Conforming finite elements with embedded strong discontinuities

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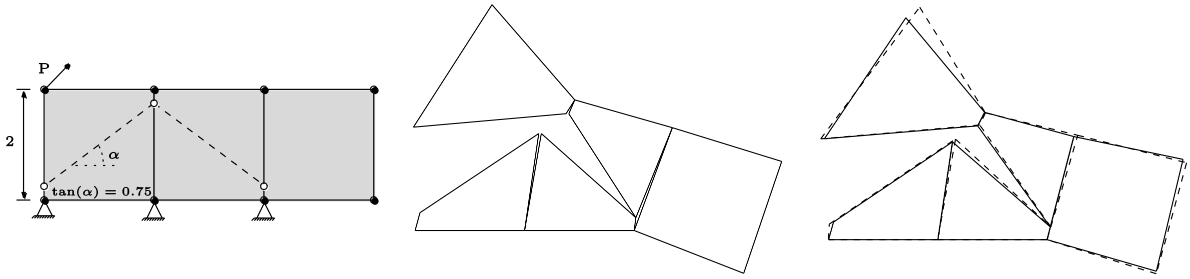
ABSTRACT

The possibility of embedding strong discontinuities into finite elements allowed the simulation of different problems, namely, brickwork masonry fracture, dynamic fracture, failure in finite strain problems and simulation of reinforcement concrete members. However, despite relevant contributions to this field, a general embedded formulation capable of dealing with strong discontinuities using conforming finite elements is still missing. Therefore a new conforming embedded formulation is herein proposed and compared with other relevant formulations, namely the Generalised Strong Discontinuity Approach (GSDA) [1] and the Generalised Finite Element Method (GFEM) [2].

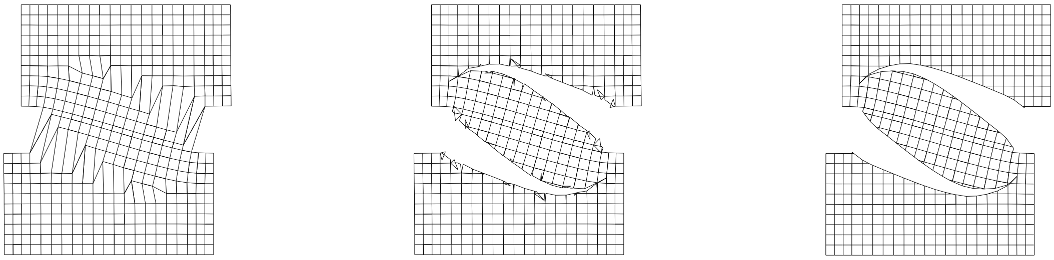
The academic example in Fig. 1 is adopted to illustrate the conforming issues in the case of crack propagation. Fig. 1(b-c), computed for $P = (1;1)N$, allows concluding that: i) although with the GSDA both the jumps and the tractions are continuous across element boundaries, incompatible displacements between elements and at the tip are obtained (Fig. 1(b)); ii) the deformed mesh obtained with the new formulation and GFEM are qualitatively better; iii) the new embedded approach is fully compatible (Fig. 1(c)); iv) the displacements obtained with both the new formulation and GFEM are similar, although the former leads to a slightly stiffer solution than the latter (Fig. 1(c)).

A structural example of a double-edged-notched specimen subjected to mixed-mode fracture is now given. In Fig. 2(a) the usual representation of embedded approaches is shown, where only the regular nodes of each element are represented. Therefore, the enriched elements remain unpartitioned and seem compatible, although distorted. Fig. 2(b) corresponds to Fig. 1(a), but now each enriched element has the discontinuity truly represented inside the parent element and the corresponding domain becomes partitioned. Therefore, the non-conformity of the elements becomes evident. In Fig. 1(c) the deformed mesh obtained with the new conforming formulation is shown.



(a) (b) (c)

Figure 1: (a) mesh (dashed line indicates the prescribed discontinuity); and deformed mesh (displacements magnified 2 times) obtained with: (b) the GSDA; (c) the new formulation (continuous) and GFEM (dashed).



(a) (b) (c)

Figure 2: Deformed mesh obtained using embedded elements (displacements magnified 200 times): (a) classic representation of (apparently compatible) deformed elements; (b) representation of the true deformed mesh revealing non-conforming elements; (c) solution with conforming elements.

Compared to previous embedded approaches (e.g. [1]): i) no additional degrees of freedom are required; and ii) the continuity of both tractions and enhanced kinematical field across elements is automatically ensured. The proposed formulation is variationally consistent and built upon the framework of the discrete crack approach. Therefore, mesh objectivity is automatically inherited.

Several structural examples allowed to conclude that the new embedded formulation is capable of providing results which are practically indistinguishable from the results obtained with GFEM.

However, in spite of the common variational framework [1] and similar results, the two formulations are built in a significantly different manner. The following main differences can be observed: i) the GFEM is nodal based whereas the present formulation is built at element level; ii) crack propagation is simpler to implement in the embedded approach, since only the crossed finite elements are enriched, instead of all nodes surrounding the discontinuity, as typically performed in GFEM; iii) with the embedded formulation, only one additional node is required at each new enriched finite element due to crack propagation, whereas with GFEM all nodes supporting the discontinuity must be enriched; iv) with the present formulation, all additional degrees of freedom are located at the discontinuity, where the quantities of interest are measured. Finally, although the observed computational cost was similar for the bi-dimensional structural problems above presented, the embedded formulation is expected to gain advantage in three-dimensional problems since significantly fewer degrees of freedom are required for each enriched finite element.

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