 2) "one of the difficulties with current capital structure theories is that they do not consider the linkage between the output market (or input market) and a firm's financial policy. It is not difficult to conceive the fact that a firm's financial policy interacts with the product market, where the firm eventually generates cash flow. Moreover, it can be argued that a firm's ultimate survival depends on how well it competes in the product market".

The existence of a link between the financial structure and output market decisions has been highlighted both on the Corporate Finance literature and the Industrial Organization literature and begins to emerge in the 80's. Riordan (2003) presents a critical survey that summarizes the existing literature on the interaction between capital structure and output market. The author argues that the capital market restrictions depend on the output market competition.

According to the literature, the relationship between capital structure and product market decisions can be divided into two types of models: the ones that emphasize the role of *limited liability* and the ones which are based on *predatory behavior*. In the first type of models, an increase in debt leads to a more aggressive behavior of the firms in the output market, i.e. when firms have limited liability [1] they tend to produce more. The models of predatory behavior defend the opposite, i.e. most indebted firms tend to adopt a more conservative approach, while firms without financial constraints tend to be more aggressive.

In the models that emphasize the role of *limited liability*, Brander and Lewis (1986) were the first to examine the relationship between financial decisions and output market competition. They consider a two stage Cournot duopoly model[2] with an uncertain environment. In the first stage, each firm decides the capital structure. In the second stage, taking into account their previously chosen financial structure, firms take their decisions on the output market. The model focuses on the effects of the limited liability in debt financing. Brander and Lewis (1986) ignore the physical investment decision. They assume that the investment decision is taken before the capital structure decision. If this assumption was not made, the debt-equity mix choice would influence the investment which would have further effects on the output market.[3] As pointed out by Brander and Lewis (1986) one possible interpretation of the capital structure choice is that the firm is initially equity financed, when the loan is taken the borrowed money is fully

[1] According to the Portal da Empresa, firms with limited liability: there is a separation between the personal assets of the entrepreneur and the assets allocated to the firm. The entrepreneur's own assets are not allocated to the operation of economic activity; the debts resulting from the economic activity respond only to the firm's assets. Translation of the site http://www.portaldaempresa.pt/CVE/pt/Criacao/escolherformajuridica/estabelecimentoindividual/

[2] There are two classical models in oligopoly theory: Cournot (1838) and Bertrand (1883). In the first model, the firms set quantities. In the last model, prices are the strategic variables. In both models the equilibrium concept used is the non-cooperative Nash equilibrium (1950).

[3] This happens in Clayton (2009) where the investment is made to reduce the marginal cost of production.

distributed among shareholders. The authors conclude that debt tends to encourage a more aggressive behavior (to produce more) in the output market, and the competitor tends to produce less. Thus, firms have an incentive to use their financial structure for strategic purposes. Maksimovic (1988) confirms the findings of Brander and Lewis (1986) regarding the aggressiveness of indebted firms' in the output market; this is due to the existence of limited liability, however the author considered a model with multiple periods of interaction and shows that debt is a barrier for firms to be able to maintain collusion outcomes.

However previous work does not consider the definition of the financial contract as a strategic variable, which that changes the results. Grimaud (2000) follows the formalization of Brander and Lewis (1986), but incorporates the definition of the financial contract as a strategic variable. According to the author, the existence of asymmetric information, between borrowers and lenders, has an important role in the relationship between financial decisions and the output market decisions. As reported by the author, the increase in debt leads to an aggressive behavior; however this is offset by the financial costs. One of the criticisms leveled at the above work, is the fact that their model does not consider the agency problems arising between creditors and shareholders. This is depicted in Clayton's (2009) work. The conclusions derived by the author go against Brander and Lewis (1986) conclusions. The paper shows that when firms have an investment option, leverage leads to a less aggressive output competition behavior and this is due to the existence of agency problems.

While Brander and Lewis (1986) present a general model, without specifying whether products are homogeneous or differentiated,[4] and whether uncertainty affects demand or costs, other authors have explored more specific models and analyzed the impact of changes in parameters such as the level of uncertainty and the level of differentiation among products, on the equilibrium output and debt levels. This type of approach is fol-

[4] According to Mathis and Koscianski (2002, 443) "The second criterion for defining a market structure is the degree of product differentiation across the goods sold by different firms in the market". An homogeneous product oligopoly means that firms produce identical products, whereas in a differentiated product oligopoly, firms produce similar products but which are not identical.

lowed by Wanzenried (2003), Frank and Le Pape (2008) and Haan and Toolsema (2008) who analyze a two-stage differentiated goods duopoly model with demand uncertainty. Frank and Le Pape (2008) only analyze Cournot competition whereas Haan and Toolsema (2008) use numerical analysis to study how the equilibrium is affected by demand uncertainty and the substitutability of products both under Cournot and Bertrand competition. Frank and le Pape (2008) conclude that the bankruptcy probability decreases with the demand uncertainty and the increase of the degree of substitutability. Haan and Toolsema (2008) reach the same conclusion in both competition model: Bertrand and Cournot. Wanzenried's (2003) conclusions are similar, but their contribution is the analysis of the welfare effects of debt issue.

The impact of debt in predatory behavior models is quite different. According to Istaitieh and Fernández (2006) in predatory models "leveraged firms decrease output, the unleveraged rival has the incentive to increase output or cut prices to drive the leveraged firm out of the market". The works of Bolton and Scharfstein (1990), Khanna and Tice (2000), Kovenock and Phillips (1997) and Glazer (1994) are highlighted. Bolton and Scharfstein (1990) defend that agency problems have a predatory effect on financing decisions; debt means that firms have a less aggressive behavior in the output market. Khanna and Tice (2000) concluded that the biggest and most profitable firms are more aggressive than the most indebted. Kovenock and Phillips (1997) argued that firms following a recapitalization and belonging to the most concentrated industries are more likely not to invest and not survive. Glazer (1994) analyzes the effect of debt maturity in the output market decisions. According to the author, firms with a debt with longer maturity tend to behave less aggressively in the output market. From the analysis of the previous works we conclude that predatory behavior models reach different conclusions from models that emphasize the role of limited liability because they incorporate in the model some aspects not considered in models that emphasize the role of limited liability. These aspects include, in particular, agency problems, imperfections in the capital market and the effect of the investment.

The work of Povel and Raith (2004) incorporates the two models (limited liability theory and predatory behavior model) the authors derive the optimal contract debt in a duopoly model with competition in prices and quantities. Aggressive behavior caused by the increase in debt depends, according to the authors, on the existing financial constraints.

Other authors analyze the relationship between the product market with investment in R&D, as can be seen in Vives (2008) and Sacco and Schmutzler's (2011) works. The authors studied the relationship between product market and capital invested in R&D. The R&D investment can be interpreted as a differentiating factor that can lead to product differentiation or the differentiation of the production process. The authors conclude that there is a U-shaped relation between competition and R&D investment, except in the case of firms that were initially less inefficient. The study by Jensen and Showalter (2004) emphasizes the link between product market, capital structure and R&D investment in patent races. This study argues that high levels of debt are associated with lower investments in patent races. This is supported in the empirical study.

Table 2.1 summarizes the differences between the aforementioned studies. From the analysis of table 2.1 we can conclude that the studies linking the capital structure with output market competition use mostly the Cournot duopoly model and develop limited liability models.

It should be highlighted that the existing empirical work relating financial and output market decisions clearly confirms the strategic role of debt on the output market. However, the sign of the impact of greater leverage on the output market is not so clear-cut. Table 2.2 summarizes the empirical literature on the strategic role of debt. From the analysis of table 2.2 we can conclude that the works of Campos (2000), Erol (2003), Lyandres (2006) are consistent with the role of limited liability i.e. debt incites a more aggressive behavior on prices or quantities. The works of Chevalier (1995b), Khanna and Tice (2000) and Zingales (1998) are more consistent with the predatory behavior models, i.e., debt leads to a less aggressive behavior. Studies of Jensen and Showalter (2004), Opler and Titman (1994), Valta (2012) and Zhang (2012) confirms the strategic role of debt.

Table 2.1: Studies relating capital market structure with product market competition.

	Market structure			Competition		Model	
	Monopoly	Duopoly	Oligopoly	Bertrand	Cournot	Limited liability	Predatory behavior
Bolton and Scharfstein (1990)		X					X
Brander and Lewis (1986)			X		X	X	
Clayton (2009)		X			X	X	
Frank and Le Pape (2008)		X			X	X	
Glazer (1994)			X		X	X*	X*
Grimaud (2000)		X			X	X	
Haan and Toolsema (2008)		X		X	X	X	
Jensen and Showalter (2004)		X			X		
Khanna and Tice (2000)**							X
Kovenock and Phillips (1997)**							X
Maksimovic (1988)			X		X	X	
Povel and Raith (2004)		X		X	X	X	X
Sacco and Schmutzler (2009)		X			X		
Vives (2008)			X	X	X		
Wanzenried (2003)		X			X	X	

* – Depends on the level of debt; ** – Empirical paper

Table 2.2: Empirical studies relating capital market structure with product market competition.

Author	Sample / Period	Competition	Main Results
Campos (2000)	1196 Spanish manufacturing firms 1991 and 1994	Bertrand and Cournot	The legal status of the firms affects their behavior. Limited liability firms behave more aggressively in prices and quantities when short-term debt increases. Long-term debt has an opposite effect.
Chevalier (1999)	Supermarket Leveraged Buyouts (LBOs)	Cournot	The announcement of a LBO leads to an increase in the expected profit of rival firms and to a less aggressive behavior in the output market
Erol (2003)	Turkish firms 1989–1999	Bertrand	Short-term debt leads to higher price. Long-term debt has the opposite effect.
Jensen and Showalter (2004)	6747 U.S firms 1991 to 2000	Cournot	Debt has a negative effect on R & D investment
Khanna and Tice (2000)	Discount department store ,862 different markets 1975 to 1996		The biggest and most profitable firms are more aggressive than most indebted.
Lyandres (2006)	Nonfinancial Firms, 1950–2003	Bertrand and Cournot	There is a positive relationship between the optimal debt structure and the extent of competitive interactions (Cournot and Bertrand).
Opler and Titman (1994)	105.075 observations of troubled industrial firms 1972 to 1991		There is a positive relationship between financial condition and performance of the firm when there is an industry downturn.
Valta (2012)	12256 loans 2900 firms, 1992 to 2007	Cournot	Firms operating in more competitive environments have higher cost of debt.
Zhang (2012)	U.S manufacturing firms 1998 to 2009	Cournot	A firm takes into account the rivals capital structure decisions to decide its own structure
Zingales (1998)	Transportation firms 1976 to 1985	Bertrand	The level of debt has a negative impact on the firms' ability to invest in the post deregulation. The level of debt prederegulation negatively affects pricing.

2.3 Bankruptcy Probability

Over time, many studies have been developed to assess the best financial distress and bankruptcy prediction model. The main objective of these studies is the classification of firms with “financial health” and the prediction of bankruptcy risk and/or insolvency. Political, social and economic consequences are the main reasons why many researchers focus on the best model(s) for the prediction of the probability of financial distress.

The definition of default is not consensual. Beaver (1966) defines default as the firm's inability to meet its obligations as they mature, this can be checked if there is bankruptcy, bond default, an overdrawn bank account, or nonpayment of a preferred stock dividend. Article 3 of the Código dos Processos de Recuperação da Empresa e da Falência (CPEREF) defines an insolvent firm as one that in the lack of own resources and lack of credit, is unable to meet its obligations punctually. There are certain facts to classify a firm as insolvent, such as: non-compliance with tax and social security, non-compliance in the payment of wages, failure to comply with the repayment of loans and interest payments, entry of the application in recovery process and bankruptcy. This setting is used by Altman (1968), Deakin (1972), Stiglitz (1972), Zavgren (1985), Brander and Lewis (1986), Frank and Le Pape (2008) and Haan and Toolsema (2008), Pindado, Rodrigues and de la Torre (2008) and Clayton (2009). According to Dwyer and Kocagil (2004, 5) “the proposals for the new Basel Capital Accord (BIS II) have stimulated debates about what constitutes an appropriate definition of default. RiskCalc applies the criteria used by most of the advanced economies in the world. Default is defined as any of the following events:

- 90 days past due
- Bankruptcy
- Placement on internal non-accrual list
- Write-down"

The study of the default probability is intended to work mostly as a warning sign for certain organizational entities. A high number of bankruptcies originate not only in a rise in unemployment as well as a substantial increase of direct and indirect firm costs. Altman (1984) estimated that the increase in these costs, in the case of U.S. firms, represent about 20% of the firm value assets. In addition to the firm, the agents that are most affected by default, are the creditors, who see their costs increase, not only the legal and administrative costs related to debt collection, but also the substantial loss of credit (capital and interest). According to Altman and Saunders (1998) the dramatic evolution of models for measuring credit risk, is not only due to the increasing of the number of bankruptcies, but also to the need to hold more and better information about the debtor, the existence of more competitive margins on loans; the decline of the value of real assets which implies lower collateral; and the dramatic growth of risk management instruments and credit derivatives. The equity holders of the firm sees probability of dilution of equity or total loss increase when the probability of default increases.

Thus, the analysis of the bankruptcy probability aims mainly to serve as a management tool or information decision support, not only for the firm, but also for all stakeholders who directly or indirectly have any relationship with the firm. For all other entities (credit institutions, suppliers, creditors, etc...) those which hold or may have some relationship with firms that have some probability of default, allows them to make a set of preventive measures and adjust.

There exists a proliferation of models of bankruptcy probability analysis. Amongst the existing studies, we highlight the work of Beaver (1966), a pioneer in the research of the insolvency through financial ratios, using univariate analysis. The use of multiple linear discriminant analysis arose from the work of Altman (1968), a crucial milestone for the study of financial distress. The Z-Score and Zeta models, developed by Altman (1968) and Altman, Haldeman and Narayanan (1977), respectively, are the most discussed techniques in empirical studies. Zavgren (1985) developed a model of bankruptcy prediction based on the logistic analysis (Logit). Zmijewski (1984) used the Probit model to estimate the probability of financial distress.

However, what distinguishes the aforementioned studies is only the applied statistic model. None of the previous studies analyzed the essence of the bankruptcy probability, i.e. how the probability varies with the financial structure and market output decisions in a competitive and uncertain market. This analysis is depicted in the following chapter.

2.4 Studies that relate capital structure, output market and bankruptcy probability

Despite the vast literature on bankruptcy probability, the existing literature is constituted essentially by prediction models. In other words, there is a lack of theoretical models to explain bankruptcy probability. The probability of default depends not only on the level of debt, but also on operational factors that allows a firm to meet its obligations. The relationship between the decisions about the financial structure, the market output and the bankruptcy probability has been analyzed theoretically by a small number of authors. This was discussed in more depth in the work of Frank and Le Pape (2008) and Haan and Toolsema (2008). Frank and Le Pape (2008) and Haan and Toolsema (2008) used numerical simulations to analyze the impact of demand uncertainty and the degree of product differentiation on the probability of bankruptcy risk. The authors come to similar conclusions; the probability of bankruptcy is decreasing with the degree of product differentiation when goods are complementary and it is increasing with the degree of product differentiation when the goods are substitutes. Moreover, the probability of default is decreasing with the level of uncertainty. However the authors assume some assumptions: firms are symmetric, demand is linear, there is a uniform distribution and marginal costs are constant. It is important to analyze all possible situations for the degree of uncertainty and see how the analysis is modified if firms are not symmetric in a competitive market.

It should be highlighted that there are some empirical works that relate financial decisions, output market decisions and the bankruptcy probability. Table 2.3 summarizes some recent empirical works. By analyzing table 2.3 we can see that empirical studies demonstrate that there is a relationship between debt and bankruptcy probability (Antunes et al, 2010 and Chacharat et al, 2010) and between debt, quantity or price delivery, market conditions (higher or lower concentration, industry performance) and bankruptcy probability (Borenstein and Rose, 1995, Evrensel, 2008 and Opler and Titman, 1994).

Table 2.3: Recent empirical work on the bankruptcy probabilty.

Authors	Relate capital and output market	Sample/Period	Main Results
Antunes et al (2011)	X	Non-financial and credit portuguese firms 1995 to 2000	Debt influences bankruptcy. Effect is higher in bankruptcy than in voluntary liquidation
Borenstein and Rose (1995)	X	7 bankruptcy US air carriers 1989 and 1992	Bankruptcy affects the pricing of bankrupt firms, they reduce on average their price by 5% or 6%
Chancharat et al (2010)	-	1081 Australians non-financial listed firms 1989 and 2005	Financial distress firms have lower debt, more efficient use of capital compared with active firms.
Evrensel (2008)	X	50 banks failures of 79 countries 1980 to 1997	Banking concentration reduces the bankruptcy risk, macroeconomic variables affect bankruptcy.
Opler and Titman (1994)	X	105075 observ. of troubled industrial firms 1972 to 1991	Positive relation between financial condition and performance of the firm in industry downturns.

2.5 Conclusion

This chapter revised the literature on the relationship between three themes essential to the success of a firm: the decisions on capital structure, market output decisions and bankruptcy probability. We found that there are already some studies that relate the first two topics. Regarding the probability of bankruptcy it appears that the vast majority of the literature presents models to predict the probability of bankruptcy, distinguishing themselves according to the technique used to forecast. However there is a gap in the literature concerning the relationship between financial decisions, output market decisions and bankruptcy probability, considering that the market is uncertain and competitive.

Chapter 3

Research Methodology

3.1 Introduction

The aim of this work is to study whether bankruptcy probability depends on the financial structure and output market decisions. In particular, we want to analyze the impact of changes in the level of demand uncertainty, the degree of product substitutability and the degree of asymmetry in level of efficiency on the equilibrium bankruptcy probability. After the analysis of the existing literature in the area we conclude that there is a shortage of studies relating financial decisions with output market decisions and bankruptcy probability for asymmetric firms. The few existing studies limit their analysis to a scenario of symmetry between firms. To achieve this objective the following guidelines were developed:

- Modeling of the problem taking into account the aforementioned studies;
- Problem resolution, dividing the resolution into two steps;
- Numerical analysis of the problem under study;
- Presentation of the main results.

3.2 Model

This dissertation considers a particular case of Brander and Lewis (1986) model, where the duopolists produce differentiated products, demand is linear, marginal costs are constant and the uncertainty in the model is on the demand side (this model has been considered by other authors, such as Haan and Toolsema (2008), Wanzenried (2003)).

We are facing a typical situation of strategic interaction, where the decisions of a firm affect their payoffs and the payoffs obtained by the other firm. This interaction is modeled using non-cooperative game theory (according to Schmalensee and Willig (2003,261) "Non-cooperative game theory is a way of modeling and analyzing situations in which each player's optimal decisions depends on his beliefs or expectations about the play of his opponents". The modeling in game theory involves the definition of players, strategies and payoffs. According to Mathis and Koscianski (2002, 476 and 477) the players in a game are defined "as rational decision makers with the goal of selecting the strategy that yields the best payoff to the player, given the strategies available to the other players in the game". The strategies are "the set of all alternative choices available to all the players in a game". The payoffs are "the return a player in a game receives from selecting a particular strategy, contingent on the strategies chosen by other players in the game".

In our case there are two firms that decide (firm i and firm j) and we consider a two stage duopoly Cournot model with product differentiation. In the first stage each firm (firm i and firm j) decides the financial structure, i.e., the level of debt and equity in the capital structure. In the second stage each firm takes its decision on the output market. We start by analyzing the case of symmetric constant marginal costs; then we analyze the case where the two firms have different marginal costs, thus we have an asymmetric duopoly model. Figure 3.1 shows the timing of the game.

Let q_i and q_j be the output of firms i and j, p_i and p_j be the price of firms i and j. q_0 represents the quantity consumed of all other products (with a price normalized to unity). The parameter γ corresponds to the degree of substitutability between the products, with $\gamma \in [0, 1]$. When $\gamma = 0$, products are completely differentiated, thus each

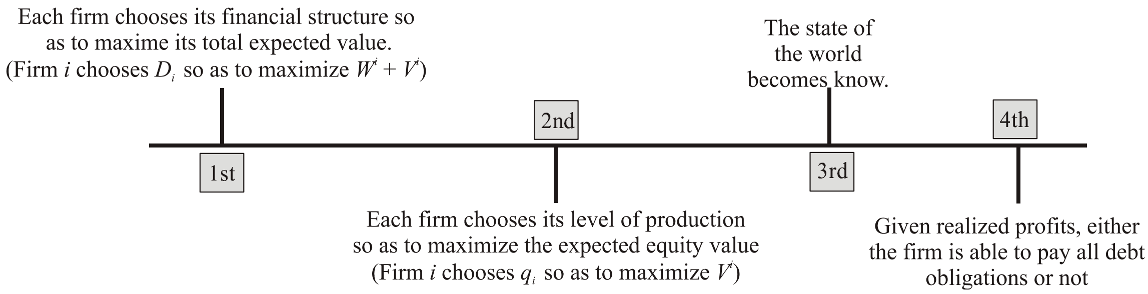

Figure 3.1: Timing of the game

firm can behave as a monopolist. When $\gamma = 1$, the two products are perfect substitutes. Parameter α_i and α_j represent the expected size of the market, α_i , α_j , β_i and β_j are positive constant. Following Dixit (1979) and Singh and Vives (1984), we assume that the utility function is quadratic:

$$U_{(q_i,q_j,q_0)} = q_0 + \alpha_i\, q_i + \alpha_j\, q_j - \frac{1}{2}\left[\beta_i q_i^2 + 2\gamma q_i q_j + \beta_j q_j^2\right] \tag{3.1}$$

Furthermore, we assume that $\alpha_i \;=\; \alpha_j \;= \alpha$, and $\beta_i = \beta_j = 1$. The consumer chooses q_i, q_j, q_0 so as to maximize U subject to $p_i q_i + p_j q_j + q_0 = M$. The budget constraint can be written as $q_0 = M - p_i q_i - p_j q_j$. Substituting this expression in the utility function, the consumer's problem can be rewritten as:

$$\max_{q_i,q_j} \;\; M - p_i q_i - p_j q_j + \alpha q_i + \alpha q_j - \frac{1}{2}\left[q_i^2 + 2\gamma q_i q_j + q_j^2\right]$$

The first order conditions of this problem are:

$$\begin{cases} -p_i + \alpha - q_i - \gamma q_j = 0 \\ -p_j + \alpha - q_j - \gamma q_i = 0 \end{cases} \Leftrightarrow \begin{cases} p_i = \alpha - q_i - \gamma q_j \\ p_j = \alpha - q_j - \gamma q_i \end{cases}$$

The first order conditions define the inverse demand, which is given by:

$$p_i = \alpha - q_i - \gamma q_j + z_i \tag{3.2}$$

The observed size of the market depends on the random variable z_i that represents the effect of an exogenous demand shock, in other words, there is uncertainty regarding the size of the market. It is assumed that this variable is distributed in the interval $[-\overline{z};\overline{z}]$ according to the uniform density function, i.e., $f(z_i) = \frac{1}{2\overline{z}}$. We assume that z_i and z_j are independent and identically distributed.

In the first stage, firms choose simultaneously their debt levels so as to maximize the value of the firm. The value of the firm is equal to the sum of the equity value and debt value. We represent the debt obligation of the firm i by b_i. b_i is the amount that the firm i pays at the end of the game to bondholders, if it has sufficient operating profits to do so. If the realized operating profits are less than b_i, all the operating profits obtained will be used to pay bondholders. The operating profits (revenue less costs) are given by:

$$R_i = (\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i \tag{3.3}$$

In the second stage of the game the manager maximizes the expected value of the firm to shareholders. The expected equity value is given by:

$$V^i(q_i, q_j, b_i, \overline{z}) = \int_{\widehat{z}_i}^{\overline{z}} ((\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i - b_i) \frac{1}{2\overline{z}} dz_i \tag{3.4}$$

where $\widehat{z}_i(q_{i,}q_{j,}b_i)$ is the critical value of z_i such that the operating profit of the firm is just enough for the firm to meet its debt obligations. This critical state of the world is implicitly defined by:

$$(\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i - b_i = 0 \tag{3.5}$$

It should be noted that the critical state of the world is influenced by the quantity choices of the two firms and by the firm's debt level. This implies that $\widehat{z}_i(q_{i,}q_{j,}b_i)$ is determined endogenously in the second stage of the game.

In the first stage firms choose simultaneously their debt levels so as to maximize the value of the firm. The value of the firm is equal to the sum of the equity value and debt value. The expected value of debt is given by:

$$W_i(q_i, q_j, b_i, \overline{z}) = \Pr(z_i > \widehat{z}_i)b_i + \int_{-\overline{z}}^{\widehat{z}_i} (\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i \frac{1}{2\overline{z}} dz_i. \tag{3.6}$$

Note that b_i is different from W_i. b_i is the amount that the firm i pays at the end of the game to bondholders, which includes capital amortization and interest. W_i is the expected value of debt, which takes into account the probability of the firm not paying in full b_i, i.e. if this probability is positive $W_i < b_i$.

Thus, the value of the firm is equal to the expected operating profits of the firm:

$$\begin{aligned} Y^i(q_i, q_j, b_{i,}\overline{z}) &= V^i(q_i, q_j, b_{i,}\overline{z}) + W^i(q_i, q_j, b_{i,}\overline{z},) \\ &= \int_{\widehat{z}_i}^{\overline{z}} ((\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i - b_i) \frac{1}{2\overline{z}} dz_i + \\ &\quad \Pr(z_i > \widehat{z}_i)b_i + \int_{-\overline{z}}^{\widehat{z}_i} (\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i \frac{1}{2\overline{z}} dz_i \\ &= \int_{-\overline{z}}^{\overline{z}} (\alpha - q_i - \gamma q_j + z_i - c_i)\, q_i \frac{1}{2\overline{z}} dz_i \end{aligned} \tag{3.7}$$

The interest rate r is defined implicitly by $W_i(1 + r) = b_i$. The welfare is given by

$$Wel = \int_{-\overline{z}}^{\overline{z}}\int_{-\overline{z}}^{\overline{z}} \left((\alpha + z_i)\, q_i + (\alpha + z_i)\, q_j - \frac{1}{2}\left(q_i^2 + 2\gamma q_i q_j + q_j^2\right)\right) \frac{1}{4\overline{z}^2} dz_i dz_j - c_i q_i - c_j q_j \tag{3.8}$$

This is a game with two decision stages and to determine the equilibrium (optimal decisions for firms) it is necessary to solve the game using the concept of subgame perfect Nash equilibrium (SPNE). The game is solved backwards, that is one starts by determining the Nash equilibrium[1] in the second stage of the game; in this case, we start by computing

[1] It is said that a combination of strategies is a Nash Equilibrium when each strategy is the best possible response to the strategies of the other players, and this is true for all players, i.e. no player gains by unilaterally deviating.

the Nash equilibrium of the second stage game as a function of the debt levels chosen by the firms in the first stage. Then we solve the first stage game. In this stage firms take their financial decisions, considering their impact on the output market equilibrium, so as to maximize the value of the firm, thus determining, the SPNE.[2]

In order to solve the model it is often necessary to use the chain rule, the fundamental theorem of integral calculus and the Leibniz rule. We are facing a maximization problem (where the maximum is obtained by solving the first order conditions) whose objective function in the first stage is given by (3.7) and in the second stage is given by (3.4). In both cases, the objective function involves parametric integrals. According to Pires (2011, 260) an integral is defined as parametric as long as in the integral only one variable is the variable of integration (the variable of integration is z_i). The following theorem Pires (2011, 261) will be frequently applied:

Theorem 3.1 *Let $\varphi_1(x)$ and $\varphi_2(x)$ be differentiable functions in $[a, b]$ defined on a closed interval $[c, b]$. Considering that $f(x, y)$ and $f'_x(x, y)$ are continuous in $[a, b] \times [c, b] \subset \mathbb{R}^2$ then the function:*

$$A(x) = \int_{\varphi_2(x)}^{\varphi_1(x)} f(x, y)dy$$

is differentiable on the interval $[a, b]$ and:

$$A'(x) = \int_{\varphi_2(x)}^{\varphi_1(x)} f'_x(x, y)dy + f(x, \varphi_2(x)\varphi'_2(x) - f(x, \varphi_1(x)\varphi'_1(x).$$

To better follow the model resolution, table 3.1 summarizes the variables used:

[2]According to Selten (1965) cited by Gibbsons (1992) “A Nash Equilibrium is a sub-game-perfect if the players’ strategies constitute a Nash equilibrium in every subgame”.

Table 3.1: Variables of the model.

Variables	**Meaning**
q_i, q_j	Output of firms i and j
b_i, b_j	Debt obligation of firms i and j
R^i	Operating profits of firm i
z_i	Random variable that represents the uncertainty regarding the size of the market
$\widehat{z}_i$	Critical value of z_i
γ	Degree of substitutability
α_i	Size of the market
Y^i, Y^j	Firm value
V^i, V^j	Expected equity value
W^i, W^j	Expected value of debt
θ_i, θ_j	Bankruptcy probability

3.3 Solving the model

3.3.1 Nash equilibrium in the second stage of the game

In the second stage of the game, firm i chooses its quantity, q_i, so as to maximize the equity value (3.4). Using the Leibniz rule, the first-order condition of this maximization problem is:

$$\int_{\widehat{z}_i}^{\overline{z}} \left(\alpha - 2q_i - \gamma q_j + z_i - c_i\right) \frac{1}{2\overline{z}} dz_i - \left[\left(\alpha - 2q_i - \gamma q_j + \widehat{z}_i - c_i\right) q_i - b_i\right] \frac{1}{2\overline{z}} dz_i = 0$$

However, taking into account the definition of $\widehat{z}_i$, the second term is equal to zero. Thus, after integrating the first term, the first-order condition is given by:

$$\overline{z} + 2\alpha - 4q_i + \widehat{z}_i - 2\gamma q_j - 2c_i = 0$$

The first order condition for firm j is derived in a similar manner. Note that the first order conditions depend on the critical states of the world, $\widehat{z}_i$ and $\widehat{z}_j$, which in turn depend on q_i and q_j. This implies that, in order to get the Nash equilibrium of the second stage game, we need to solve simultaneously the system of first order conditions and the two conditions that define the critical states of the world. In other words, for an interior solution (i.e., for $-\overline{z} < \widehat{z}_i < \overline{z}$), the Nash equilibrium is given by the solution of the following system:

$$\begin{cases} \overline{z} + 2\alpha - 4q_i + \widehat{z}_i - 2\gamma q_j - 2c_i = 0 \\ \overline{z} + 2\alpha - 4q_j + \widehat{z}_j - 2\gamma q_i - 2c_j = 0 \\ \left(\alpha - q_i - \gamma q_j + \widehat{z}_i - c_i\right) q_i - b_i = 0 \\ \left(\alpha - q_j - \gamma q_i + \widehat{z}_j - c_j\right) q_j - b_j = 0 \end{cases}$$

It turns out that this system does not have an analytical solution. In fact, through substitution, it can be shown that solving this system is equivalent to solving a polynomial equation of the fourth order, which does not have a closed form solution.

It should be noted that, for some values of (b_i, b_j) one or both of the critical states of the world may be equal to $-\overline{z}$ or equal to $\overline{z}$. In these cases the third and/or fourth need to be substituted by $\widehat{z}_i = -\overline{z}$ or $\widehat{z}_i = \overline{z}$. A complete analysis of all the possible Nash equilibria involve computing these corner solutions.

Since the model cannot be solved analytically, we developed a GAUSS code to solve the model numerically (the code is presented in the Appendix). Considering the various types of possible equilibria, we ran simulations for many values of the parameters γ and $\overline{z}$ (so as to analyze how the equilibrium changes with the parameter values). For each set of parameter values, we determine the Nash Equilibrium of the second stage game, for many possible combinations of the debt levels (b_i, b_j). Let $q_i^*(b_i, b_j)$, $q_j^*(b_i, b_j)$, $\widehat{z}_i^*(b_i, b_j)$ and $\widehat{z}_j^*(b_i, b_j)$ be the Nash equilibrium quantities and critical states of the world for given debt levels (b_i, b_j).

3.3.2 Subgame Perfect Nash equilibrium

After computing the Nash equilibrium of the second stage game as a function of the debt levels chosen by the firms in the first stage we solved the first stage game using backwards induction. In this stage, firms take their financial decisions, considering their impact on the output market equilibrium, so as to maximize the value of the firm, thus determining the Subgame Perfect Nash Equilibrium (SPNE).

As mentioned above, we developed a GAUSS code to solve the model numerically, considering the various types of possible equilibria, for many values of the parameters γ and $\overline{z}$ (so as to analyze how the equilibrium changes with the parameter values). The program first determines the Nash Equilibrium of the second stage game, for given debt levels (b_i, b_j), and then for each (b_i, b_j) the equilibrium value of each firm (Y_i, Y_j), is computed. This is repeated for many (b_i, b_j) and the equilibrium values of Y_i and Y_j are saved in two matrices. The equilibrium of the first stage game is then determined. We identify, for a given debt level of the other firm, the firm's level of debt that maximizes its value, thus determining the firm's best response. The Nash equilibrium of the debt game occurs when we find a vector (b_i^{**}, b_j^{**}) such that the two firms are simultaneously in their best responses. Thus (b_i^{**}, b_j^{**}) denotes the SPNE levels of debt. Finally, considering (b_i^{**}, b_j^{**}) the corresponding SPNE quantities (q_i^{**}, q_j^{**}) of the second stage game are computed as well as other equilibrium variables like the bankruptcy probabilities $(\theta_i^{**}, \theta_j^{**})$,the equilibrium interest rate (r_i^{**}, r_j^{**}) and so on.

Applying the Leibniz rule to the expected value of debt, the equilibrium expected value of debt in the first stage of the game is given by:

$$W_i^{**} = (1-\theta_i^{**})b_i^{**} + \frac{1}{4\overline{z}} q_i^{**} \left[\left(\widehat{z}^{**} + \overline{z} \right) \left(2\alpha - 2c_i - 2q_i^{**} - 2\gamma q_j^{**} \right) + \left(\left(\widehat{z}_i^{**} \right)^2 - \overline{z}^2 \right) \right] \quad (3.9)$$

Applying the Leibniz rule to the expected value of equity, the equilibrium expected value of equity in the first stage of the game is given by:

$$V_i^{**} = \frac{1}{4\overline{z}} \left[q_i^{**} \left[\left(\overline{z} - \widehat{z}_i^{**} \right) \left(2\alpha - 2q_i^{**} - 2c_i - 2\gamma q_j^{**} \right) + \left(\overline{z}^2 - \left(\widehat{z}_i^{**} \right)^2 \right) \right] - 2b_i^{**} \left(\overline{z} - \widehat{z}_i^{**} \right) \right] \quad (3.10)$$

The interest rate r is defined implicitly by $W_i(1+r) = b_i$ so, in the SPNE:

$$r^{**} = \frac{b_i^{**}}{W_i^{**}} - 1 \quad (3.11)$$

The bankruptcy probability is given by

$$\theta_i^{**} = \frac{\widehat{z}^{**} + \overline{z}}{2\overline{z}} \tag{3.12}$$

Applying the Leibniz rule in the expected value of welfare, the equilibrium expected welfare level in the first stage of the game is given by:

$$Wel^{**} = \alpha(q_i^{**} + q_j^{**}) - \frac{1}{2}\left((q_i^{**})^2 + 2\gamma q_i^{**} q_j^{**} + \left(q_j^{**}\right)^2\right) - c_i q_i^{**} - c_j q_j^{**} \tag{3.13}$$

There are certain combinations of the parameters γ and $\overline{z}$ that originate multiple equilibria. Such situation occurs mainly for γ close to 0 and $\overline{z} < 1$, so we decided to eliminate these observations. We chose to use $\alpha = 5$ and $\overline{z}_{\max} = 2$. The chosen values do not affect the results qualitatively. To avoid areas where b_i does not intersect within the feasible areas, we defined an upper bound of debt. The debt cannot be higher than the expected monopoly profits. We defined a value that is between the monopoly profit and the duopoly profit as the upper bound of debt.

Chapter 4

Analysis of the results

4.1 Introduction

In this chapter we present the results of our numerical simulations. The analysis is divided into two parts. The first part analyzes the symmetric case ($c_i = c_j = 0$), where both firms are equally efficient. This case replicates Haan and Toolsema (2008) but using a more general numerical model as we also study cases where the equilibrium involves corner solutions for the critical states of the world. The second part examines the effect of cost asymmetries on the equilibrium debt levels, output levels, the value of each firm, the implicit interest rates, the bankruptcy probabilities and the welfare. To the best of our knowledge, the second part of the work is new and thus presents an interesting contribution to the literature relating capital market structure, output market competition and bankruptcy risk.

In our numerical simulation, we consider $\overline{z}$ values in the interval $[0, 2]$ and γ values in the interval $[0, 1]$. After the analysis of the values obtained, we concluded that low values of $\overline{z}$ ($\overline{z} < 0.8$) and γ ($\gamma < 0.1$) that originate multiple equilibria, some with inconsistent results, so we decided to restrict our analysis to feasible areas ($\overline{z} \geq 0.8$ and $\gamma \geq 0.1$). It should be noted that this type of restriction was also considered by Haan and Toolsema (2008) who excluded from the analysis small values of $\overline{z}$ and γ, values that are near to the extremes. They also did not consider large values for the parameters, to avoid corner solutions.

We chose to use $\alpha = 5$, the choice of values has not a significant influence on the final qualitative results. For the amount of debt, debt cannot be higher than expected monopoly profits, so we decided to use a weighted average of monopoly and duopoly profits as the upper bound of debt.

4.2 Symmetric duopoly

This subsection studies the results of a symmetric two stage duopoly model where firms first decide their financial structure and next choose the quantity to produce. Our analysis is focused on the equilibrium of the whole game (the subgame perfect equilibrium).[1] The objective is to analyze how the equilibrium values of the variables change with the uncertainty level, $\overline{z}$, and with the degree of product substitutability, γ.

Figures 4.1 and 4.2 show the equilibrium levels of debt as function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. These figures allow us to conclude the following:

Result 4.1 *The SPNE level of debt, b^{**}, is strictly positive and decreasing with the level of demand uncertainty, $\overline{z}$. On the other hand, the degree of product substitutability does not have a monotonic impact on b^{**}. For small values of demand uncertainty, b^{**} is decreasing with product substitutability. However, for higher values of demand uncertainty, b^{**} initially increases with γ but after a certain point follows a U relationship with γ.*

The results are consistent with the Frank and Le Pape (2008) work.

Figures 4.3 shows the equilibrium ouput level as function of the degree of product substitutability, γ and as a function of the level of demand uncertainty, $\overline{z}$. The figure allow us to conclude the following:

[1] It should be noted that one could also analyze the Nash equilibrium of the second stage, which is contingent on the debt levels chosen in the first stage of the game and study how it changes with the level of debt.

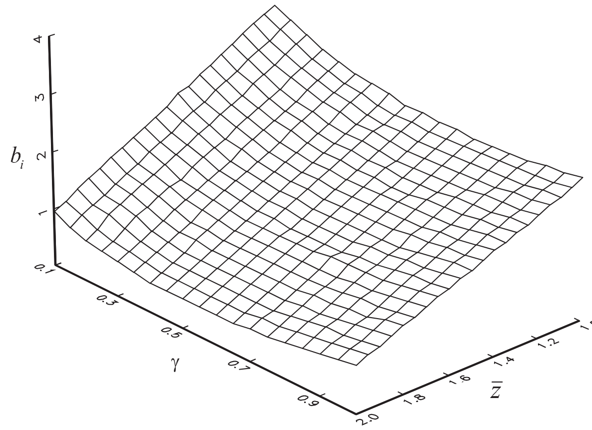

Figure 4.1: SPNE debt level as a function of the degree of product substitubility and the level of demand uncertainty.

Result 4.2 *The SPNE level output, q^{**}, is decreasing with the degree of product substitutability, γ and increasing with the level of demand uncertainty $\overline{z}$. However, the impact of demand uncertainty is relatively small.*

We found similar results to Frank and Le Pape (2008).

Regarding the effect of the demand uncertainty level, $\overline{z}$,on the equilibrium debt levels and on the output levels, the intuition of the results presented above is that, when the level of uncertainty is higher, for fixed debt level, firms have a more aggressive behavior in the output market. Intuitively, the increase in the uncertainty level implies that there are more good states of the world with positive marginal profits, thus the expected marginal profits conditional on $z_i > \widehat{z}_i$ increase, hence it is optimal to produce a higher quantity (note that increasing $\overline{z}$ also means that there are more states of the world with more negative marginal profits, but equityholders do no care about these states of the world, unless the firm is all equity financed, i.e. since they are protected by limited liability).

However, considering the result 4.1, firms can get the same strategic effect with a lower level of debt. Therefore, firms act in a more conservative manner in the debt market when uncertainty increases.

Figures 4.4 and 4.5 show that the equilibrium interest rate depends of the degree of

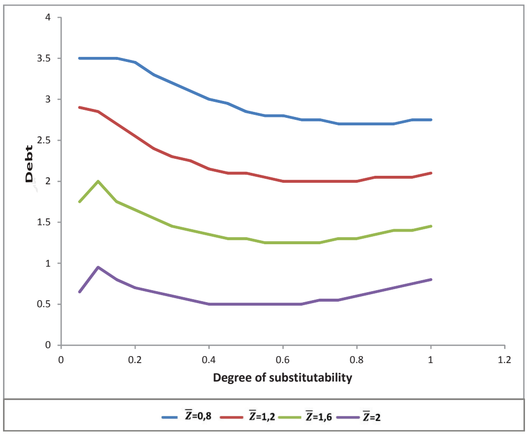

Figure 4.2: SPNE debt level as a function of the degree of product substitutability, for several values of demand uncertainty.

product substitutability, γ, and on the level of demand uncertainty, $\overline{z}$. These figures allow us to conclude the following:

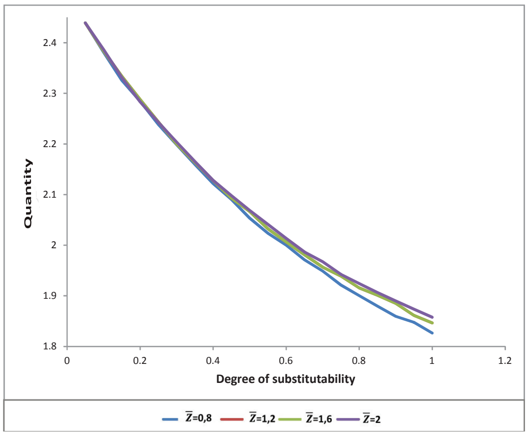

Figure 4.3: SPNE output level as a function of the degree of product substitutability.

Result 4.3 *The SPNE interest rate, r^{**}, is increasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$. The impact is more significant for high values of γ ($\gamma > 0.6$) and $\overline{z}$ ($\overline{z} > 1.6$).*

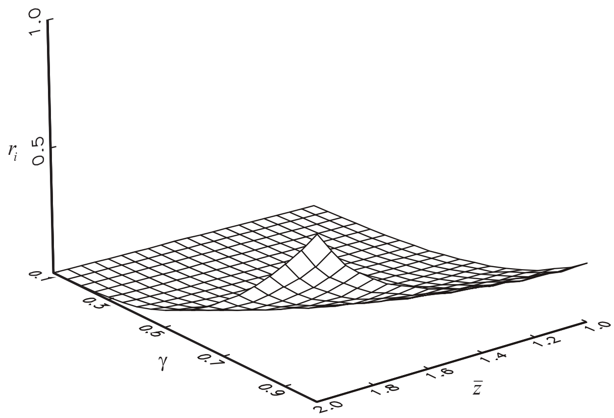

Figure 4.4: SPNE interest rate as a function of the degree of product substitutability and the level of demand uncertainty.

Figure 4.6 shows the equilibrium bankruptcy probability as function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. The figure allow us to conclude the following:

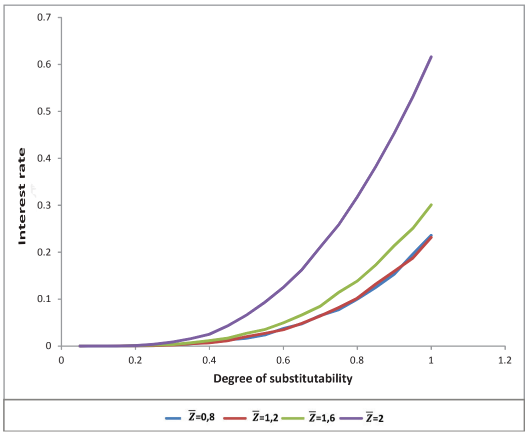

Figure 4.5: SPNE interest rate as a function of the degree of product substitutability for various levels of demand uncertainty.

Result 4.4 *The SPNE bankruptcy probability, θ^{**}, is increasing with the degree of product substuitability, γ, and it is decreasing with the level of demand uncertainty, $\overline{z}$.*

These results are consistent with the Haan and Toolsema (2008) work. With regard to the demand uncertainty the results confirm those obtained by Frank and Le Pape (2008).

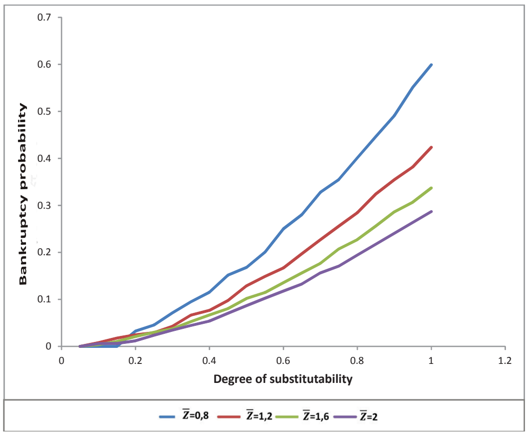

Figure 4.6: SPNE bankruptcy probability as a function of the degree of product substitutability for various levels of demand uncertainty.

Regarding the effect of the demand uncertainty level, $\overline{z}$,on the bankruptcy probability, if there is an increase in the level of uncertainty, the bankruptcy probability decreases.

We have three effects. The direct effect is that, for given debt and quantity levels, the increase in the uncertainty level, increases the bankruptcy probability. Regarding the indirect effects the fact that there is larger uncertainty leads firms to behave in a more aggressive manner in the output market. This effect tends to increase the bankruptcy probability. However, the greater uncertainty leads firms to be more conservative in the debt market, thus issuing less debt. A lower debt lowers the bankruptcy probability, directly and indirectly, through its influence on the second period equilibrium quantities. We conclude that the third effect dominates, i.e. firms behave less aggressively in the debt market when uncertainty is higher, which leads to lower equilibrium bankruptcy probabilities.

Figure 4.7 shows the equilibrium expected equity value depends of the degree of product substitutability, γ, and it depends on the level of demand uncertainty, $\overline{z}$. The figure allow us to conclude the following:

Result 4.5 *The SPNE expected equity value, V^{**}, is decreasing with the degree of product substitutability, γ, and it is increasing with the level of demand uncertainty, $\overline{z}$.*

The results confirm the results obtained by Frank and Le Pape (2008).

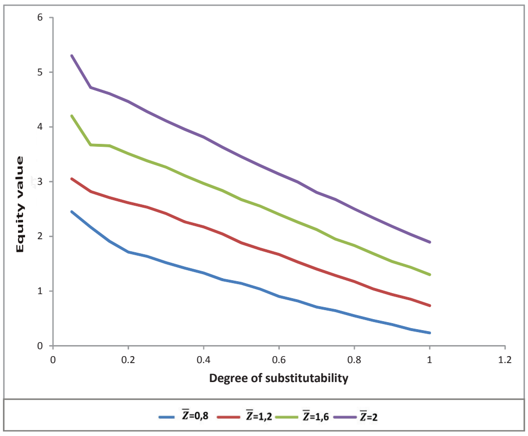

Figure 4.7: SPNE expected equity value as a function of the degree of product substitutability for various levels of demand uncertainty.

Figure 4.8 shows the equilibrium expected debt value as function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. The figure allows us to conclude the following:

Result 4.6 *The SPNE expected debt value, W^{**}, is decreasing with the level of demand uncertainty, $\overline{z}$. On the other hand, the degree of product substitutability does not have a monotonic impact on W^{**}. For small values of demand uncertainty, W^{**} is decreasing with product substitutability. However, for higher values of demand uncertainty, W^{**} initially increases with γ but after a certain point follows a U relationship with γ.*

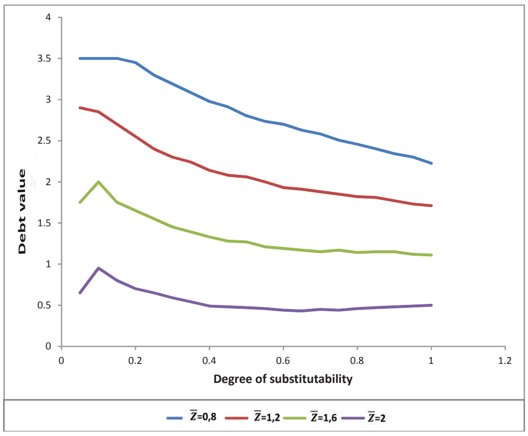

Figure 4.8: SPNE expected debt value as a function of the degree of product substitutability for various levels of demand uncertainty.

Figure 4.9 shows the equilibrium welfare depends on the degree of product substitutability, γ. The figure allow us to conclude the following:

Result 4.7 *The SPNE welfare level, Wel^{**}, is decreasing with the degree of product substitutability, γ, and decreasing with the level of demand uncertainty, $\overline{z}$. However the impact of demand uncertainty is relatively small.*

Figure 4.10 shows the equilibrium firm value as function of the degree of product substitutability, γ, and as a function of the level of demand uncertainty, $\overline{z}$. This figure allows us to conclude the following:

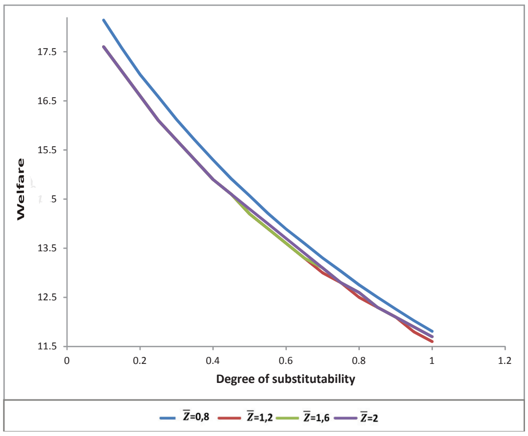

Figure 4.9: SPNE welfare as a function of the degree of product substitutability for various levels of demand uncertainty.

Result 4.8 *The SPNE firm value, Y^{**}, is decreasing with the degree of product substitutability, γ, and with the level of demand uncertainty, $\overline{z}$. However, the impact of the level of demand uncertainty, $\overline{z}$, is relatively small.*

The results confirm the results obtained by Haan and Toolsema (2008) and Franck and Le Pape (2008).

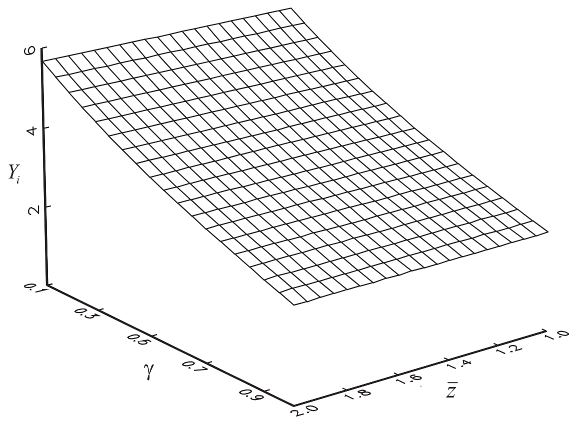

Figure 4.10: SPNE expected firm value as a function of the degree of product substitutability and the level of demand uncertainty.

4.3 Asymmetric duopoly

This subsection studies the results of an asymmetric two stage duopoly model where firms first decide their financial structure and next choose the quantity to produce. We consider that firms differ in their marginal production cost. Firm j has a null production cost while firm i has marginal cost c_i. We study what happens as firm i becomes less efficient by analyzing the SPNE as the marginal cost of firm i, c_i, varies between 0 (symmetric case) and 2.

This section aims to examine how the variables equilibrium levels (debt, output, implicit interest rates, bankruptcy probabilities, equity value, value of the firm and welfare) vary as the marginal cost of firm i increases (x-axis), considering three possible values for the degree of product substitutability, γ ($\gamma = 0.2, \gamma = 0.6$ and $\gamma = 1$). Three graphs are presented for each variable (the first corresponds $\overline{z} = 0.8$, the second $\overline{z} = 1.2$ and the third one with $\overline{z} = 1.6$), this subdivision allows to check if the behavior is stable with the level of demand uncertainty, $\overline{z}$.

Figures 4.11 and 4.12 show the debt level of the firm i and the debt level of the firm j as function of the marginal cost of production of the firm i, c_i. Figures allow us to conclude the following:

Result 4.9 *The SPNE level of debt of the firm i, b_i^{**}, is decreasing with the marginal cost of production of the firm i, c_i. The decrease is more pronounced for high levels of demand uncertainty, $\overline{z}$. However the SPNE level of debt of the firm j, b_j^{**}, is increasing with the marginal cost of production of the firm i, c_i. The increase is more pronounced for high levels of demand uncertainty, $\overline{z}$.*

Figures 4.13 and 4.14 show the output level of the firm i and the output level of the firm j as function of the marginal cost of production of the firm i, c_i. Figures allow us to conclude the following:

Result 4.10 *The SPNE level of output firm i, q_i^{**}, is decreasing with the firm's marginal cost of production, c_i. On the contrary, the SPNE level of ouput of firm j, q_j^{**}, is increasing*

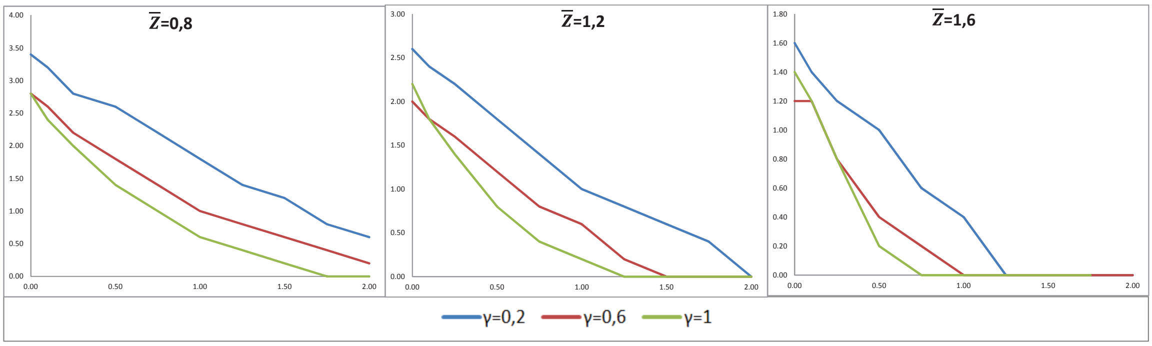

Figure 4.11: SPNE debt level of the more inefficient firm as a function of its marginal costs.

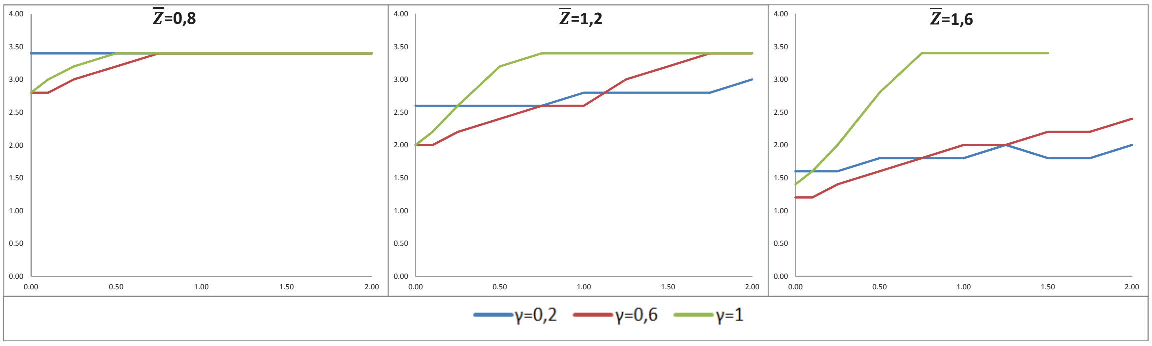

Figure 4.12: SPNE debt level of the more efficient firm as a function of the marginal costs of the rival.

with the rival's marginal cost of production.

Regarding the effect of the marginal production cost of firm i, c_i in the debt level and in the output level of the two firms, the results presented above show that as the firm becomes less efficient (i.e. their marginal production costs increases), the inefficient firm adopts a more conservative approach in the debt market and in the output market. The intuition for this result is that, an increase in the marginal production cost leads to a decrease in the marginal profit which implies a decrease in the debt and output levels.

The more efficient firm has the opposite behavior, i.e. it becomes more aggressive in the debt market and in the output market. These effects are more pronounced for high levels of uncertainty, which increases the volatility of marginal profit.

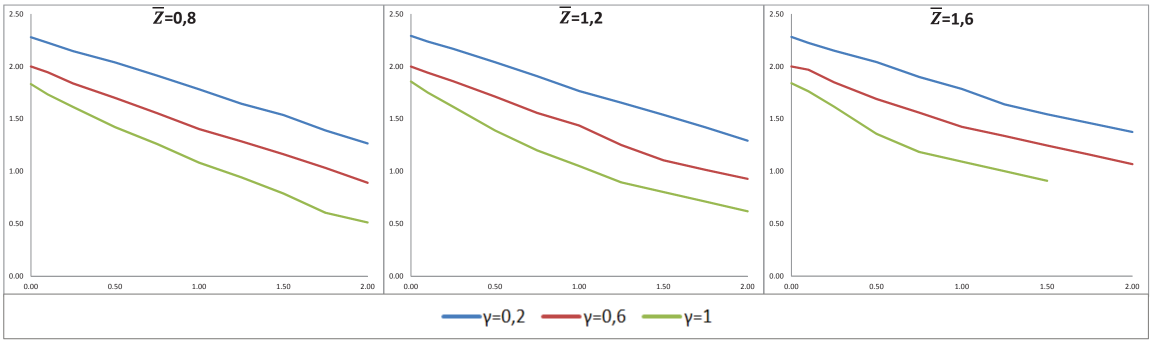

Figure 4.13: SPNE output level of the more inefficient firm as a function of its marginal costs.

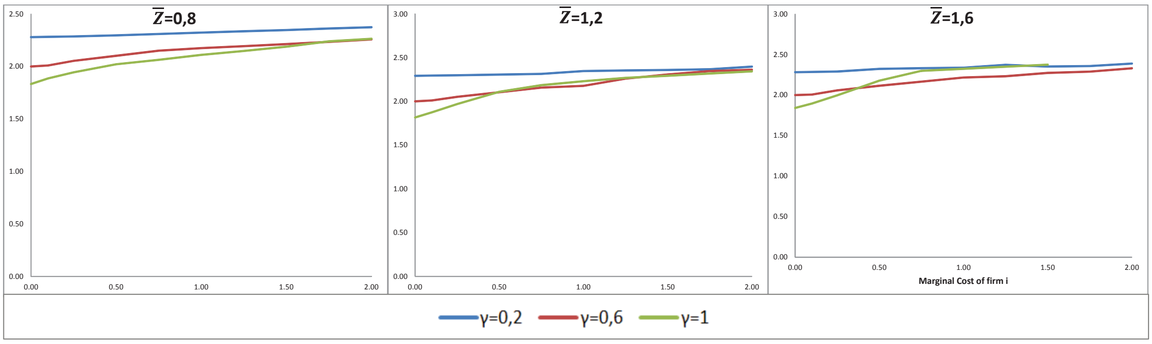

Figure 4.14: SPNE output level of the more efficient firm as a function of rival's marginal costs.

Figures 4.15 and 4.16 show the interest rate of the firm i and the interest rate of the firm j as function of the marginal cost of production of the firm i, c_i. Figures allow us to conclude the following:

Result 4.11 *The SPNE interest rate of firm i, r_i^{**}, is increasing with the marginal cost of production of the firm i, c_i. On the contrary, the SPNE interest rate of firm j, r_j^{**}, is decreasing with the marginal cost of production of the firm i, c_i. The change (increase or decrease) is more pronounced for high levels of degree of product substitutability, γ.*

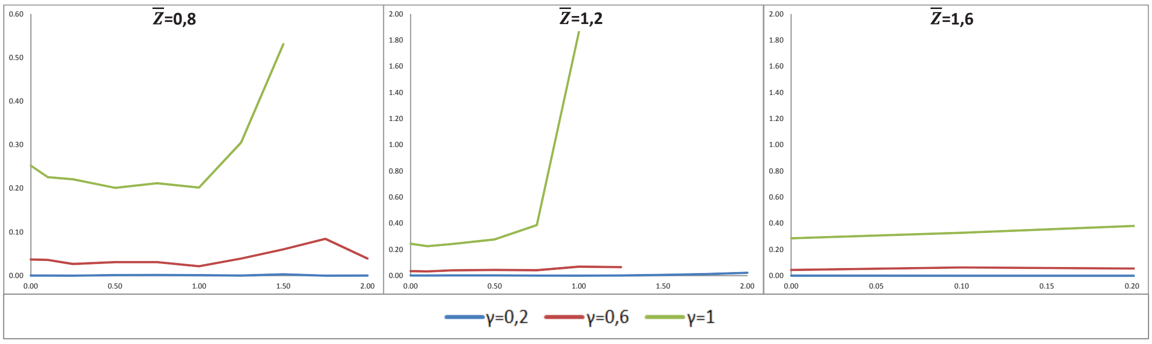

Figure 4.15: SPNE interest rate level of the less efficient firm as a function of its marginal costs.

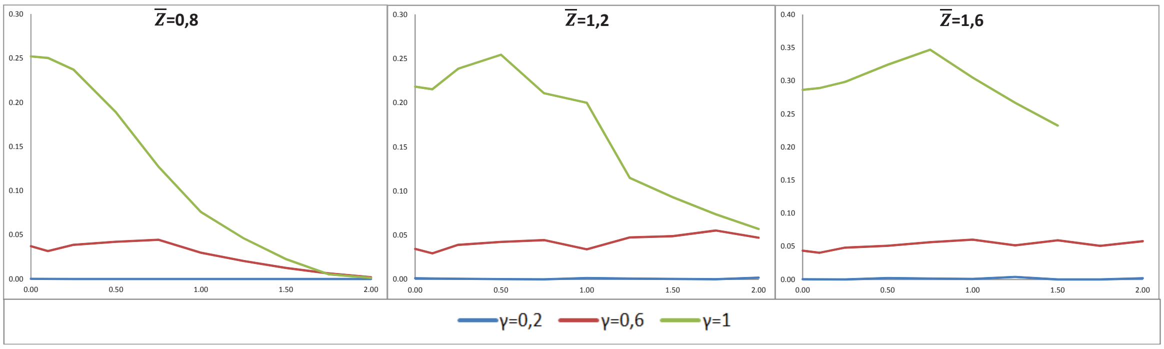

Figure 4.16: SPNE interest rate level of the more efficient firm as a function of rival's marginal costs.

Figures 4.17 and 4.18 show the bankruptcy probability of firm i and the bankruptcy probability of firm j as function of the marginal cost of production of the firm i, c_i. These figures allow us to conclude the following:

Result 4.12 *The SPNE bankruptcy probability of firm i, θ_i^{**}, follows a U relationship with the marginal cost of production of the firm i, c_i and the SPNE of the bankruptcy probability of the firm j, θ_j^{**}, follows a inverted U relationship with the marginal cost of production of the firm i, c_i. The change of behavior occurs for intermediate levels of marginal cost of production of the firm i, c_i, in the firm i case and low values of marginal cost of production of the firm i, c_i, in firm j case.*

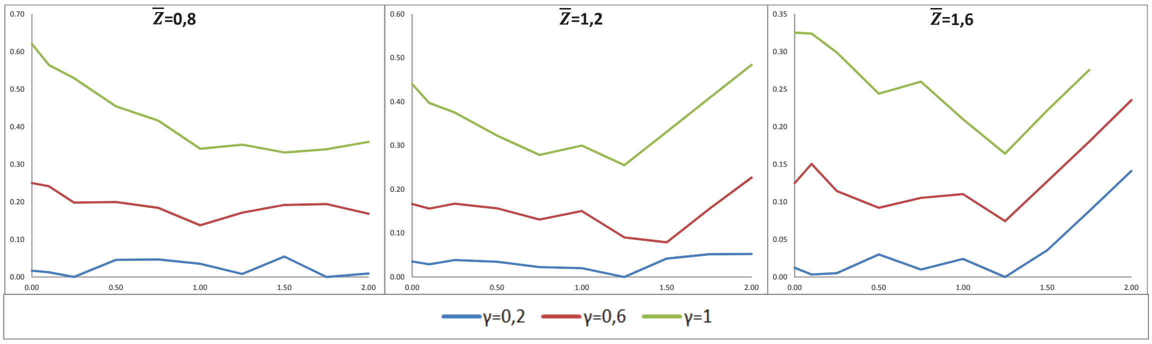

Figure 4.17: SPNE bankruptcy probability of the less efficient firm as a function of its marginal costs.

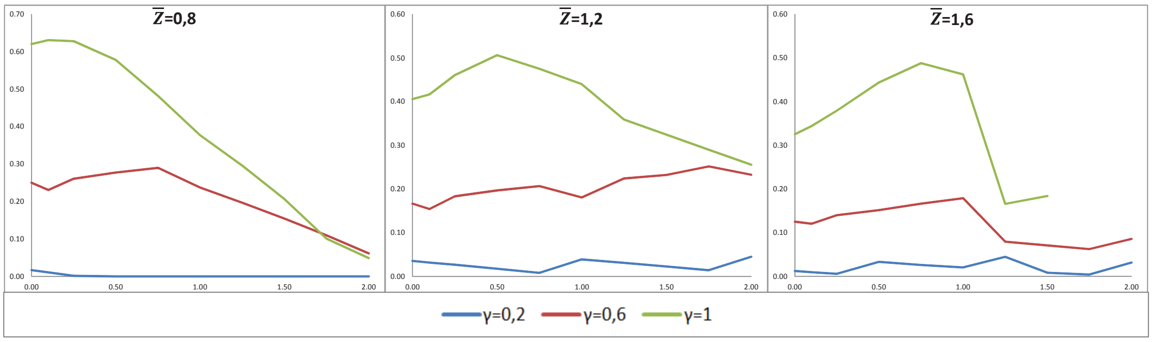

Figure 4.18: SPNE bankruptcy probability of the more efficient firm as a function of the rival's marginal costs.

Regarding the effect of the marginal production cost of firm i, c_i in the bankruptcy probabilities, the result presented above shows that the effect of the increasing c_i has the opposite effect on the less efficient firm, firm i (U relationship) and the more efficient firm, firm j (inverted U relationship). Intuitively, it can be stated that there is a direct and an indirect effect. The direct effect is that the increasing of the marginal cost of the firm i, increases its bankruptcy probability, having the opposite effect on the more efficient firm. This direct effect dominates for high levels of asymmetry ($c_i > 1$). The indirect effect results from the fact that the increase in the marginal cost of the firm i leads to a more conservative behavior in the debt and output markets which implies a decrease in the bankruptcy probability (as explained in the symmetric case). On the contrary, the behavior of the more efficient firm is more aggressive which leads to an increase in the bankruptcy probability. This indirect effect dominates for low levels of asymmetry.

Figures 4.19 and 4.20 show the expected equity values of firm i and firm j, respectively, as function of the marginal cost of production of the firm i, c_i. These figures allow us to conclude the following:

Result 4.13 *The SPNE expected equity value of the firm i, V_i^{**}, is decreasing with the firm's marginal cost of production of the firm i, c_i. On the contrary, the SPNE expected equity value of firm j, V_j^{**}, is increasing with the rivals' marginal cost of production.*

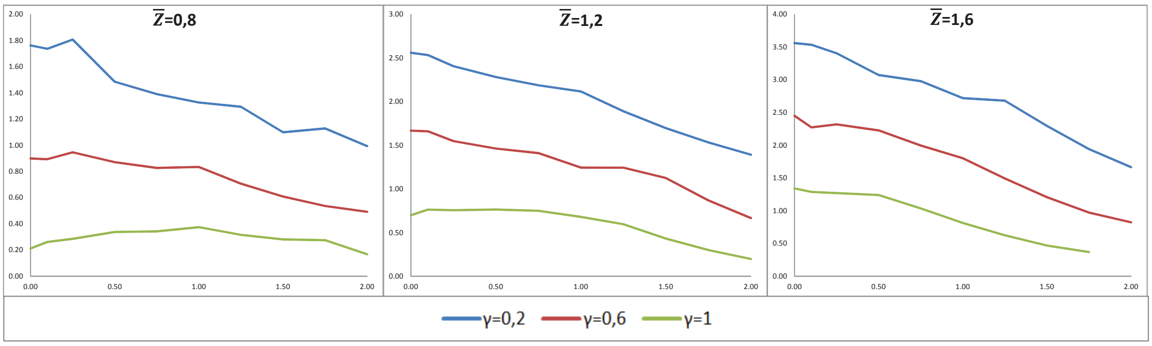

Figure 4.19: SPNE expected equity value of the less efficient firm as a function of its marginal costs.

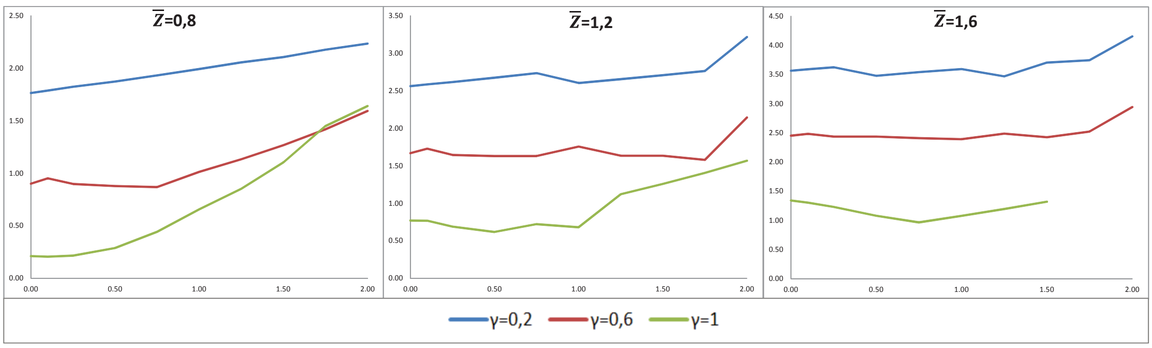

Figure 4.20: SPNE expected equity value of the more efficient firm as a function of the rival's marginal costs.

Figures 4.21 and 4.22 show the expected debt values of firm i and firm j as function of the marginal cost of production of firm i, c_i. The figures allow us to conclude the following:

Result 4.14 *The SPNE expected debt value of the firm i, W_i^{**}, is decreasing with marginal cost of production of the firm i, c_i. On the contrary, the SPNE expected debt value of firm j, W_j^{**}, is increasing with the marginal cost of production of firm i, c_i.*

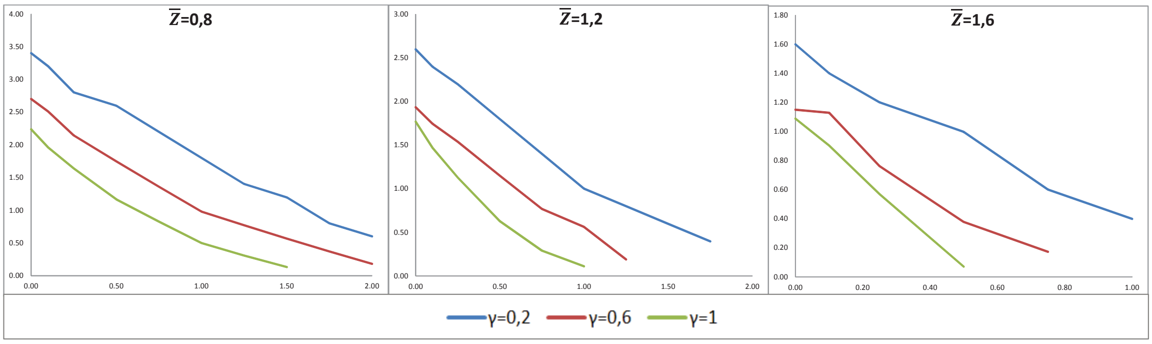

Figure 4.21: SPNE expected debt value of the less efficient firm as a function of its marginal costs.

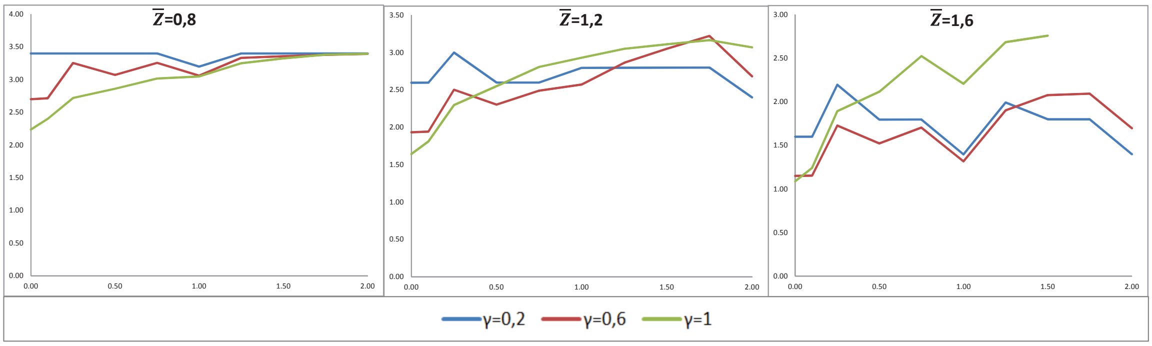

Figure 4.22: SPNE expected debt value of the more efficient firm as a function of the rival's marginal costs.

Figure 4.23 shows the welfare level as function of the marginal cost of production of firm i, c_i. The figure allow us to conclude the following:

Result 4.15 *The SPNE of the welfare, Wel^{**}, is decreasing with the marginal cost of production of the firm i, c_i.*

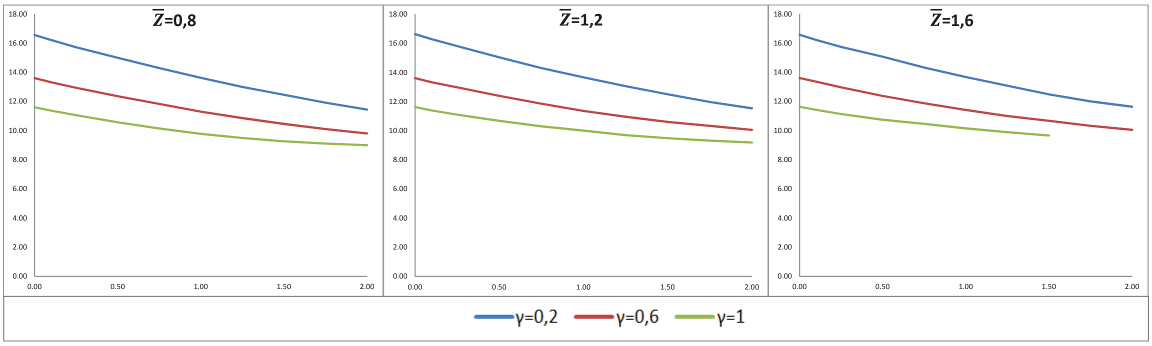

Figure 4.23: SPNE expected social welfare as a function of less efficient firm's marginal costs.

Results 4.13 and 4.14 allows us to conclude the following:

Result 4.16 *The SPNE expected value of the firm i, Y_i^{**}, is decreasing with marginal cost of production of the firm i, c_i. On the contrary, the SPNE expected value of firm j, Y_j^{**}, is increasing with the marginal cost of production of firm i, c_i.*

After analyzing the results we conclude that in the symmetric case we confirmed some results obtained by Toolsema and Haan (2008) and Frank and Le Pappe (2008). In the asymmetric case, our result provide an interesting contribution to the literature. In particular, we emphasize the more conservative behavior of the inefficient firm and the more aggressive behavior of the rival firm. The effect of increasing the marginal cost of the less efficient firm on the bankruptcy probabilities depends on whether the direct effect or the indirect effect dominate. The direct effect dominates for high levels of asymmetry while the indirect effect dominates for low levels of asymmetry. With regard to the variables interest rate, expected equity value, expected debt value and welfare, the results obtained are consistent with the literature.

Chapter 5

Conclusions

The present work examined, analytically and numerically, how the market structure influences financial and product market decisions and, consequently, how it affects the bankruptcy risk. We analyzed the impact of changes in the level of demand uncertainty, the degree of product differentiation and the asymmetry in the marginal production costs on the risk of bankruptcy. Although some specific conclusions were already reported throughout the text and at the end of the previous chapter, we now summarize the most important results of this work.

Regarding the literature review we conclude that there are several studies that analyze the strategic interaction between financial decisions and output market decisions. In addition, we find that the existing literature on the bankruptcy probability has as its main objective the identification of the best model to predict the bankruptcy risk. After the literature review we conclude that the study of the interaction between the main decisions of a firm (financial decisions and output market decisions) and the probability of a firm not meeting its obligations or going bankrupt is important. Considering the importance of the subject, there is a need for interconnecting financial decisions, output market decisions and bankruptcy probabilities and furthermore ascertain how the variables analyzed change when some important market structure parameters vary (such as changes in the level of demand uncertainty, the level of product differentiation and the cost asymmetry between firms).

We proceeded to the resolution of a model, using as a basis the fundamental concepts of game theory and the few existing studies. We analyzed a two stage duopoly game model. In the first stage, firms decide the level of debt that maximizes the firm value and in the second stage of the game, firms decide on the optimal quantity that maximizes firm value for the shareholders. The model was solved backwards. We first determined the Nash equilibrium of the quantity decision game and then determined the equilibrium levels of debt. Due to the complexity of the problem we had to solve the model analytically using GAUSS. We determined the SPNE of some variables: debt levels, output levels, the value of each firm, the implicit interest rates, the bankruptcy probabilities and the welfare. The numerical model was run for many values of the parameter of the model in order to allow us to study the impact of changes in the level of demand uncertainty, in the degree of product substitutability and in the level of asymmetry in marginal production costs. We studied two scenarios: one where the two firms have the same marginal costs (symmetric case) and another one where firms differ in their marginal cost of production (asymmetric case).

After the analysis of the results, we conclude that, in the symmetric model, the debt decreases with uncertainty whereas the degree of product substitutability does not have a monotonic impact on the equilibrium debt levels. Regarding the bankruptcy probability, it is increasing with the degree of product substitutability and decreasing with uncertainty.

The result of the asymmetric case reveal that the SPNE output decreases with the degree of product substitutability and with the firm's marginal cost of production and, on the contrary, it is increasing with the rival firm marginal production cost. These results are similar to the ones obtained in traditional oligopoly models where the equilibrium quantity of a firm is decreasing with its marginal costs and increasing with the rival's marginal production costs. Moreover, the equilibrium debt level of the less efficient firm is decreasing with its marginal cost of production while the most efficient firm has the opposite behavior. This is a quite interesting result as it tells us that the less efficient firm is more cautious and finances less with debt while the more efficient firm becomes «more aggressive» in the debt market. A very interesting result is that there is a U shaped relationship between a firm's marginal cost and its bankruptcy probability. This result can be explained by the existence of direct and indirect effects. On the one hand, for the same debt level, increasing the marginal cost of the firm is expected to lead to an increase on the bankruptcy probability. On the other hand, since the a decrease in efficiency leads to lower levels of debt, this leads to a decrease on the bankruptcy probability. For small levels of cost asymmetry the indirect effect of decreasing the debt level is larger than the direct effect, thus we get the counterintuitive result that an increase in marginal production costs lead to a lower probability of bankruptcy. For larger levels of inefficiency, the direct effect is larger than the indirect effect and hence increasing inefficiency increases the bankruptcy probability.

One interesting extension of this work would be to incorporate in the model the bankruptcy costs that directly and indirectly affect the firm.

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Appendix

Gauss Program

```
/* This program computes the SPNE of the Brander and Lewis model,*/
/* considering linear demands with differentiated products, constant */
/* marginal costs which may be asymmetric, demand uncertainty with */
/*a uniform distribution of the uncertainty parameter and Cournot */
/* competition. Alpha is the dimension of the market, gama is the
/* differentiation parameter zbarra is the uncertianty parameter and */
/*c1 and c2 are marginal costs. */
/* The program determines first the NE of the second stage game,*/
/* for given debt levels (b1,b2) and for each (b1,b2) the equilibrium*/
/* value of each firm, (Y1,Y2), is computed. This is repeated for many*/
/* (b1,b2) and the equilibrium values of Y1 and Y2 are saved in two*/
/* matrices. Next the NE of the first stage game is determined*/
/* (b1eq,b1eq) and the corresponding NE of the second stage game*/
/* and equilibrium bankruptcy probabilities are determined. */
/* This procedure is repeated for many values of the parameter values*/
/* so as to analyze how the equilibrium changes with changes in the */
/* parameter values */
library co;
#include co.ext;
coset;

/************** Inicial parameters of the model **************/

alpha=5; /* market dimension */
zbarmax=1.6; /* maximum value of the uncertainty degree */
zbarmin=1.6; /* minimum value of the uncertainty degree */
c1=0.5; /* marginal cost of firm 1 */
c2=0; /* marginal cost of firm 2 */
gamamax=1; /* maximum value of the differentiation parameter */
```

```
gamamin=0.95; /* minimum value of the differentiation parameter */
zbarra=zbarmin;
saltozbar=0.10; /* step size for the iterations on the uncertainty degree */
saltgama=0.05; /* step size for the iterations on the uncertainty degree */
niterzbar=int((zbarmax-zbarmin)/saltozbar)+1; /*number of iterations for uncertainty
                                                 degree */
nitergam=int((gamamax-gamamin)/saltgama)+1; /*number of iterations for
                                              differentiation parameter */

/************Create matrices to keep the SPNE values of quantities, *************/
/*******************debt and default probabilities ***********************/

b1eqmat=zeros(niterzbar,nitergam);
b2eqmat=zeros(niterzbar,nitergam);
teta1eqmat=zeros(niterzbar,nitergam);
teta2eqmat=zeros(niterzbar,nitergam);
q1eqmat=zeros(niterzbar,nitergam);
q2eqmat=zeros(niterzbar,nitergam);
zbarmat=zeros(niterzbar,1);
gamamat=zeros(1, nitergam);
w1eqmat=zeros(niterzbar,nitergam);
w2eqmat=zeros(niterzbar,nitergam);
v1eqmat=zeros(niterzbar,nitergam);
v2eqmat=zeros(niterzbar,nitergam);
y1eqpmat=zeros(niterzbar,nitergam);
y2eqpmat=zeros(niterzbar,nitergam);
r1eqmat=zeros(niterzbar,nitergam);
r2eqmat=zeros(niterzbar,nitergam);
welfeqmat=zeros(niterzbar,nitergam);
numberENmat=zeros(niterzbar,nitergam);

/***********Start iterations of level of uncertainty (zbarra) ***********/
/****************and differentiation level (gama)****************/

iterzb=1;
do while zbarra <=zbarmax;
gama=gamamin;
iterga=1;
do while gama <=gamamax;

/*****************************************************************/
/***Finding the second stage NE for various levels of (b1,b2) and saving ***/
/***the NE value of each firm in a matrix which will be use to find SPNE ***/
/*****************************************************************/
```

```
/******* This is to obtain the lower and upper bounds for b1 and b2 ********/

b1min=0;
b2min=0;
b1max=3.5; /*Debt cannot be higher than expected monopoly profits. Here we are using an */
b2max=3.5; /*weighted average of monopoly and duopoly profits as the upper bound of debt */
saltob=0.10; /* step size for the iterations on the debt levels */
niterb1=int((b1max-b1min)/saltob)+1;/*number of iterations for debt level of firm 1*/
niterb2=int((b2max-b2min)/saltob)+1;/*number of iterations for debt level of firm 2*/
y1mat=ones(niterb1,niterb2); /* create matrix to save the NE total value of firm 1 */
y1mat=y1mat*(-500);
y2mat=ones(niterb1,niterb2); /* create matrix to save the NE total value of firm 2 */
y2mat=y2mat*(-500);
b2mat=zeros(1,niterb2);
b1mat=zeros(niterb1,1);
b1=b1min;

/***********Start iterations of level of uncertainty (zbarra) ***********/
/****************and differentiation level (gama)****************/

iterb1=1;
do while b1<= b1max;
b2=b2min;
iterb2=1;
do while b2<= b2max;

/****************** Check if NE is interior for all 4 variables ******************/

x0 = {3, 3,0,0}; /* starting values of variables to be used in the constrained optimization routine*/
_co_IneqProc=&ineqlim; /* ineqlim is the procedure where the inequality constraints are defined */
_co_MaxIters=100; /* maximum number of iterations in the constrained optimization */
{x,f,g,ret}=co(&fob,x0); /* this calls the routine to solve constrained optimization problem. Objective function defined in procedure fob */
call coprt(x,f,g,ret);
if f<0.00001; /* if the optimal value of the objective function is very close to zero it means that we found an interior NE */
goto nefound; /* if previous condition true can «jump» to the end of if cycles, since
```

```
                    NE was already found. Jump to line with level «nefound» */
else;

/***** Check if NE is z^1 = -zbar and z^2 = -zbar, q1 and q2 interior *********/

_co_IneqProc=&ineqlim1;
_co_MaxIters=100;
{x,f,g,ret}=co(&fob1,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound;
else;

/***** Check if NE is z^1 = -zbar and z2 interior, q1 and q2 interior *********/

_co_MaxIters=100;
_co_IneqProc=&ineqlim2;
{x,f,g,ret}=co(&fob2,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound;
else;

/***** Check if NE is z^1 interior and z2= -zbar , q1 and q2 interior *********/

_co_MaxIters=100;
_co_IneqProc=&ineqlim3;
{x,f,g,ret}=co(&fob3,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound;
else;
goto nefound2; /* If we arrive here it means that no NE was found in the feasible
                  region (z^1 and z^2 cannot be in the upper limit) */
endif;
endif;
endif;
endif;
nefound:

/****** Compute the NE total value of each firm and save it in the matrix *******/

y1=(alpha-x[1]-gama*x[2]-c1)*x[1]; /* The NE value of firm 1 is equal to the
                                      equilibrium expected profit (considering
                                      the NE values of q1 and q2) */
```

```
y2=(alpha-x[2]-gama*x[1]-c2)*x[2]; /* The NE value of firm 2 is equal to the
                                      equilibrium expected profit (considering
                                      the NE values of q1 and q2) */
y1mat[iterb1,iterb2]=y1; /* save the NE value of firm 1 in a matrix, where each row
                            corresponds to a value of b1, and each columnn to
                            the value of b2 */
y2mat[iterb1,iterb2]=y2; /* save the NE value of firm 1 in a matrix, where each row
                            corresponds to a value of b1, and each columnn to
                            the value of b2 */
nefound2:
b2mat[1,iterb2]=b2;
b2=b2+saltob;
iterb2=iterb2+1;
endo;
b1mat[iterb1,1]=b1;
b1=b1+saltob;
iterb1=iterb1+1;
endo;

/***** The iterations for the NE of the second stage game end here *****/
/*****************************************************************/
/****************Find the SPNE levels of b1 and b2 ****************/
/*****************************************************************/

iterb1=1;
numberEN=0;
do while iterb1 <= niterb1;
iterb2=1;
do while iterb2 <= niterb2;
y1col=y1mat[.,iterb2];
y2row=y2mat[iterb1,.];
y2col=y2row';
if y1mat[iterb1,iterb2]==maxc(y1col) and y2mat[iterb1,iterb2]==maxc(y2col); /* this
                                                   checks if a given (b1,b2) is a
NE */
if y1mat[iterb1,iterb2]==(-500) or y2mat[iterb1,iterb2]==(-500); /* if we are in the
                                            region where no NE of 2nd stage game
                                        was found, jump to line with level notane */
goto notane;
else;
b1eq=b1min+saltob*(iterb1-1); /* if NE is in feasible region, this gives us SPNE
                                 value of b1 */
b2eq=b2min+saltob*(iterb2-1); /* if NE is in feasible region, this gives us SPNE
```

```
                                           value of b2 */
print "SPNE is equal to" b1eq b2eq;
numberEN=numberEN+1;
else;
endif;
endif;
notane:
iterb2=iterb2+1;
endo;
iterb1=iterb1+1;
endo;
numberENmat[iterzb,iterga]=numberEN;
b1eqmat[ iterzb,iterga]=b1eq; /* save the SPNE of b1 in a matrix */
b2eqmat[ iterzb,iterga]=b2eq; /* save the SPNE of b2 in a matrix */
b1=b1eq;
b2=b2eq;

/*************************************************************************/
/**Compute the SPNE levels of q1, q2, theta1, theta2. W1, W2, V1, V2, r1, r2***/
/**and welfareThis is done by compute NE of the 2nd stage game, for the SPNE**/
/*************************value of (b1,b2)*****************************/
/*************************************************************************/

x0 = {3, 3,0,0}; /* starting values */
_co_IneqProc=&ineqlim;
_co_MaxIters=100;
{x,f,g,ret}=co(&fob,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound1;
else;
_co_IneqProc=&ineqlim1;
_co_MaxIters=100;
{x,f,g,ret}=co(&fob1,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound1;
else;
_co_MaxIters=100;
_co_IneqProc=&ineqlim2;
{x,f,g,ret}=co(&fob2,x0);
call coprt(x,f,g,ret);
if f<0.00001;
```

```
goto nefound1;
else;
_co_MaxIters=100;
_co_IneqProc=&ineqlim3;
{x,f,g,ret}=co(&fob3,x0);
call coprt(x,f,g,ret);
if f<0.00001;
goto nefound1;
else;
goto nefound3;
endif;
endif;
endif;
endif;
nefound1:


teta1eq=(x[3]+zbarra)/(2*zbarra); /* compute the SPNE of theta1 */
teta2eq=(x[4]+zbarra)/(2*zbarra); /* compute the SPNE of theta2 */
teta1eqmat[ iterzb,iterga]=teta1eq; /* save the SPNE of theta1 in a matrix */
teta2eqmat[ iterzb,iterga]=teta2eq; /* save the SPNE of theta2 in a matrix */
w1eq=(1-teta1eq)*b1+1/(4*zbarra)*x[1]*((x[3]+zbarra)*(2*alpha-2*c1-2*x[1]-
2*gama*x[2])+(x[3]^2-zbarra^2)); /* compute the SPNE of w1 */
w2eq=(1-teta2eq)*b2+1/(4*zbarra)*x[2]*((x[4]+zbarra)*(2*alpha-2*c2-2*x[2]-
2*gama*x[1])+(x[4]^2-zbarra^2)); /* compute the SPNE of w2 */
w1eqmat[ iterzb,iterga]=w1eq; /* save the SPNE of w1 in a matrix */
w2eqmat[ iterzb,iterga]=w2eq; /* save the SPNE of w2 in a matrix */
v1eq=(1/(4*zbarra))*(x[1]*((zbarra-x[3])*(2*alpha-2*x[1]-2*c1-2*gama*x[2])+
(zbarra^2-x[3]^2))-2*b1*(zbarra-x[3])); /* compute the SPNE of v1 */
v2eq=(1/(4*zbarra))*(x[2]*((zbarra-x[4])*(2*alpha-2*x[2]-2*c2-2*gama*x[1])+
(zbarra^2-x[4]^2))-2*b2*(zbarra-x[4])); /* compute the SPNE of v2 */
v1eqmat[ iterzb,iterga]=v1eq; /* save the SPNE of v1 in a matrix */
v2eqmat[ iterzb,iterga]=v2eq; /* save the SPNE of v2 in a matrix */
y1eqp=(alpha-x[1]-gama*x[2]-c1)*x[1]; /* compute the SPNE of y1 */
y2eqp=(alpha-x[2]-gama*x[1]-c1)*x[2]; /* compute the SPNE of y1 */
y1eqpmat[ iterzb,iterga]=y1eqp; /* save the SPNE of y1 in a matrix */
y2eqpmat[ iterzb,iterga]=y2eqp; /* save the SPNE of y2 in a matrix */
r1eq=(b1/w1eq)-1; /* compute the SPNE of r1 */
r2eq=(b2/w2eq)-1; /* compute the SPNE of r2 */
r1eqmat[ iterzb,iterga]=r1eq; /* save the SPNE of r1 in a matrix */
r2eqmat[ iterzb,iterga]=r2eq; /* save the SPNE of r2 in a matrix */
welfeq=alpha*(x[1]+x[2])-(1/2)*(x[1]^2+2*gama*x[1]*x[2]+x[2]^2)-c1*x[1]-c2*x[2];
                                        /* compute the SPNE of welfare */
```

```
welfeqmat[ iterzb,iterga]=welfeq; /* save the SPNE of welfare in a matrix */
q1eqmat[ iterzb,iterga]=x[1]; /* save the SPNE of q1 in a matrix */
q2eqmat[ iterzb,iterga]=x[2]; /* save the SPNE of q1 in a matrix */
nefound3:
gamamat[1,iterga]=gama;
gama=gama+saltgama;
iterga=iterga+1;
endo;
zbarmat[ iterzb,1]=zbarra;
zbarra=zbarra+saltozbar;
iterzb=iterzb+1;
endo;

/************* Here end the iteractions for zbarra and gama ****************/
/*****************************************************************/
/********************* Output Section ****************************/
/*****************************************************************/

output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\matb1eq.out reset; /* output file just the
                                                      matrix b1eq*/
outwidth 150;                  /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print b1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\matb2eq.out reset; /* output file just the
                                                      matrix b2eq*/
outwidth 150;                /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print b2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\teta1eq.out reset; /* output file just the
```

```
                                                    matrix teta1eqmat*/
outwidth 150;                    /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print teta1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                       /* print number formatation */
output file=d:\Mestrado\gaussres\teta2eq.out reset; /* output file just the
                                                    matrix teta2eqmat*/
outwidth 150;                    /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print teta2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                       /* print number formatation */
output file=d:\Mestrado\gaussres\w1eq.out reset; /* output file just the
                                                   matrix w1eqmat*/
outwidth 150;                    /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print w1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                       /* print number formatation */
output file=d:\Mestrado\gaussres\w2eq.out reset; /* output file just the
                                                   matrix w2eqmat*/
outwidth 150;                    /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print w2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                       /* print number formatation */
```

```
output file=d:\Mestrado\gaussres\v1eq.out reset; /* output file just the
                                                   matrix v1eqmat*/
outwidth 150;                     /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print v1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                        /* print number formatation */
output file=d:\Mestrado\gaussres\v2eq.out reset; /* output file just the
                                                    matrix v2eqmat*/
outwidth 150;                     /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print v2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                        /* print number formatation */
output file=d:\Mestrado\gaussres\y1eqp.out reset; /* output file just the
                                                    matrix y1eqpmat*/
outwidth 150;                     /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print y1eqpmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                        /* print number formatation */
output file=d:\Mestrado\gaussres\y2eqp.out reset; /* output file just the
                                                    matrix y2eqpmat*/
outwidth 150;                     /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print y2eqpmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
```

```
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\r1eq.out reset; /* output file just the
                                                    matrix r1eqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print r1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\r2eq.out reset; /* output file just the
                                                    matrix r2eqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print r2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\welfeq.out reset; /* output file just the
                                                      matrix welfeqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print welfeqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\q1eq.out reset; /* output file just the
                                                    matrix q1eqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print q1eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
```

```
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\q2eq.out reset; /* output file just the
                                                        matrix q2eqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print q2eqmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;
output off;
format /rdt 8,5;                    /* print number formatation */
output file=d:\Mestrado\gaussres\numberEN.out reset; /* output file just the
                                                        matrix welfeqmat*/
outwidth 150;                 /* dimension of output print columns*/
iterzb=1;
do while iterzb <= niterzbar;
print numberENmat[iterzb,.];
iterzb= iterzb+1;
endo;
output off;

/**************** Procedures for the constrained optimization *****************/
/******************** Procedures for an interior NE **********************/
/****************** Procedure for objective function *********************/

proc fob(x);
local x1,x2,x3,x4,y1,y2,y3,y4;
x1 = x[1];
x2 = x[2];
x3 = x[3];
x4 = x[4];
y1 = -x1+0.25*zbarra+0.5*alpha+0.25*x3-0.5*gama*x2-0.5*c1;
y2 = -x2+0.25*zbarra+0.5*alpha+0.25*x4-0.5*gama*x1-0.5*c2;
y3 = (alpha-x1-gama*x2+x3-c1)*x1-b1;
y4 = (alpha-x2-gama*x1+x4-c2)*x2-b2;
retp (y1^2+y2^2+y3^2+y4^2);
endp;

/*** Procedure for inequality constraints *****/
proc ineqlim(x);
local limits;
limits=zeros(6,1);
```

```
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
retp (limits);
endp;

/****** Procedures for an z^1 = -zbar and z^2 = -zbar, q1 and q2 interior ******/
/**************** Procedure for inequality constraints *******************/

proc fob1(x);
local x1,x2,x3,x4,y1,y2,y3,y4;
x1 = x[1];
x2 = x[2];
x3 = x[3];
x4 = x[4];
y1 = -x1+0.25*zbarra+0.5*alpha+0.25*x3-0.5*gama*x2-0.5*c1;
y2 = -x2+0.25*zbarra+0.5*alpha+0.25*x4-0.5*gama*x1-0.5*c2;
y3 = x3+zbarra;
y4 = x4+zbarra;
retp (y1^2+y2^2+y3^2+y4^2);
endp;

/*** Procedure for inequality constraints *****/

proc ineqlim1(x);
local limits;
limits=zeros(8,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
limits[7]=(alpha-x[1]-gama*x[2]+x[3]-c1)*x[1]-b1;
limits[8]=(alpha-x[2]-gama*x[1]+x[4]-c2)*x[2]-b2;
retp (limits);
endp;

/****** Procedures for an z^1 = -zbar and z2 interior, q1 and q2 interior *******/
/**************** Procedure for objective function ********************/
```

```
proc fob2(x);
local x1,x2,x3,x4,y1,y2,y3,y4;
x1 = x[1];
x2 = x[2];
x3 = x[3];
x4 = x[4];
y1 = -x1+0.25*zbarra+0.5*alpha+0.25*x3-0.5*gama*x2-0.5*c1;
y2 = -x2+0.25*zbarra+0.5*alpha+0.25*x4-0.5*gama*x1-0.5*c2;
y3 = x3+zbarra;
y4 = (alpha-x2-gama*x1+x4-c2)*x2-b2;
retp (y1^2+y2^2+y3^2+y4^2);
endp;

/*************** Procedure for inequality constraints *******************/

proc ineqlim2(x);
local limits;
limits=zeros(7,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
limits[7]=(alpha-x[1]-gama*x[2]+x[3]-c1)*x[1]-b1;
retp (limits);
endp;

/****** Procedures for an z^1 interior and z2= -zbar , q1 and q2 interior ******/
/*************** Procedure for objective function ********************/

proc fob3(x);
local x1,x2,x3,x4,y1,y2,y3,y4;
x1 = x[1];
x2 = x[2];
x3 = x[3];
x4 = x[4];
y1 = -x1+0.25*zbarra+0.5*alpha+0.25*x3-0.5*gama*x2-0.5*c1;
y2 = -x2+0.25*zbarra+0.5*alpha+0.25*x4-0.5*gama*x1-0.5*c2;
y3 = (alpha-x1-gama*x2+x3-c1)*x1-b1;
y4 = x4+zbarra; /* equation 4 */
retp (y1^2+y2^2+y3^2+y4^2);
endp;
```

```
/**************** Procedure for inequality constraints *****************/

proc ineqlim3(x);
local limits;
limits=zeros(7,1);
limits[1]=x[1];
limits[2]=x[2];
limits[3]=x[3]+zbarra;
limits[4]=-x[3]+zbarra;
limits[5]=x[4]+zbarra;
limits[6]=-x[4]+zbarra;
limits[7]=(alpha-x[2]-gama*x[1]+x[4]-c2)*x[2]-b2;
retp (limits);
endp;
```