Portuguese Study Groups' Reports

Report on "Modelling fiber flow in fiberboard manufacturing"

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Abstract

Resin weighs substantially on the costs associated with the manufacturing of MDF (medium-density fiberboard). It is thus important to ensure the efficiency of resin usage in production lines with the purpose of ensuring a given internal bond strength (IB) for specific products. Sonae has two MDF production lines using similar technologies and materials but which need different resin percentages to reach the same IB. It was asked of the study group to find out what were the determining factors responsible for this difference and, if possible, suggest alternatives which will improve resin efficiency.

1.1 Introduction

2.1 Basic estimates

In this section we will collect some basic data about the system and estimate some of the important parameters. We start with some intrinsic properties of the fiber.

Length of fiber	$l = 2.5 \times 10^{-3}\, m$
Diameter of fiber	$d = 3 \times 10^{-5}\, m$
Volume of fiber	$v = \frac{\pi}{4} d^2 l = 1.8 \times 10^{-12}\, m^3$
Density of dry fiber	$\rho = 430\, kg/m^3$
Mass of dry fiber	$m = \rho v = 7.6 \times 10^{-10}\, kg$

The factory under study works in several regimes with rather different mass flux in the blowline. For the purpose of these estimates we will consider a flux of $20\, ton/hour$ of dry fiber.

Mass flux of dry fibers	$F = 20\, ton/h = 5.6\, kg/s$
Flux of fibers	$f = \frac{F}{m} = 7.2 \times 10^9\, fibers/s$
Diameter of blowline	$D = 0.1\, m$
Cross section (area) of blow line	$A = \frac{\pi}{4} D^2 = 7.9 \times 10^{-3}\, m^2$
Flow velocity	$u = 30\, m/s$
Kinetic energy of dry fiber	$E = \frac{1}{2} m u^2 = 3.4 \times 10^{-7}\, J$
Momentum of dry fiber	$p = mu = 2.3 \times 10^{-8}\, kg\, m/s$
Number of fibers per unit volume	$n = \frac{f}{uA} = 3 \times 10^9\, m^{-3}$
Fraction of volume occupied by the fibers	$nv = 0.054 = 5.4\%$
Average distance between centers of fibers	$n^{-1/3} = 0.3\, mm$

From this table we can start to form a picture of the structure of the fluid flowing in the blowline. The ratio between the length of the fiber and the average distance between centers of fibers is rather large,

$$l n^{1/3} = 10 \ , \tag{2.1}$$

which shows that the fibers are entangled. On the other hand, the ratio of the fiber diameter to the average distance between centers of fibers is rather small,

$$d n^{1/3} = \frac{1}{10} \ , \tag{2.2}$$

which shows that the fibers are rather spread out.

Imagine cutting the blowline by a plane orthogonal to the pipe. How many fibers intercept this plane? The answer is given by the number of fibers with center inside the volume equal to the area A of the pipe times the lenght l of the fiber,

$$N = nAl = 6 \times 10^5 \tag{2.3}$$

This can be shown exactly if we assume an uniform random distribution for the centers of the fibers and for their angular orientation.

It is also useful to compare the size of the resin droplets with the size of the fibers. The size of the resin droplets depends on the type of injector used. We shall assume that the average radius of injected resin droplet is $r = 200\,\mu m$. Moreover, we assume that the final product has a resin content (in mass) of 15%.

average radius of injected resin droplet	$r = 200\,\mu m$
Volume of resin droplet	$v_r = \frac{4}{3}\pi r^3 = 3.4 \times 10^{-11}\,m^3$
Density of resin	$\rho_r = 1270\,kg/m^3$
Mass of resin droplet	$m_r = \rho_r v_r = 4.3 \times 10^{-8}\,kg$
Mass ratio between resin droplet and fiber	$m_r/m = 56$
Resin content (in mass) in final MDF	$0.15 = \frac{F_r}{F_r+F}$
Resin mass flow	$F_r = 0.99\,kg/s$
Resin droplet flow	$f_r = F_r/m_r = 2.3 \times 10^7\,/s$
Number of fibers per resin droplet injected	$x = f/f_r = 310$

Finally, let us estimate the distance between the center of the fiber and the center of the resin droplet for which they are in contact. Consider a cylindrical fiber oriented at an angle θ relative to the direction of the incoming spherical resin droplet. The interaction cross section is

$$\begin{aligned} \sigma(\theta) &= \pi r^2 + dl\cos\theta + r(2d + 2l\cos\theta) && (2.4) \\ &= (d+2r)l\cos\theta + \pi r^2 + 2rd && (2.5) \end{aligned}$$

This is the area of the region defined by the set of points at a distance smaller or equal to r from a rectangle of sides d and $l\cos\theta$. We now average over all angles with the uniform distribution on the sphere $\sin\theta\, d\theta d\phi$,

$$\langle\sigma\rangle = \int_0^{\pi/2} d\theta \sin(\theta)\sigma(\theta) = \frac{1}{2}(d+2r)l + \pi r^2 + 2rd = 6.8 \times 10^{-7}\,m^2 \tag{2.6}$$

The effective distance of interaction is then given by

$$\langle\sigma\rangle = \pi\ell_{int}^2 \qquad \Rightarrow \qquad \ell_{int} = 460\,\mu m \tag{2.7}$$

The ratio between the effective distance of interaction and the diameter of the pipe is

$$\frac{\ell_{int}}{D} = 4.6 \times 10^{-3} \tag{2.8}$$

After the resin droplets captures some fibers their effective interaction distance grows to approximately l. In this case, $\frac{l}{D} = 2.5 \times 10^{-2}$.

3.1 Discrete-time stochastic models

4.1 Injectors' area of influence

In this section we shall assume that each injector has an area that it will be able to reach in order to cover the fibers there and which we shall call it's *area of influence*. As we saw above, this assumes that most of the resin will stick to the fibers before most of the mixing that will take place along the blow–line. If this hypothesis holds, then the area which is covered by a single injector becomes of extreme importance.

Furthermore, under these conditions it is also clear that it would be better to have the injectors in a ring configuration at the same point along the blow–line, as this would ensure that there would be no resin acting on an area which had already been touched upon by the resin sent in by a previous injector.

We shall consider these areas of influence and determine the optimal number of injectors that should be used to cover the largest possible area in each case. Clearly this will depend on the shape of the area of influence which, by its own nature, will be quite complex and depend on many factors. We shall thus just give an idea of what may be worked out in this context, by considering two different fairly simple configurations for these sets, namely, a ball and a circular sector. Although simple, these two examples cover two different situations. The first when the area of influence is tangent to the outside circle and the second when it *opens up* at a point on the circle's boundary with some specific angle which we shall assume to be sufficiently wide.

The problem we shall consider is thus how to cover the largest possible part of a disk D (the cross–section of the blow-line) with k copies of one domain A denoted by $A_i, i = 1 \ldots k$ (the different areas of influence), while satisfying the following conditions:

H1: $A_i \subset D$, $i = 1 \ldots k$ (all A_j's are inside D)

H2: $A_i \cap A_j = \emptyset$ for all $i \neq j$ (the A_j's are mutually disjoint)

H3: $\partial A_i \cap \partial D \neq \emptyset$ for all $i = 1, \ldots, k$ (every copy intersects the boundary of D at least in one point)

4.1.1 Disk-shaped areas of influence

We shall first consider the case where A is also a disk. In order to have the copies A_i satisfying the above conditions, they must all be tangent to the boundary of the cross–section ∂D and it thus follows that they will form a ring around this boundary touching each of the two neighbours at one (and only one) point. It also follows that all disks A_i have their centres at the same distance from the centre of D; we shall denote this distance by R_1, while R_2 will denote the radius of the A_i's.

We thus have that, for k disks, each will be of the form

$$\left[x - R_1 \cos\left(\frac{2\pi i}{k}\right)\right]^2 + \left[y - R_1 \sin\left(\frac{2\pi i}{k}\right)\right]^2 = R_2^2, \quad i = 0, 1, \ldots, k-1.$$

In order to determine R_1 and R_2, it is enough to consider two of these disks is sucession (consecutive values of i), determine their intersection, and ensure that this reduces to one and only one point. We thus want to solve the system ($i = 0, 1$)

$$\left\{ \begin{array}{lcl} (x - R_1)^2 + y^2 & = & R_2^2 \\ \left[x - R_1 \cos\left(\frac{2\pi}{k}\right)\right]^2 + \left[y - R_1 \sin\left(\frac{2\pi}{k}\right)\right]^2 & = & R_2^2 \end{array} \right.$$

This yields solutions of the form

$$\left\{ \begin{array}{lcl} x & = & R_1 \cos^2\left(\frac{\pi}{k}\right) \pm \cos\left(\frac{\pi}{k}\right)\sqrt{\Delta} \\ y & = & \frac{1}{2} R_1 \sin\left(\frac{2\pi}{k}\right) \pm \sin\left(\frac{\pi}{k}\right)\sqrt{\Delta} \end{array} \right. ,$$

where

$$\Delta = R_2^2 - R_1^2 \sin^2\left(\frac{\pi}{k}\right).$$

The two disks will be tangent at one single point if and only if Δ vanishes which is equivalent to

$$R_2 = R_1 \sin\left(\frac{\pi}{k}\right).$$

Since $R_1 + R_2 = R$, the radius of the cross–section, we finally obtain

$$\left\{ \begin{array}{l} R_1 = \dfrac{1}{1 + \sin\left(\dfrac{\pi}{k}\right)} R \\ \\ R_2 = \dfrac{\sin\left(\dfrac{\pi}{k}\right)}{1 + \sin\left(\dfrac{\pi}{k}\right)} R \end{array} \right.$$

The ratio between the area occupied by the k copies and the area of the disk is thus given by

$$\rho_k = k \frac{\pi R_2^2}{\pi R^2} = k \frac{\sin^2\left(\dfrac{\pi}{k}\right)}{\left[1 + \sin\left(\dfrac{\pi}{k}\right)\right]^2}.$$

The plot of ρ_k as a function of the number of disks is displayed in Figure 4.1, from which we see that the maximum is attained for some k between

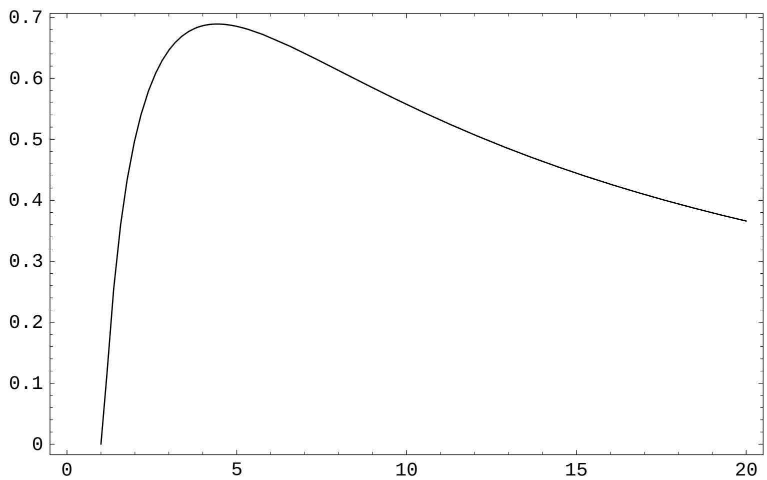


Figure 4.1: Ratio between the area occupied by the disks and the total area of the cross–section.

4 and 5. Table 4.1 presents the values for $k = 2, \ldots, 5$ and it turns out that the optimal integer value corresponds to $k = 4$. However, we also see that the difference between this value and those corresponding to $k = 3$ and $k = 5$ is not very large.

<table>
<tr><td>k</td><td>2</td><td>3</td></tr>
<tr><td>$\rho_{\mathbf{k}}$</td><td>$\frac{1}{2} = 0.5$</td><td>$63 - 36\sqrt{3} \approx 0.6462$</td></tr>
<tr><td>k</td><td>4</td><td>5</td></tr>
<tr><td>$\rho_{\mathbf{k}}$</td><td>$12 - 8\sqrt{2} \approx 0.6863$</td><td>$\frac{10(5-\sqrt{5})}{\left(4+\sqrt{10-2\sqrt{5}}\right)^2} \approx 0.6852$</td></tr>
</table>

Table 4.1: *Ratio function ρ_k for disks and low values of k.*

The different configurations for these values of k are shown in Figure 4.4.

4.1.2 Sector-shaped areas of influence

In order to have a different shape for the area of influence for comparison, we shall now consider the case where this is a circular sector with the vertex on the boundary of the cross–section and the end points of the side segments also on this boundary. Whether or not this last requirement is realistic is not possible to ascertain at this point. However, the main issue here is to see that for two different shapes, disks and sectors, the optimal number of injectors does not change considerably. A further study of the dependence on the angle opening can, in any case, be easily carried out along the same

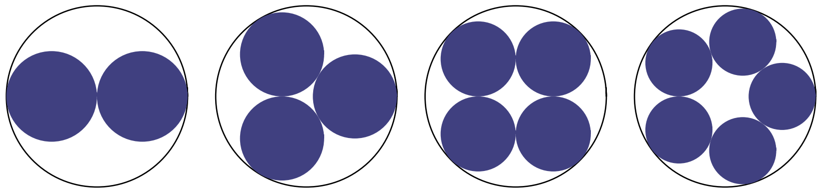


Figure 4.2: Optimal coverings with $2, 3, 4$ and 5 disks.

lines.

In this case, and in a similar fashion to what was done in the previous example, we have to solve the following system of equations

$$\begin{cases} (x-R)^2+y^2 & = & R_2^2 \\ \left[x-R\cos\left(\frac{2\pi}{k}\right)\right]^2+\left[y-R\sin\left(\frac{2\pi}{k}\right)\right]^2 & = & R_2^2 \end{cases}$$

yielding solutions of the form

$$\begin{cases} x & = & R\cos^2\left(\frac{\pi}{k}\right)\pm\cos\left(\frac{\pi}{k}\right)\sqrt{\Delta} \\ y & = & \frac{1}{2}R\sin\left(\frac{2\pi}{k}\right)\pm\sin\left(\frac{\pi}{k}\right)\sqrt{\Delta} \end{cases},$$

where

$$\Delta = R_2^2 - R^2\sin^2\left(\frac{\pi}{k}\right).$$

As before, this will have one and only one solution if Δ vanishes and we obtain

$$R_2 = R\sin\left(\frac{\pi}{k}\right).$$

The intersection points of this particular sector with the boundary of the cross–section is at points (x_0, y_0) with

$$x_0 = \frac{1}{4}R\left(3+\cos\left(\frac{\pi}{k}\right)\right)$$

and two possible values for y, namely,

$$y_0^{\pm} = \pm\frac{1}{2\sqrt{2}}\sqrt{7+\cos\left(\frac{2\pi}{k}\right)}R\sin\left(\frac{\pi}{k}\right).$$

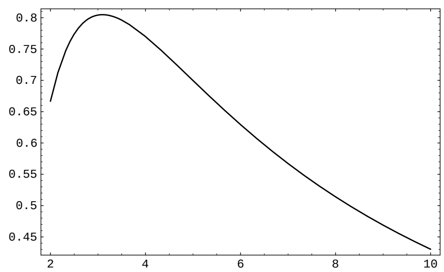
From this we may determine the sector opening α and the ratio between the occupied area and the total area which are given by

$$\alpha_n = 2\arctan\left[\frac{1}{\sqrt{2}}\sqrt{7+\cos\left(\frac{2\pi}{k}\right)}\csc\left(\frac{\pi}{k}\right)\right]$$

and

$$\rho_n = \frac{n}{\pi} \arctan\left[\frac{1}{\sqrt{2}}\sqrt{7+\cos\left(\frac{2\pi}{k}\right)}\csc\left(\frac{\pi}{k}\right)\right]\sin\left(\frac{\pi}{k}\right),$$

respectively. These functions are shown in Figure 4.3, from which we see



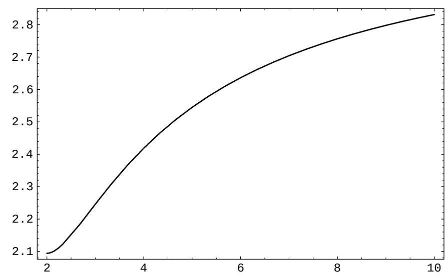


Figure 4.3: Ratio between the area occupied by the sectors and the total area of the cross–section (left) and angle opening for each sector (right).

that now the maximum ratio is achieved very close to $k = 3$. The actual values for $k = 2, 3, 4$ and 5 are shown in Table 4.2 and the maximum over the integers is indeed achieved at $k = 3$ with a corresponding angle of

$$\alpha_3 = 2\arctan\sqrt{\frac{11}{3}} \approx 129^{\mathrm{o}}.$$

k	**2**	**3**
$\rho_{\mathbf{k}}$	$\frac{2}{3} \approx 0.6667$	$\frac{9}{4\pi}\arctan\sqrt{\frac{11}{3}} \approx 0.8043$
k	**4**	**5**
$\rho_{\mathbf{k}}$	$\frac{2}{\pi}\arctan\sqrt{7} \approx 0.7699$	$\frac{5(5-\sqrt{5})}{8\pi}\arctan\sqrt{7+\frac{8}{\sqrt{5}}} \approx 0.6997$

Table 4.2: *Ratio function ρ_k for sectors and low values of k.*

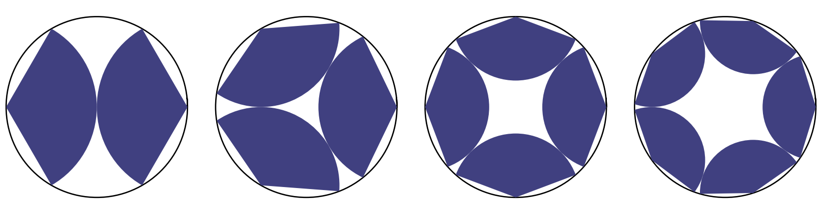


Figure 4.4: Optimal coverings with 2, 3, 4 and 5 circular sectors.

5.1 Equivalent blow lines

6.1 Conclusions and recommendations