



UNIVERSIDADE DE ÉVORA
DEPARTAMENTO DE ECONOMIA



DOCUMENTO DE TRABALHO Nº 2005/09
May

**How to Classify a Government?
Can a Neural Network do it?**

1st version: February 09, 2005

This version: April 25, 2005

António Caleiro *

Universidade de Évora, Departamento de Economia

* Paper given at the XII Jornadas de Classificação e Análise de Dados, Universidade dos Açores, Ponta Delgada (April 22, 2005).

I would like to thank Steffen Hörnig for an extremely useful suggestion. Obviously, the usual disclaimer applies.

UNIVERSIDADE DE ÉVORA
DEPARTAMENTO DE ECONOMIA
Largo dos Colegiais, 2 – 7000-803 Évora – Portugal
Tel.: +351 266 740 894 Fax: +351 266 742 494
www.decon.uevora.pt wp.economia@uevora.pt

Abstract:

An electoral cycle created by governments is a phenomenon that seems to characterise, at least in some particular occasions and/or circumstances, the democratic economies.

As it is generally accepted, the short-run electorally-induced fluctuations prejudice the long-run welfare. Since the very first studies on the matter, some authors offered suggestions as to what should be done against this electorally-induced instability. A good alternative to the obvious proposal to increase the electoral period length is to consider that voters abandon a passive and naive behaviour and, instead, are willing to learn about government's intentions.

The electoral cycle literature has developed in two clearly distinct phases. The first one considered the existence of non-rational (naive) voters whereas the second one considered fully rational voters. It is our view that an intermediate approach is more appropriate, *i.e.* one that considers learning voters, which are boundedly rational. In this sense, one may consider neural networks as learning mechanisms used by voters to perform a classification of the incumbent in order to distinguish opportunistic (electorally motivated) from benevolent (non-electorally motivated) behaviour of the government. The paper explores precisely the problem of how to classify a government showing in which, if so, circumstances a neural network, namely a perceptron, can resolve that problem.

Keywords: Classification, Elections, Government, Neural Networks, Output Persistence, Perceptrons

JEL Classification: C450, D720, E320

1. Introduction and Motivation

An electoral cycle created by governments is a phenomenon that seems to characterise, at least in some particular occasions and/or circumstances, the democratic economies. As it is generally accepted, the short-run electorally-induced fluctuations prejudice the long-run welfare. Since the very first studies on the matter, some authors offered suggestions as to what should be done against this electorally-induced instability. For some authors, ever since the seminal paper of NORDHAUS (1975), a good alternative to the obvious proposal to increase the electoral period length is to consider that voters abandon a passive and naive behaviour and, instead, are willing to learn about government's intentions.

The electoral cycle literature has developed in two clearly distinct phases. The first one, which took place in the mid-1970s, considered the existence of non-rational (naive) voters. In accordance with the rational expectations revolution, in the late 1980s the second phase of models considered fully rational voters. It is our view that an intermediate approach is more appropriate, *i.e.* one that considers learning voters, which are boundedly rational. In this sense, one may consider neural networks as learning mechanisms used by voters to perform a classification of the incumbent in order to distinguish opportunistic (electorally motivated) from benevolent (non-electorally motivated) behaviour of the government. The main objective of this paper consists precisely on studying the problem of how to classify a government showing in which, if so, circumstances a neural network, namely a perceptron, can resolve that problem. To achieve this objective we will consider a quite recent version of a stylised model of economic policy, *i.e.* a version based on an aggregate supply curve embodying *output* persistence. See GÄRTNER (1996,1997,1999,2000).

The rest of the paper is structured as follows. Section 2 offers the analysis of the bounded rationality approach as a motivation for the use of neural networks as learning devices. Section 3 then presents the characteristics of the particular neural network, *i.e.* the perceptron, which will be used to perform the classification of the government task. Section 4 explores the problem of how to classify a government showing in which, if so, circumstances the perceptron can resolve that problem. Section 5 concludes.

2. The Bounded Rationality Approach

In the spirit of the bounded rationality research program, which is really to put the economist and the agents in his model on an equal behavioral footing, we expect that, in searching these literatures for ways to model our agents, we shall find ways to improve ourselves. in SARGENT (1993), pg. 33.

Generally speaking, learning models have been developed as a reasonable alternative to the unrealistic informational assumption of rational expectations models. Moreover, through learning models it is possible to study the dynamics of adjustment between equilibria which, in most rational expectations models, is ignored. In fact, rational expectations hypotheses are, in some sense and with some exceptions, a limiting property of a dynamic system which evolves from one equilibrium to another, this being possible because it is assumed that agents know the true model of the economy and use it to form their expectations which, in turn, implies that agents are also able to solve the model.

Interestingly, learning models also deal with another difficulty of rational expectations models, namely the existence of multiple equilibria. It is well known that for linear models, where only expectations of current variables are considered, the rational expectations equilibrium is unique. Conversely, when expectations about the future endogenous variables are required, multiple rational expectations equilibria can occur. Moreover, this is also a common feature of stochastic control/decision problems. In this case, the lack of equilibrium uniqueness arises from an imperfectly specified intertemporal decision problem under uncertainty. The analysis of learning processes can, in fact, provide a way of selecting the ‘reasonable’ equilibrium or sub-set of equilibria. On the one hand, if the learning mechanism is chosen *optimally*, then a *desirable rational equilibrium* is selected from the set of the rational expectations equilibria; see MARCET and SARGENT (1988,1989a,1989b). On the other hand, if the learning mechanism is viewed under an *adaptive* approach, in particular in *expectational stability models*, it can also act as a selection criterion in multiple equilibria models involving *bubbles* and *sunspots*; see EVANS (1986), EVANS and GUESNERIE (1993), EVANS and HONKAPOHJA (1994,1995). To sum up, learning mechanisms, whether optimally or adaptively chosen, ‘select’ the particular steady state

as, in some sense, terminal conditions do.

Through this last point, one can already anticipate the usual distinction between learning mechanisms. Although a number of different studies modelling learning have been presented, two main classes of models can be distinguished: rational learning and boundedly rational learning models.³ In rational learning models, it is assumed that agents know the true *structural form* of the model generating the economy, but not some of the parameters of that model. In boundedly rational learning models, it is assumed that agents, while learning is taking place, use a ‘reasonable’ rule, for instance, by considering the *reduced form* of the model.

Rational learning, which some authors identify with Bayesian learning, thus assumes that the model structure is known by the agents while the learning process is taking place. Given the difficulties that arise in modelling this kind of learning, the bounded rationality approach has the appealing advantage of being (at least) more tractable. Moreover, the assumption that agents use a misspecified model during the learning process makes the bounded rationality approach less controversial.

Obviously, the use of a misspecified model during learning has its consequences on the formation of expectations. In fact, under the bounded rationality approach, agents are modelled as using an ‘incorrect’ rule, derived from backward-looking reduced form equations, to generate expectations while they are learning about the true structural form.

In the bounded rationality approach, various notions of *expectational stability* and of *econometric learning* procedures have been the main formulations. Interestingly, the distinction between these two main procedures has to do with the ‘notion’ of time where learning takes place. While the *expectational stability principle* assumes that learning takes place in ‘notional’, ‘virtual’ or meta-time, *econometric learning procedures*

³ WESTAWAY (1992) prefers to distinguish closed-loop learning, where agents learn about the parameters of the decision rule, from open-loop learning, where agents form an expectation of the path for a particular variable which they sequentially update. As is pointed out, closed-loop learning will be virtually identical to the parameter updating scheme using Kalman filtering.

⁵ If agents never discount past information, then Kalman filtering can be seen as a rolling least-squares regression with an increasing sample. On the contrary, if past information becomes less important, then a ‘forgetting factor’ can be included which gives a rolling window, or more precisely a form of weighted

assume real-time learning.

The *expectational stability* approach considers the influence of – and thus the distinction between – perceived laws on actual laws of motion of the economic system. The actual law of motion results from the substitution of the perceived law of motion in the structural equations of the true model. It is then possible to obtain a mapping $L(\theta)$ from the perceived to the actual law of motion, where θ denotes the set of parameters. Rational expectations solutions $\bar{\theta}$ are then the fixed points of $L(\theta)$. Finally, a given rational expectations solution $\bar{\theta}$ is said to be *expectationally-stable* if the differential equation:

$$\frac{d\theta}{d\tau} = L(\theta) - \theta$$

is locally asymptotically stable at $\bar{\theta}$, where τ denotes meta-time.

In adaptive real-time learning, agents are assumed to use an *econometric procedure* for estimating the perceived law of motion. Least-squares learning is widely used in this formulation in spite of its apparent drawbacks; see SALMON (1995) for a criticism of this issue. A more sophisticated application of these econometric procedures is the consideration of the Kalman filter which, as is well known, nests least squares learning and recursive least squares.⁵

Given the above discussion we can question, as WESTAWAY (1992) clearly and naturally points out:

“How do policymakers react to the fact that the private sector is learning?”

Paradoxically, this pertinent question has been almost ignored. In fact, the analysis of the implications of learning mechanisms in policy-making is far from being complete. Some exceptions are BARRELL *et al.* (1992), BAŞAR and SALMON (1990a,1990b),

least squares.

⁷ A more formal definition would consider a neural network $\langle \mathbf{P}, \prec \rangle$ to be a directed graph over the set \mathbf{P} of processors (neurons), where a processor is a mapping from an input to an output space.

CRIPPS (1991), EVANS and HONKAPOHJA (1994), FUHRER and HOOKER (1993), MARIMON and SUNDER (1993,1994), SALMON (1995) and WESTAWAY (1992). SALMON (1995) is, to the best of my knowledge, one of the very few references where an innovative bounded rationality approach such as *neural networks learning* has been applied in a policy-making problem. We propose to use this approach within a political business cycles context. That being said, we will consider that *bounded rationality voters* have to classify economic policies and outcomes as coming from opportunistic or from benevolent government behaviour.

3. The Neural Networks Methodology

Given the characteristics of *neural networks* it, thus, seems appropriate to consider that the above mentioned classification task can be performed under this formulation of bounded rationality agents. Given that (artificial) neural networks are simulations of how biological neurons are supposed to work, the structure of human brains, where processing units, the so-called *neurons*, are connected by *sinapses*, is approximated by these (artificial) neural networks. As such, the interconnected network of processing units describes a model which maps a set of given *inputs* to an associated set of *outputs* values.⁷ As the number of inputs does not have to be equal to the number of outputs, a neural network can, alternatively, be described as mapping one set of variables onto another set of a possibly different size.

The knowledge of the values for the input and output variables constitutes, then, the major part of the information needed to implement a neural network. Despite the minimal information requirement, this constitutes no motive for questioning the results obtained; see SALMON (1995). In fact, this characteristic makes neural networks particularly appropriate for cases where the structure connecting inputs to outputs is unknown.⁸ In this sense, neural networks can be classified as ‘non-structural’ *procedural* models. Furthermore, they are in good agreement with a typical characteristic of bounded rationality: the *adaptive* behaviour. Indeed, the adaptation to

⁸ Take, for instance, WALL (1993) which pretends to bridge the gap between *substantive rationality* and *procedural rationality*. The fact that it is considered that the exact form of the objective function is unknown is what makes this *bounded rationality* model a good example of a possible application of neural networks.

the environment as a crucial characteristic of a neural network makes it distinct from many (standard) models of learning.⁹

Neural networks are used mainly to learn in two types of tasks; see SWINGLER (1996):

1. Continuous numeric functions – When the task is to approximate some continuous function, as in the case of a *signal extraction*.
2. Classification – When the input is a description of an object to be recognised and the output is an *identification* of the class to which the object belongs. The most common kind of neural network for classification purposes is the so-called *perceptron*.¹⁰ In what follows it will be shown how perceptrons would classify policies and outcomes as ‘electoralist’ or not, using a recent stylised model of economic policy.

Let us then clarify the *modus operandi* of neural networks by a simple formalisation as follows.¹¹ Given an input vector x , the neural network determines a particular parameterisation, say β , which, in conjunction with a function g – also possibly determined by the neural network – leads to the output vector $y = g(x, \beta)$ ‘closest’ to some target y^* . In other words, the output units $y(k)$, ($k = 1, \dots, t$), process, using a function g , the inputs $x(i)$, ($i = 1, \dots, r$), previously amplified or attenuated by the connection strengths $\beta(i, k)$.¹²

The simplest neural network structure described above is usually relaxed to obtain flexibility by considering a layer of, so-called, *hidden units*. In this case, the transformation of inputs into outputs includes an intermediate processing task performed by the hidden units. Each hidden unit, then, produces, by the consideration of

⁹ In particular, neural networks relax the constant linear reduced form assumption of *least squares learning* by considering a time varying possibly non-linear stochastic approximation of that reduced form.

¹⁰ For a clear explanation of the link between perceptrons and the statistical discriminant analysis see CHO and SARGENT (1996).

¹¹ To all of those whose scepticism only disappears with a sound mathematical presentation see ELLACOTT and BOSE (1996). More advanced references include WHITE (1989).

¹² Implicitly assumed is a *feedforward model* where *signals* flow only from $x(i)$ to $y(k)$. Nevertheless, it is also possible to consider *feedback effects*.

an *activation* or *transfer function* $f(\cdot)$, an intermediate output $s(j)$, ($j = 1, \dots, s$), which is finally sent to the output layer.¹³ This situation can be illustrated as follows.

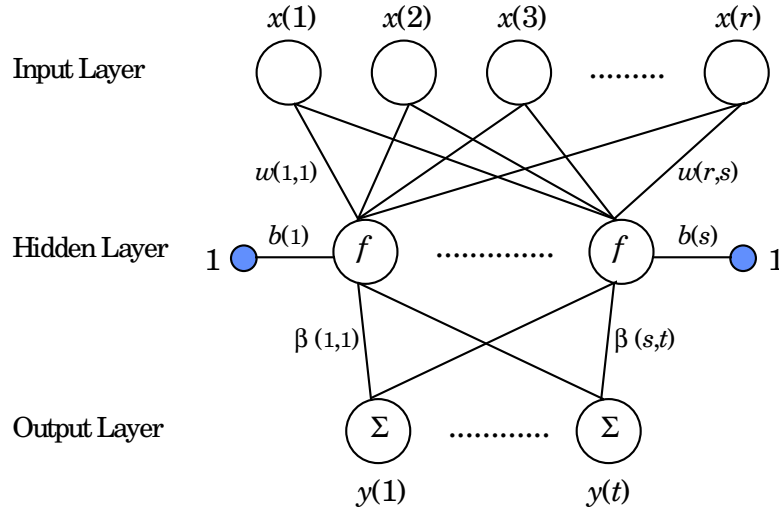


Figure 1 – The neural network structure

Mathematically the network then computes:

1. The input(s) to the hidden layer, $h(j)$, as a weighted sum

$$h(j) = b(j) + \sum_{i=1}^r w(i, j)x(i) \quad j = 1, \dots, s;$$

2. The output(s) of the hidden layer, which are the input(s) to the output layer, are subject to an output activation

$$s(j) = f(h(j)),$$

where f is the so-called *activation function*.

3. The output(s) of the output layer¹⁴

$$y(k) = \sum_{j=1}^s \beta(j, k)s(j) \quad k = 1, \dots, t.$$

Plainly the two crucial elements of a neural network are the parameter set $\theta = (w, \beta)$ and the activation/transfer function $f(\cdot)$. The transfer function usually has the role of

¹³ It is also (and generally) possible to consider a *bias node* ‘shifting’ the weighted sum of inputs by some factor $b(j)$. See Figure 1.

¹⁴ It is possible to consider an activation function and/or a bias before the determination of the ‘final’ outputs.

normalising a node's output signal strength between 0 and 1.¹⁵ The most used are the tanh or some sigmoid function $f(h) = (1 + \exp(-h))^{-1}$ and a gaussian function or some radial basis function. The differences between these functions are made clear by considering the following figure.

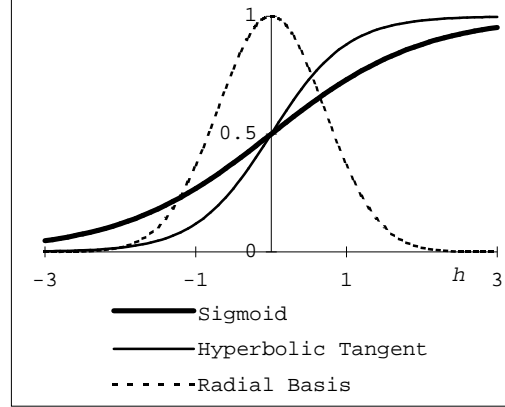


Figure 2 – Some squashing functions

As is clear, the sigmoid and the tanh functions are similar. Moreover, while the radial basis function amplifies in-between values, the sigmoid and tanh functions attenuate or amplify extreme values. Clearly, their differences in practice result also from the specific parameterisation θ .

As usual, once the parameters have been set, say $\hat{\theta}$, the neural network is able to predict outputs $\hat{y} = g(\bar{x}, \hat{\theta})$ for input values \bar{x} which were not included in the training data.

3.1. The learning process

As pointed out in WHITE (1989), the output vector $y = g(x, \theta)$ can be viewed as generating a family of approximations (as θ ranges over the set Θ , say) for the *unknown* relation between inputs x and their corresponding outputs y . The best approximation can be determined by a *recursive learning* procedure known as back-propagation. The learning process – *training* – is then an iterative procedure of processing inputs through the neural network, determining the errors and back-

¹⁵ This is why some authors designate these functions as *squashing functions*.

propagating the errors through the network to adjust the parameters in order to minimise the error between the predicted and observed outputs. This method of learning is referred to as *gradient descent* as it involves an attempt to find the lowest point in the error space by a process of gradual descent along the error surface.¹⁶

In our case, a single-layer network known as *perceptron* will be used to perform the classification task or, in other words, will be used to determine the vector of weights and bias specifying a line on the space output-inflation such that two sub-sets of points – obviously the opportunistic and benevolent ones – are defined. At this stage, a short explanation about how the neural network will determine the above-mentioned vector seems appropriate.

In the particular case under study, the *learning process* conducing to the above-mentioned vector of weights and bias can thus be described as follows:

1. Initial weights, w , and bias, b , are generated in an interval with enough range;¹⁷
2. Given some target vector y^* , with binary values associated with the two considered categories of governments, the error, e , is computed as the difference between y^* and the perceptron output y .
 - i) If there is no error in the classification, that is $e = 0$, then $\Delta w = \Delta b = 0$;
 - ii) If some pair of economic policies/states is classified as belonging to category 1, say benevolent, and should have been classified as belonging to category 0, say opportunistic, then $e = -1$. Therefore, in order to increase the chance that the input vector x will be classified correctly, the weight vector w is ‘put farther away’ from x by subtracting x from it; this meaning that $\Delta w = -x^T$;
 - iii) If some pair of economic policies/states is classified as belonging to category 0 and should have been classified as belonging to category 1, then $e = 1$. Therefore, in order to increase the chance that the input vector x will be

¹⁶ Two factors are used to control the training algorithm’s adjustment of the parameters: the *momentum factor* and the *learning rate coefficient*. The *momentum term*, which is quite useful to avoid local minima, causes the present parameter changes to be affected by the size of the previous changes. The *learning rate* dictates the proportion of each error which is used to update parameters during learning.

¹⁷ Note that a *hard limit transfer function* will be used and this gives $y = 1$ when $w x + b > 0$ and $y = 0$ when $w x + b \leq 0$

classified correctly, the weight vector w is ‘put closer’ to x by adding x to it; this meaning that $\Delta w = x^T$.

To sum up, the perceptron learning rule will be based upon the following updating rules:

$$\Delta w = (y^* - y)x^T = ex^T, \quad (1)$$

and

$$\Delta b = (y^* - y)\mathbf{1}^T = e. \quad (2)$$

Using (1) and (2) repeatedly – the so-called *training process* – the perceptron will eventually find a vector of weights and bias, such that all the pairs of inflation and output are classified correctly. Indeed, it is well known that, if those pairs are linearly separable, the perceptron will always be able to perform the classification by determining a linear decision boundary.

4. The Classification of the Government

In the electoral business cycle literature, one of the most crucial conclusions is that the short-run electorally-induced fluctuations prejudice the long-run welfare. In fact, because the electoral results depend on voters’ evaluation, we can consider that if electoral business cycles do exist it is because voters, through ignorance or for some other reason, allow them to exist. This point introduces a well-known problem of *electorally-induced behaviour punishment* and its related *problem of monitoring*. In reality, voters often cannot truly judge/classify if an observed state/policy is the result of a *self-interested/opportunistic government* or, on the contrary, results as a *social-planner/benevolent* outcome, simply because voters do not know the structure, the model or the transmission mechanism connecting policy values to state values. Moreover, a constant monitoring of government behaviour seems *not* to be considered a crucial practice by the electorate.

Even so, voters do ‘anticipate’ the possible economic damage resulting from such *myopic* behaviour by governments and, especially closer to the elections, start to *classify*

policies and outcomes as potentially being the result of an ‘electoralist’ strategy. This is done in order not to be ‘fooled’ by the incumbent government or simply to punish the incumbent government in case of clear *signals* of electorally-induced policies. In other words, a classification is made, so that for a sufficiently small sub-set of policies classified as ‘electoralist’, voters usually do not take that as a serious motive for punishment, but others, regarded as serious deviations, are punished.¹⁸ In general, this classification task is made difficult by ignorance of the structural form of the model transforming policies in outcomes and also simply because information gathering costs money and time.

4.1. The model

Recently some authors have assumed an extended version of the standard aggregate supply curve $y_t = \bar{y} + \beta(\pi_t - \pi_t^e)$, where y_t denotes the level of output (measured in logarithms) that deviates from the natural level, \bar{y} , whenever the inflation rate, π_t , deviates from its expected level π_t^e , by considering

$$y_t = (1 - \eta)\bar{y} + \eta y_{t-1} + \delta(\pi_t - \pi_t^e), \quad (3)$$

where η measures the degree of output persistence. See GÄRTNER (1999) for an output persistence case and/or JONSSON (1997) for an unemployment persistence case.¹⁹

When normalizing the natural level of output such that $\bar{y} = 0$ the aggregate supply curve reduces to:

$$y_t = \phi y_{t-1} + \alpha(\pi_t - \pi_t^e), \quad (4)$$

where, following the hypothesis of adaptive expectations,

$$\pi_t^e = \gamma \pi_{t-1}, \quad (5)$$

¹⁸ Note the difference between this approach and the one considered, for instance, in MINFORD (1995). Here, it is assumed that “voters penalise absolutely any evidence that monetary policy has responded to anything other than news”, by ‘absolutely’ meaning that there is enough withdrawal of voters to ensure electoral defeat.

¹⁹ As acknowledged in GÄRTNER (1999), only quite recently authors have started to pay due attention to the consequences of considering that relevant macroeconomic variables, *in reality*, show some degree of persistence over time. In fact, a casual observation on reality shows that Europe has been facing a problem in what concerns unemployment which indeed reflects persistence. Given the close connection between unemployment and output, it should be possible to ‘translate’ our results in terms of output to results in terms of unemployment.

where $0 \leq \phi \leq 1$ and $0 \leq \gamma \leq 1$.

As said before, a most common kind of neural network for classification purposes is the so-called perceptron. In order to perform the task of classifying the government, in what concerns its behaviour during the mandate, it is required the determination of the opportunistic and benevolent solutions. These solutions differ in accordance with the way time periods are discounted: whereas for society, therefore also for a benevolent government, future periods should be less important than present ones, this is not the case with an opportunistic government, as future moments, *i.e.* those closer to the election day, are more vital than present ones, in order to explore the decay in the memory of voters.

Having said that, concerning the government's objective function, we make the standard assumption that the incumbent faces a mandate divided into two periods, $t = 1, 2$, such that society's welfare during the mandate, *i.e.* the benevolent government's objective function is given by:

$$U = U_1 + \rho U_2, \quad (6)$$

where ρ is the social rate of discount, whereas opportunistic government's objective function is :

$$V = \mu V_1 + V_2, \quad (7)$$

where μ is the degree of memory of the electorate. In (6) and (7) we also admit that

$$U_t = V_t = -\frac{1}{2}\pi_t^2 + \beta y_t. \quad (8)$$

In these circumstances it is worth immediately noticing that, in general, excepting if $\mu\rho = 1$, the policies that maximise social welfare (6) are not the ones that maximise popularity (7). As it plausible to assume that both ρ and μ do not exceed 1, it is immediately clear that only in the case of perfect memory, *i.e.* $\mu = 1$, and both periods being equally important for society, *i.e.* $\rho = 1$, an opportunistic government will behave exactly as a benevolent one. This fact allows for making it plausible to ask the question: *how to classify a government?*, whose answer is supposed to be given by a neural network when separating optimal outcomes into two parts: the opportunistic and the benevolent ones. In other words, the opportunistic and benevolent solutions (policies

and outcomes) will constitute the necessary inputs for the neural network application. Given the classification task format, let us precisely define what will be called *opportunistic* or ‘electoralist’ *inputs*, that is policies, and *opportunistic outputs*, that is outcomes, to be compared with *benevolent inputs* and *benevolent outputs*.

Clearly, the opportunistic policy and outcomes will be, respectively, the values of inflation and output which result from the maximisation of (6) and (7) subject to (4) and (5). This immediately leads to the optimal policies:²⁰

$$\pi_1^B = \alpha\beta(1 - \rho(\gamma - \phi)), \quad (9)$$

$$\pi_2^B = \alpha\beta, \quad (10)$$

$$\pi_1^O = \alpha\beta \left(1 - \frac{\gamma - \phi}{\mu} \right), \quad (11)$$

$$\pi_2^O = \alpha\beta. \quad (12)$$

Those policies lead to the optimal output levels:

$$y_1^B = \phi y_0 + \alpha(\alpha\beta(1 - \rho(\gamma - \phi)) - \gamma\pi_0), \quad (13)$$

$$y_2^B = \phi(\phi y_0 + \alpha(\alpha\beta(1 - \rho(\gamma - \phi)) - \gamma\pi_0)) + \alpha(\alpha\beta - \gamma\alpha\beta(1 - \rho(\gamma - \phi))), \quad (14)$$

$$y_1^O = \phi y_0 + \alpha \left(\alpha\beta \left(1 - \frac{\gamma - \phi}{\mu} \right) - \gamma\pi_0 \right), \quad (15)$$

$$y_2^O = \phi \left(\phi y_0 + \alpha \left(\alpha\beta \left(1 - \frac{\gamma - \phi}{\mu} \right) - \gamma\pi_0 \right) \right) + \alpha \left(\alpha\beta - \gamma\alpha\beta \left(1 - \frac{\gamma - \phi}{\mu} \right) \right). \quad (16)$$

Before proceeding with the classification task, it is relevant to note that there are, in fact, two possible patterns for the political business cycle: *i*) a typical one, where inflationary expansions take place immediately before the elections and *ii*) an atypical one, where the inflationary expansions take place immediately after the elections.²¹

Given that:

²⁰ From this point onwards, the superscripts *B* and *O* identify an element as, respectively, concerning the benevolent and the opportunistic government.

²¹ This means that, in general, not possible to always use the observed pre-elections expansions as empirical evidence supporting the existence of an opportunistic behaviour of the government as, in fact, even some experienced scholars incorrectly do.

$$\pi_2^B - \pi_1^B = \alpha\beta\rho(\gamma - \phi), \quad \pi_2^O - \pi_1^O = \alpha\beta\frac{\gamma - \phi}{\mu},$$

the typical pattern will be observed when $\gamma > \phi$ and the atypical one when $\gamma < \phi$.

Plainly, when $\gamma = \phi$ there will be no cycle at all.

Given the optimal solutions, (9) to (16), it is straightforward to verify that, because

$$\pi_1^B - \pi_1^O = \alpha\beta(\gamma - \phi)\frac{1 - \mu\rho}{\mu}, \quad \pi_2^B - \pi_2^O = 0,$$

$$y_1^B - y_1^O = \alpha^2\beta(\gamma - \phi)\frac{1 - \mu\rho}{\mu}, \quad y_1^B - y_1^O = -\alpha^2\beta(\gamma - \phi)^2\frac{1 - \mu\rho}{\mu},$$

the typical pattern will then be characterised by:

$$\pi_2^B > \pi_1^B, \quad \pi_2^O > \pi_1^O, \quad \pi_1^B > \pi_1^O, \quad \pi_2^B = \pi_2^O$$

and

$$y_1^B > y_1^O, \quad y_2^B < y_2^O,$$

whereas the atypical pattern will be characterised by:

$$\pi_2^B < \pi_1^B, \quad \pi_2^O < \pi_1^O, \quad \pi_1^B < \pi_1^O, \quad \pi_2^B = \pi_2^O$$

and

$$y_1^B < y_1^O, \quad y_2^B < y_2^O.$$

Given that, in the previous mandate, no matter the kind of government,

$$\pi_0 = \alpha\beta, \tag{17}$$

it is possible to further simplify the optimal output levels expressions, (13) to (16), to:

$$y_1^B = \phi y_0 + \alpha^2\beta(1 - \rho(\gamma - \phi) - \gamma), \tag{18}$$

$$y_2^B = \phi^2 y_0 + \alpha^2\beta(\phi - 2\phi\rho\gamma + \rho\phi^2 - \phi\gamma + 1 - \gamma + \gamma^2\rho), \tag{19}$$

$$y_1^O = \phi y_0 + \alpha^2\beta\frac{\mu + \phi - \gamma - \gamma\mu}{\mu}, \tag{20}$$

$$y_2^O = \phi\left(\phi y_0 + \alpha^2\beta\frac{\mu + \phi - \gamma - \gamma\mu}{\mu}\right) + \alpha\left(\alpha\beta - \gamma\alpha\beta\left(1 - \frac{\gamma - \phi}{\mu}\right)\right). \tag{21}$$

4.2. The classification task

The optimal inflation rates, (9) to (12), and output levels, (18) to (21), define the

coordinates of four points in the (y, π) plane. This space is to be partitioned, *if possible*, in two sub-spaces by a linear decision boundary – in that consists the classification task – by the neural network. See figure 3.

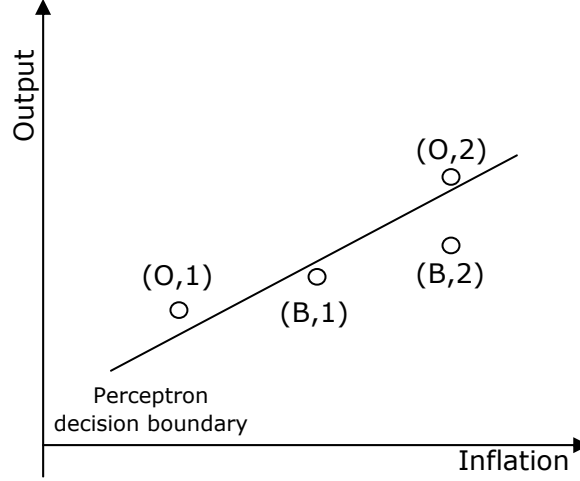


Figure 3 – The neural network classification

Figure 3 allows visualising the opportunistic and benevolent trajectories in the inflation-output, (y, π) , space, showing an example where the classification of the government is possible to be achieved by that kind of neural network.

There are, therefore, four points located in the (y, π) space, two of each type, O and B . This makes possible to draw two line segments connecting the two points of each kind. If these two line segments cross, it is impossible to obtain a decision boundary. This can be checked by a system of equations involving two convex combinations between these points defining the intersection between the straight line segments. They cannot be separated if the two parameters, λ_1, λ_2 in the convex combinations:

$$\lambda_1 \begin{bmatrix} y_1^B \\ \pi_1^B \end{bmatrix} + (1 - \lambda_1) \begin{bmatrix} y_2^B \\ \pi_2^B \end{bmatrix} = \lambda_2 \begin{bmatrix} y_1^O \\ \pi_1^O \end{bmatrix} + (1 - \lambda_2) \begin{bmatrix} y_2^O \\ \pi_2^O \end{bmatrix}, \quad (22)$$

are both between 0 and 1.

Given the optimal inflation rates, (9) to (12), and output levels, (18) to (21), the solutions for λ_1, λ_2 in (22) are:

$$\lambda_1 = \frac{\alpha^2 \beta}{\phi \mu} \frac{(\phi - \gamma)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}, \quad (23)$$

$$\lambda_2 = \frac{\alpha^2 \beta \rho}{\phi} \frac{(\phi - \gamma)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}.^{22} \quad (24)$$

Plainly, in general, the possibility to classify the government depends upon the initial level of output, y_0 .²³ Figure 4 thus represents those two solutions (23) and (24) as a function of y_0 .

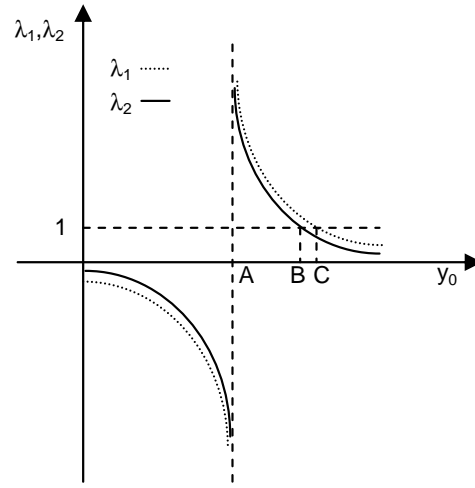


Figure 4 – The influence of initial output level

In order to have $\lambda_1 = 1$ in (23), – point C in figure 4 – the initial level of output must be:

$$y_0 = \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1 - \gamma)\mu}{\phi(1 - \phi)\mu}, \quad (25)$$

whereas, in order to have $\lambda_2 = 1$ in (24), – point B in figure 4 – the initial level of output must be:

$$y_0 = \alpha^2 \beta \frac{\rho(\phi - \gamma)^2 + \phi(1 - \gamma)}{\phi(1 - \phi)}. \quad (26)$$

²² $\lambda_1 - \lambda_2 = \frac{1 - \rho\mu}{\mu} \frac{\alpha^2 \beta}{\phi} \frac{(\gamma - \phi)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}$

²³ When $\phi = \gamma$, both λ_1, λ_2 are equal to zero, meaning that both types of governments behave the same.

As y_0 given by (25) is higher than y_0 given by (26),²⁴ this means that for

$$y_0 > \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1 - \gamma)\mu}{\phi(1 - \phi)\mu}, \quad (27)$$

$\lambda_1 < 1$ and, therefore, also that $\lambda_2 < 1$. Moreover,

$$y_0 > \alpha^2 \beta \frac{1 - \gamma}{1 - \phi} \quad (28)$$

guarantees that both λ_1, λ_2 are positive. See point A in figure 4. After noticing that y_0 given by (25) is higher than y_0 given by (28),²⁵ it is possible to consider an initial condition

$$y_0 > \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1 - \gamma)\mu}{\phi(1 - \phi)\mu}, \quad (29)$$

such that it is impossible to associate *all* the observed behaviours to the correct type of government. In all the other cases, the classification task can be resolved by the perceptron. See figure 5.

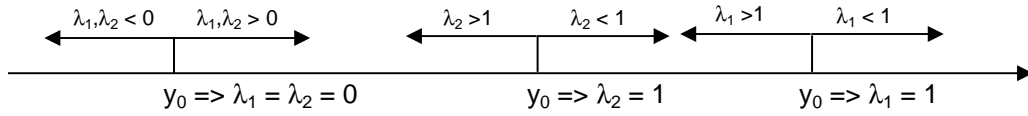


Figure 5 – The classification regions

Notwithstanding that conditionally, there is a fundamental exception. When output does not show any persistence over time, *i.e.* $\phi = 0$, which is, indeed, the most considered case in the literature, it is possible to show that a straight line with intercept between $\alpha^2 \beta \gamma(\gamma\mu - 2)$ and $\alpha^2 \beta \gamma \frac{\gamma - 2\mu}{\mu}$ and slope equal to $\alpha(\gamma + 1)$ will always divide the space in a correct way, this being eventually the result of the perceptron classification. See the Annex.

Plainly, in practical terms, given that a learning process takes place, from the training of

²⁴ $\alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1 - \gamma)\mu}{\phi(1 - \phi)\mu} - \alpha^2 \beta \frac{\rho(\phi - \gamma)^2 + \phi(1 - \gamma)}{\phi(1 - \phi)} = \alpha^2 \beta (\phi - \gamma)^2 \frac{1 - \mu\rho}{\phi\mu(1 - \phi)} > 0.$

²⁵ $\alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1 - \gamma)\mu}{\phi(1 - \phi)\mu} - \alpha^2 \beta \frac{1 - \gamma}{1 - \phi} = \alpha^2 \beta \frac{(\phi - \gamma)^2}{\phi\mu(1 - \phi)} > 0.$

the perceptron does not usually result a straight line with the above mentioned characteristics. Most importantly, given that the two straight lines connecting the two pairs of points in the output-inflation space are parallel, this guarantees that the space is linearly separable. Figure 6 shows this situation.

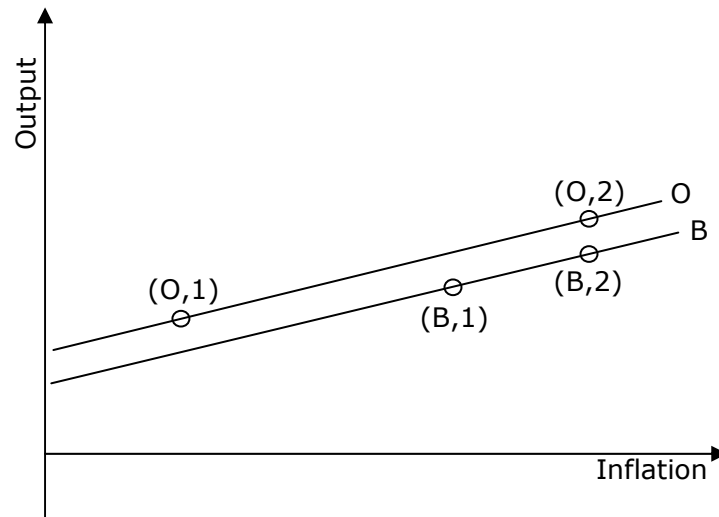


Figure 6 – A particular(ly interesting) case

As it is well known, if the space can be linearly separable, as it is the case when output does not show any persistence, the perceptron will always determine a vector of values for the weights and a bias value such that the straight line associated with these values divide the space in a correct way. By the use of MATLAB, a simple exercise was performed, whose results are shown in figure 7.

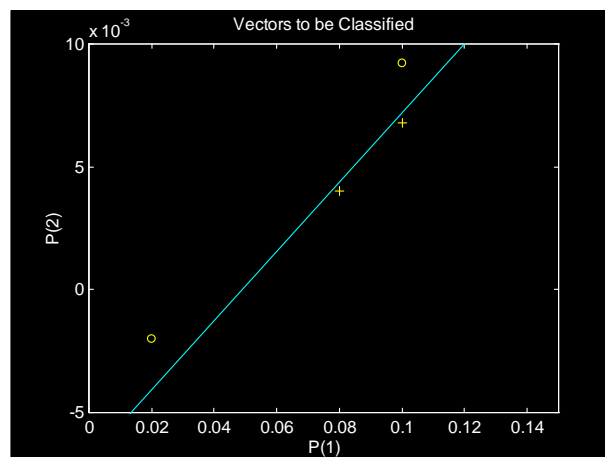


Figure 7 – An example using Matlab

5. Concluding Remarks

This paper explores a crucial aspect in the issues of political business cycles by considering the effects of bounded rationality, an aspect that has been largely ignored. I hope that this paper has contributed to fill some, even if in a little amount, of the gap.

As a direction for future improvements I would like to explore the possible dynamics of convergence for output in order to check, for the long-run, the real importance of the initial level of output. Furthermore, other types of neural networks may also be considered.

References

BARRELL, Ray, Guglielmo Maria CAPORALE, Stephen HALL, and Anthony GARRATT (1992), “Learning about Monetary Union: An Analysis of Boundedly Rational Learning in European Labour Markets”, *National Institute of Economic and Social Research Discussion Paper No. 22*, August.

BAŞAR, Tamer, and Mark H. SALMON (1990a), “Inflation and the Evolution of the Credibility with Disparate Beliefs”, in CHRISTODOULAKIS, N.M. (ed.), **Dynamic Modelling and Control of National Economies 1989: Selected papers from the 6th IFAC Symposium**, Pergamon Press, Oxford, 75-81.

BAŞAR, Tamer, and Mark SALMON (1990b), “Credibility and the Value of Information Transmission in a Model of Monetary Policy and Inflation”, *Journal of Economic Dynamics and Control*, **14**, 97-116.

CHO, In-Koo, and Thomas J. SARGENT (1996), “Neural Networks for Encoding and Adapting in Dynamic Economies”, in AMMAN, Hans M., David A. KENDRICK, and John RUST (eds.), **Handbook of Computational Economics**, Vol. **I**, Elsevier Science, Amsterdam, 441-470.

CRIPPS, Martin (1991), “Learning Rational Expectations in a Policy Game”, *Journal of Economic Dynamics and Control*, **15**, 297-315.

ELLACOTT, Steve, and Deb ROSE (1996), **Neural Networks: Deterministic Methods of Analysis**, International Thomson Computer Press, London.

EVANS, George W. (1986), “Selection Criteria for Models with Non-uniqueness”, *Journal of Monetary Economics*, **18**, 147-157.

EVANS, George W., and Roger GUESNERIE (1993), “Rationalizability, Strong

Rationality and Expectational Stability”, *Games and Economic Behavior*, Special Issue on Learning Dynamics, **5**, October, 632-646.

EVANS, George W., and Seppo HONKAPOHJA (1994), “Learning, Convergence, and Stability with Multiple Rational Expectations Equilibria”, *European Economic Review*, **38**, 1071-1098.

EVANS, George W., and Seppo HONKAPOHJA (1995), “Expectational Stability and Adaptive Learning: An Introduction”, in KIRMAN, Alan, and Mark SALMON (eds.), **Learning and Rationality in Economics**, Basil Blackwell, Oxford, 102-126.

FUHER, J.C., and M.A. HOOKER (1993), “Learning about Monetary Regime Shifts in an Overlapping Wage Contract Model”, *Journal of Economic Dynamics and Control*, **17**, Issue **4**, July, 531-553.

GÄRTNER, Manfred, (1996), “Political business cycles when real activity is persistent”, *Journal of Macroeconomics*, **18**, 679-692.

GÄRTNER, Manfred (1997), “Time-consistent monetary policy under output persistence”, *Public Choice*, **92**, 429-437.

GÄRTNER, Manfred (1999), “The Election Cycle in the Inflation Bias: Evidence from the G-7 countries”, *European Journal of Political Economy*, **15**, 705-725.

GÄRTNER, Manfred (2000), “Political Macroeconomics: A Survey of Recent Developments”, *Journal of Economic Surveys*, **14**, Issue **5**, 527-561.

JONSSON, Gunnar (1997), “Monetary Politics and Unemployment Persistence”, *Journal of Monetary Economics*, **39**, **2**, June, 303-325.

MARCET, Albert, and Thomas J. SARGENT (1988), “The Fate of Systems With “Adaptive” Expectations”, *The American Economic Review (Papers and Proceedings)*, **78**, No. **2**, May, 168-172.

MARCET, Albert, and Thomas J. SARGENT (1989a), “Convergence of Least Squares Learning Mechanisms in Self-Referential Linear Stochastic Models”, *Journal of Economic Theory*, **48**, No. **2**, August, 337-368.

MARCET, Albert, and Thomas J. SARGENT (1989b), “Convergence of Least-Squares Learning in Environments with Hidden State Variables and Private Information”, *Journal of Political Economy*, **97**, No. **6**, 1306-1322.

MARIMON, Ramon, and Shyam SUNDER (1993), “Indeterminacy of Equilibria in a Hyperinflationary World: Experimental Evidence”, *Econometrica*, **61**, No. **5**, 1073-1108.

MARIMON, Ramon, and Shyam SUNDER (1994), “Expectations and Learning under Alternative Monetary Regimes: An Experimental Approach”, *Economic Theory*, **4**, 131-162.

MINFORD, Patrick (1995), "Time-Inconsistency, Democracy, and Optimal Contingent Rules", *Oxford Economic Papers*, **47**, No. **2**, April, 195-210.

NORDHAUS, William D. (1975), "The Political Business Cycle", *The Review of Economic Studies*, **42**(2), No. **130**, April, 169-190.

SALMON, Mark (1995), "Bounded Rationality and Learning: Procedural Learning", in KIRMAN, Alan, and Mark SALMON (eds.), **Learning and Rationality in Economics**, Basil Blackwell, Oxford, 236-275.

SARGENT, Thomas J. (1993), **Bounded Rationality in Macroeconomics**, Clarendon Press, Oxford.

SWINGLER, Kevin (1996), **Applying Neural Networks: A Practical Guide**, Academic Press Limited, London.

WALL, Kent D. (1993), "A Model of Decision Making Under Bounded Rationality", *Journal of Economic Behavior and Organization*, **20**, 331-352.

WESTAWAY, Peter (1992), "A Forward-Looking Approach to Learning in Macroeconomic Models", *National Institute Economic Review*, **2**, May, 86-97.

WHITE, Halbert (1989), "Some Asymptotic Results for Learning in Single Hidden-Layer Feedforward Network Models", *Journal of the American Statistical Association*, **84**, No. **408**, December, 1003-1013.

Annex – Mathematical details

In the case $\pi_0 = \alpha\beta$, the solutions are:

$$\begin{aligned}\pi_1^B &= \alpha\beta(1 - \rho(\gamma - \phi)), & y_1^B &= \phi y_0 + \alpha^2\beta(1 - \rho(\gamma - \phi) - \gamma), \\ \pi_2^B &= \alpha\beta, & y_2^B &= \phi^2 y_0 + \alpha^2\beta(\phi - 2\phi\rho\gamma + \rho\phi^2 - \phi\gamma + 1 - \gamma + \gamma^2\rho), \\ \pi_1^O &= \alpha\beta\left(1 - \frac{\gamma - \phi}{\mu}\right), & y_1^O &= \phi y_0 + \alpha^2\beta\frac{\mu - \gamma + \phi - \gamma\mu}{\mu}, \\ \pi_2^O &= \alpha\beta, & y_2^O &= \phi\left(\phi y_0 + \alpha^2\beta\frac{\mu - \gamma + \phi - \gamma\mu}{\mu}\right) + \alpha\left(\alpha\beta - \gamma\alpha\beta\left(1 - \frac{\gamma - \phi}{\mu}\right)\right).\end{aligned}$$

This means that:

$$\begin{aligned}\pi_1^B - \pi_1^O &= \alpha\beta(\gamma - \phi) \frac{1 - \mu\rho}{\mu}, & \pi_2^B - \pi_2^O &= 0, \\ y_1^B - y_1^O &= \alpha^2\beta(\gamma - \phi) \frac{1 - \mu\rho}{\mu}, & y_2^B - y_2^O &= -\alpha^2\beta(\gamma - \phi)^2 \frac{1 - \mu\rho}{\mu}.\end{aligned}$$

Given the previous expressions, the slopes and intercepts of the straight lines, $y_t^i = a^i + b^i\pi_t^i$, for $i = B, O$ and $t = 1, 2$ are:

$$\begin{aligned}b^B &= \frac{\phi(\phi - 1) y_0 + \alpha^2\beta(-\gamma\phi + \gamma^2\rho - 2\phi\rho\gamma + \rho\gamma + \rho\phi^2 + \phi - \rho\phi)}{\beta\alpha\rho(\gamma - \phi)}, \\ a^B &= \frac{\phi(-1 + \phi + \rho\phi^2 - \phi\rho\gamma) y_0 + \alpha^2\beta(\phi\rho\gamma^2 - \gamma\rho\phi^2 + \phi - 4\phi\rho\gamma + 2\rho\phi^2 - \gamma\phi + 2\gamma^2\rho + 3\phi\rho^2\gamma^2 - 3\phi^2\rho^2\gamma + \rho^2\phi^3 - \gamma^3\rho^2)}{(\phi - \gamma)\rho}, \\ b^O &= \frac{\phi\mu(\phi - 1) y_0 - \alpha^2\beta(\phi\gamma\mu - \gamma^2 + 2\gamma\phi - \gamma - \phi^2 - \phi\mu + \phi)}{\beta\alpha(\gamma - \phi)}, \\ a^O &= \frac{\mu\phi(\phi^2 + \phi\mu - \gamma\phi - \mu) y_0 - \alpha^2\beta(4\phi\gamma\mu - 3\phi\gamma^2 + 3\phi^2\gamma + \gamma^3 - \phi^3 - 2\mu\gamma^2 + \mu^2\phi\gamma + \mu\phi^2\gamma - \mu\phi\gamma^2 - \mu^2\phi - 2\phi^2\mu)}{(\phi - \gamma)\mu}.\end{aligned}$$

Those two lines will cross at

$$\pi = \alpha\beta\left(1 - \frac{\alpha^2\beta\rho(\phi - \gamma)^3}{((\phi - 1)y_0 + \alpha^2\beta(1 - \gamma))\mu\phi}\right).$$

Moreover,

$$b^B - b^O = \phi(1 - \rho\mu) \frac{(\phi - 1) y_0 - \alpha^2 \beta (\gamma - 1)}{\alpha \beta \rho (\gamma - \phi)},$$

$$a^B - a^O = (1 - \rho\mu) \frac{\phi\mu(1 - \phi) y_0 + \alpha^2 \beta (-\phi\mu + \phi\gamma\mu - \rho\gamma^3 + 3\phi\rho\gamma^2 - 3\gamma\rho\phi^2 + \phi^3\rho)}{(\gamma - \phi) \mu\rho}.$$

In the particular case of an initial output level $y_0 = \alpha^2 \beta \frac{1 - \gamma}{1 - \phi}$ we have

$$b^B = b^O = \alpha(1 - \phi + \gamma) > 0,$$

$$a^B - a^O = -(1 - \rho\mu) \alpha^2 \beta \frac{(\gamma - \phi)^2}{\mu} < 0.$$

In case of no persistence at the output level, that is when $\phi = 0$, the solutions are:

$$\pi_1^B = \alpha\beta(1 - \rho\gamma), \quad \pi_2^B = \alpha\beta,$$

$$y_1^B = \alpha^2 \beta (1 - \gamma - \gamma\rho), \quad y_2^B = \alpha^2 \beta (1 - \gamma + \gamma^2\rho),$$

$$\pi_1^O = \alpha\beta \left(1 - \frac{\gamma}{\mu}\right), \quad \pi_2^O = \alpha\beta,$$

$$y_1^O = \alpha^2 \beta \frac{\mu - \gamma - \gamma\mu}{\mu}, \quad y_2^O = \alpha^2 \beta \frac{\mu + \gamma^2 - \gamma\mu}{\mu}.$$

It is then straightforward to verify that:

$$\pi_1^B - \pi_1^O = \alpha\beta\gamma \frac{1 - \mu\rho}{\mu} > 0, \quad \pi_2^B - \pi_2^O = 0,$$

$$y_2^B - y_1^B = \alpha^2 \beta \gamma (1 + \gamma) \rho > 0, \quad y_2^O - y_1^O = \alpha^2 \beta \gamma (1 + \gamma) \frac{1}{\mu} > 0,$$

$$y_1^B - y_1^O = \alpha^2 \beta \gamma \frac{1 - \mu\rho}{\mu} > 0, \quad y_2^B - y_2^O = -\alpha^2 \beta \gamma^2 \frac{1 - \mu\rho}{\mu} < 0,$$

$$y_2^O - y_1^B = \alpha^2 \beta \gamma \frac{\gamma + \mu\rho}{\mu} > 0.$$

Given the previous expressions, the slopes and intercepts of the straight lines, $y_t^i = a^i + b^i \pi_t^i$,

for $i = B, O$ and $t = 1, 2$ are:

$$b^B = \alpha(\gamma + 1)$$

$$a^B = \alpha^2 \beta \gamma (\gamma \rho - 2)$$

$$b^O = \alpha(\gamma + 1)$$

$$a^O = \alpha^2 \beta \gamma^{\frac{\gamma - 2\mu}{\mu}}.$$

Plainly,

$$a^O - a^B = \alpha^2 \beta \gamma^2 \frac{1 - \mu \rho}{\mu} > 0.$$