


**Proceedings of the 2013
International Conference on
Computational and Mathematical
Methods in Science and Engineering**

Cabo de Gata, Almería, Spain

June 24-27, 2013

A stylized, golden eagle logo with its wings spread, positioned behind the text.

CMMSE
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ISBN 978-84-616-2723-3

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Allee effects models in randomly varying environments

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Abstract

Based on a deterministic model of population growth with Allee effects, we propose a general stochastic model that incorporates environmental random fluctuations in the growth process. We study the model properties, existence and uniqueness of solution and the stationary behavior. We also obtain expressions for the first passage times, in particular, the mean and standard deviation of extinction times for the population.

Key words: Allee effects, population growth, random environments, extinction times

1 Introduction

Warder Clyde Allee (1885/1955) was an American zoologist and ecologist who taught animal ecology. The Allee effects were first described by Allee and colleagues in 1949, as we can see in [2]. There are two kinds of Allee effects, strong Allee effects and weak Allee effects. When a population has a "critical size or density" below which the population decreases on average and above which it increases on average, it is called a strong Allee effect. On the other hand, when a population does not exhibit a "critical size or density" but at low densities the population growth rate increases with increasing density, we say there is a weak Allee effect. Allee effects show up in many wildlife populations, particularly when low population size hinders the efficacy of collective defensive behavior from predators or results in individuals being far apart, which makes it difficult to find mating partners.

We introduce a quite general deterministic model of population growth with strong Allee effect, particular cases of which can be seen in [6], [8], [10] or [1]. The stochastic model we propose is based on this deterministic model with an added term to account for the effect of environmental fluctuations on the growth rate. Other stochastic Allee