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Fertility in Portugal
How persistent is it?[1]

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Abstract:

The decline in fertility that has been observed in Portugal is an apparent fact. From 1960 to 2002, the average number of children by woman has decreased from 3.1 to 1.5.
Not ignoring this strong evidence of a sustainable decrease in fertility, the fact is that the numbers on the fertility rates by women' ages show different realities. At the first sight, the decline in fertility of younger women has been the result of a postponement of births given that a general increase in fertility rates has been observed for older women. A question that then comes up is the following: are these observed trajectories sustainable in the sense of reflecting persistence in time or are just mere phases of a cycle in fertility? The paper intents to start giving an answer to that question by the use of statistical techniques, in a univariate approach, which are adequate to measure the degree of persistence over time.

Keywords: Fertility, Persistence, Portugal

JEL Classification: C22, J11, J13

1. Introduction and motivation

This paper considers the fertility rates for Portuguese women aged between 16 and 48 years old for the period 1971-2002.[3] Figure 1 gives a picture of the evolution registered by fertility.

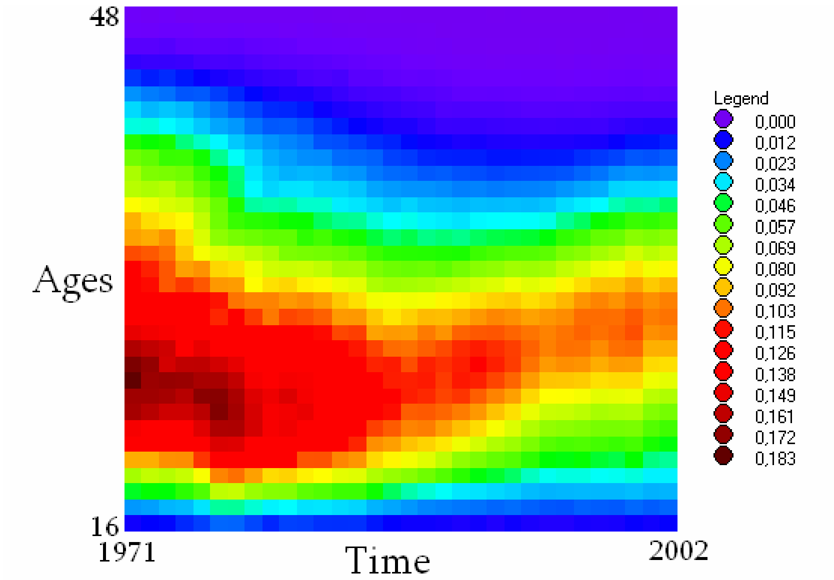

Figure 1 – The evolution of fertility in Portugal [1971-2002]

Clearly, in the overall, there seems to exist evidence of a sustainable decrease in fertility but it is also true that the numbers on the fertility rates by women ages show different realities. At the first sight, the decline in fertility of younger women has been the result of a postponement of births given that a general increase in fertility rates has been observed for older women. A question that then comes up is the following: are these observed trajectories sustainable in the sense of reflecting persistence in time or are just mere phases of a cycle in fertility? The paper intents to start giving an answer to that question by the use of statistical techniques, in a univariate approach, which are adequate to measure the degree of persistence over time. Before doing that, an aspect of apparent importance, as it is the correlation between fertility rates, is to be presented.

In fact, in accordance to figure 1, it seems that, despite the inevitable high correlation, in fertility, for women with close ages, given the usual time lag between births, the correlation may increase after a certain amount of time. This is certainly important to

[3] The data source is the Eurostat.

be taken into account in the assessment of the results in what concerns the detection of the women's ages for which fertility is more persistent. Figure 2 plots the correlation coefficient of fertility among the distinct ages.

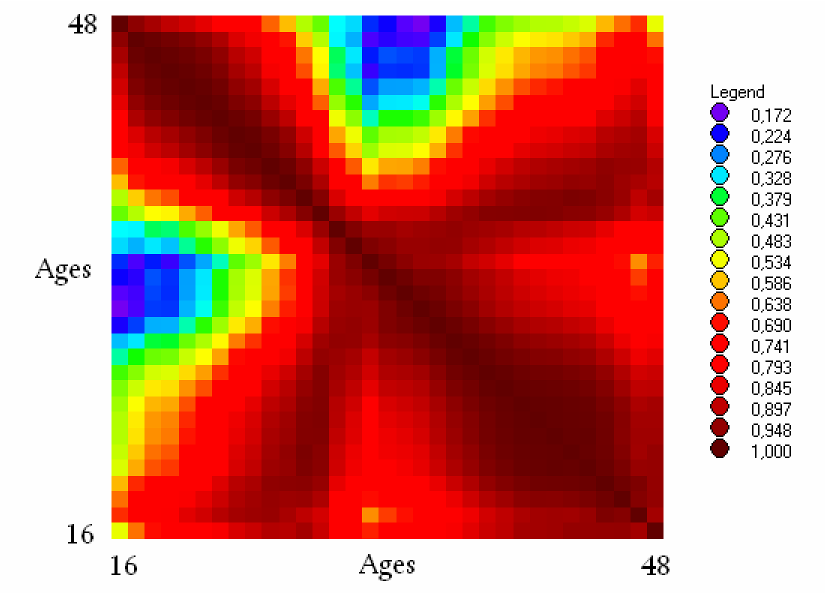

Figure 2 – The correlation between fertility rates

Generally speaking, what figure 2 shows is that the correlation coefficient of fertility rates decreases when the distance in ages increases but only to a certain distance, as for certain ages, women sufficiently separated in age are more similar, in terms of fertility.

That being said, the rest of the paper has the following structure. Section 2 analyses the fertility persistence. Section 3 concludes.

2. Measuring fertility persistence in Portugal

As the previous section has shown, in accordance to the women's ages, the fertility rates in Portugal have gone through evolutions that are clearly distinct. A question that then comes up is the following: are these observed trajectories sustainable in the sense of reflecting persistence in time or are just mere phases of a cycle in fertility? We start giving an answer to this question by the use of statistical techniques, in a univariate approach, which are adequate to measure the degree of persistence over time.

Since some time ago, some authors have started to pay attention to persistence in (economic) time series as a phenomenon that reveals to be crucial to policy measures, namely at the inflation level. In fact, to the best of our knowledge, all the applications of the statistical techniques to measure the level of persistence have considered the inflation rate case. See Hondroyiannis and Lazaretou (2004), Levin and Piger (2002), Marques (2004), Minford *et al.* (2004), and Pivetta and Reis (2004). We propose to apply those statistical techniques, developed by Andrews and Chen (1994), Dias and Marques (2004) and Marques (2004), to fertility rates in Portugal. In this sense, the novelty in the approach is supposed to be a contribution to filling the gap in the demographics literature.

Starting with a simple definition, fertility persistence is the speed with which fertility returns to baseline (its previous level) after, say a shock, *i.e.* some event (for instance, a demographic policy measure) that provoked an increase (or decrease) in the fertility rate. This definition, in other words, implies that the degree of fertility persistence is associated with the speed with which fertility responds to a shock. When the value is high, fertility responds quickly to a shock. On the contrary, when the value is small, the speed of adjustment by fertility is low. To put it clearer, a variable is said to be the more persistent the slower it converges or returns to its previous level, after the occurrence of a shock.

Quantifying the response of fertility to a shock is indeed important not only because it may allow assessing the effectiveness of demographic policy measures but also because it may, indeed, show at what time is more essential to act, through those measures, in order to overwhelm a harmful effect of a shock over fertility. By definition, quantifying the response of fertility to shocks implies evaluating the persistence of fertility.

As the estimates of persistence at time t will express how long we expect that a shock to fertility will take to die off (if ever), given present and *past* fertility, authors have proposed to obtain those estimates by the use of *autoregressive models*. As it is well known, a univariate AR(k) process is characterised by the following expression:

(1) $$f_t = \mu + \sum_{j=1}^{k} \alpha_j f_{t-j} + \varepsilon_t$$

where f_t denotes the fertility rate at moment *t*, which is explained by a constant μ, by past values up to lag *k*, as well as by a number of other factors, whose effect is captured by the random variable ε_t. Plainly, (**1**) can also be written as:

(**2**) $$\Delta f_t = \mu + \sum_{j=1}^{k-1} \delta_j \Delta f_{t-j} + (\rho - 1) f_{t-1} + \varepsilon_t$$

where

(**3**) $$\rho = \sum_{j=1}^{k} \alpha_j$$

and

$$\delta_j = -\sum_{i=j+1}^{k} \alpha_i \ .$$

In the context of the above model (**1**), or (**2**), persistence can be defined as the speed with which fertility converges to its previous level after a shock in the disturbance term that raises fertility at moment *t* by 1%.[4]

The techniques allowing for measuring the persistence are based on the analysis of the autoregressive coefficients α_j in (**1**) or (**2**), which are subject to a statistical estimation. Plainly, the most simple case of the models (**1**) or (**2**) is the so-called AR(1) model, that is:

(**4**) $$f_j = \mu + \alpha_1 f_{t-1} + \varepsilon_t \, .$$

Clearly, the variable ε_t in this kind of models has a particular importance given that it may be associated with policy measures leading to a shock in the fertility rates. A positive shock, at moment *t*, will significantly last for future moments the higher is the

[4] Given that the persistence is a long-run effect of a shock to fertility, this concept is intimately linked to a concept usually associated to autoregressive models such as (**1**) or (**2**), *i.e.* the impulse response function of fertility, which, in fact, is not a useful measure of persistence since its infinite length.

autoregressive coefficient α_1. Following this approach, Andrews and Chen (1994) proposed the sum of the autoregressive coefficients, $\rho = \sum_{j=1}^{k} \alpha_j$, as a measure of persistence.[5] The rationale for this measure comes from realizing that for $|\rho| < 1$, the cumulative effect of a shock on fertility is given by $\frac{1}{1-\rho}$.

Following the approach above described, some autoregressive models (**1**) were estimated for Portugal, considering all women's ages between 16 and 48, for the period 1971 to 2002. The results can be consulted in the Appendix 1.

A general comment on the obtained results is that, not surprisingly, for many ages, a simple AR(1) (with or without constant) *appears* to be a congruent model in explaining the fertility rate. Yet there are a non ignorable number of cases where a higher order autoregressive model is suggested by the data. This fact poses a problem from the viewpoint of measuring persistence in the fertility rates, as it will be described below.

Quite recently, Marques (2004) has suggested a non-parametric measure of persistence, γ, based on the relationship between persistence and mean reversion. In particular, Marques (2004) suggested using the statistic:

(**5**) $$\gamma = 1 - \frac{n}{T},$$

where n stands for the number of times the series crosses the mean during a time interval with $T + 1$ observations, to measure the absence of mean reversion of a given series, given that it may be seen as the unconditional probability of that given series not crossing its mean in period t.[6]

[5] Authors have, indeed, proposed other alternative measures of persistence, such as the largest autoregressive root, the spectrum at zero frequency, or the so called half-life. For a technical appraisal of these other measures see, for instance, Marques (2004) and Dias and Marques (2004).

[6] As acknowledged in Marques (2004), values close to 0.5 indicate the absence of any significant persistence (white noise behaviour) while figures significantly above 0.5 signal significant (positive) persistence.

As Dias and Marques (2004) have shown, there is a one-to-one correspondence between the sum of autoregressive coefficients, ρ, given by (**3**) and the non-parametric measure, γ, given by (**5**), when the data are generated by an AR(1) process, but such a one-to-one correspondence ceases to exist once higher order autoregressive processes are considered. In other words, only in the particular case of a first-order autoregressive model, AR(1), either one of the two measures can be used to quantify the level of persistence, as both transmit the same result, but as soon as higher order autoregressive models are considered, *i.e.*, AR(k) with $k \geq 2$, the monotonic relationship between ρ and γ no longer exists, therefore leading to possibly crucial differences when measuring persistence in the series.

As Dias and Marques (2004) plainly show, using the alternative measure of persistence, γ, given by (**5**), has some important advantages.[7] Given its nature, such measure of persistence does not impose the need to assume a particular specification for the data generation process, therefore does not require a model for the series under investigation to be specified and estimated.[8] This is so given that γ is indeed extracting all the information about the persistence from the data itself. As it measures how often the series reverts to its means and (high/low) persistence exactly means that, after a shock, the series reverts to or crosses its means more (seldom/frequently), one does not need to specify a particular form for the data generation process.

That being said, it resulted clear that, in order to measure the persistence in the fertility rates in Portugal, one should rely on the use of the non-parametric measure γ. Clearly, in order to compute the estimative for each women's age, the mean of each series has to be computed. As suggested in Marques (2004), a time varying mean is more appropriate than the simple average for all the period under investigation. In our case we followed that suggestion by using the well known Hodrick-Prescott (HP) filter in order to compute the mean.

As it is well known, the HP filter defines the trend or mean, g_t, of a time series, f_t, as the solution to the minimisation problem:

[7] The statistical properties of γ are extensively analysed in Marques (2004) and Dias and Marques (2004).

[8] In technical terms, this means that the measure is expected to be robust against potential model misspecifications and given its non-parametric nature also against outliers in the data.

$$\min_{\{g_t\}}\left\{\sum_{t=1}^{T}(f_t-g_t)^2+\lambda\sum_{t=2}^{T-1}((g_{t+1}-g_t)-(g_t-g_{t-1}))^2\right\}$$

i.e. the HP-filter seeks to minimise the cyclical component (f_t-g_t) subject to a smoothness condition reflected in the second term. The higher the parameter λ, the smoother will be the trend and the less deviations from trend will be penalised. In the limit, as λ goes to infinity, the filter will choose $(g_{t+1}-g_t)=(g_t-g_{t-1})$, for $t=2,\ldots,T-1$, which just amounts to a linear trend. Conversely, for $\lambda=0$, we get the original series.

Plainly, the HP-filter is a very flexible device since it allows us to approximate many commonly used filters by choosing appropriate values of λ. Given that the data is of yearly frequency, authors have suggested using values for λ between 7 and 13. In order to check the robustness of the results we considered all these values when computing the estimates of γ. See the Appendix 2, where the pictures corresponding to the $\lambda=10$ case are also shown.

From the results, one can conclude that there are two particular groups of women that reveal to possess higher levels of fertility persistence. The first group is composed by women with ages between 22 and 25 years old, notably 24 years old, and a second group composed by women whose age is 30 or 31 years old.

3. Conclusion and directions for further research

This paper has explored the question of fertility persistence in Portugal. The main conclusion is that there are two groups of women that are of particular relevance for demographic policy measures, namely women between 22 and 25 years and those aged between 30 and 31 years old.

As directions for further research we would like to further explore the lagged correlation analysis in order to discern about the moment in time where women started to change their relative behaviour towards fertility.

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Appendix 1 – The statistical results for the autoregressive models

```
Modelling FRate16 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate16_1        0.99399     0.019432   51.153  0.0000  0.9890

R^2 = 0.989039  \sigma = 0.00141509  DW = 1.63
* R^2 does NOT allow for the mean *
RSS = 5.807175301e-005 for 1 variables and 31 observations
```

```
Modelling FRate17 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate17_1         1.3524      0.17607    7.681  0.0000  0.6782
FRate17_2       -0.36100      0.17505   -2.062  0.0486  0.1319

R^2 = 0.994091  \sigma = 0.00221059  DW = 2.02
* R^2 does NOT allow for the mean *
RSS = 0.0001368276735 for 2 variables and 30 observations
```

```
Modelling FRate18 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate18_1         1.3615      0.17450    7.802  0.0000  0.6849
FRate18_2       -0.37369      0.17269   -2.164  0.0392  0.1433

R^2 = 0.993794  \sigma = 0.0037565  DW = 1.95
* R^2 does NOT allow for the mean *
RSS = 0.0003951165252 for 2 variables and 30 observations
```

```
Modelling FRate19 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate19_1         1.4354      0.16430    8.736  0.0000  0.7316
FRate19_2       -0.44900      0.16231   -2.766  0.0099  0.2146

R^2 = 0.995284  \sigma = 0.00469408  DW = 1.80
* R^2 does NOT allow for the mean *
RSS = 0.0006169637452 for 2 variables and 30 observations
```

```
Modelling FRate20 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate20_1        0.98171     0.013405   73.232  0.0000  0.9944

R^2 = 0.994437  \sigma = 0.00652368  DW = 1.40
* R^2 does NOT allow for the mean *
RSS = 0.001276750225 for 1 variables and 31 observations
```

```
Modelling FRate21 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate21_1         1.3325      0.17443    7.639  0.0000  0.6758
FRate21_2       -0.34853      0.17141   -2.033  0.0516  0.1287

R^2 = 0.996334  \sigma = 0.00624478  DW = 1.94
```

```
* R^2 does NOT allow for the mean *
RSS = 0.001091925057 for 2 variables and 30 observations
```

```
Modelling FRate22 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate22_1         1.4493      0.16504    8.782  0.0000  0.7336
FRate22_2       -0.46168      0.16134   -2.862  0.0079  0.2263

R^2 = 0.998078  \sigma = 0.00505135  DW = 1.85
* R^2 does NOT allow for the mean *
RSS = 0.0007144505379 for 2 variables and 30 observations
```

```
Modelling FRate23 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate23_1        0.97275    0.0084521  115.089  0.0000  0.9977

R^2 = 0.99774  \sigma = 0.00579306  DW = 1.68
* R^2 does NOT allow for the mean *
RSS = 0.001006785791 for 1 variables and 31 observations
```

```
Modelling FRate24 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate24_1        0.97557    0.0069129  141.122  0.0000  0.9985

R^2 = 0.998496  \sigma = 0.00491198  DW = 1.74
* R^2 does NOT allow for the mean *
RSS = 0.0007238265099 for 1 variables and 31 observations
```

```
Modelling FRate25 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate25_1        0.97227    0.0065120  149.304  0.0000  0.9987

R^2 = 0.998656  \sigma = 0.00467743  DW = 1.68
* R^2 does NOT allow for the mean *
RSS = 0.0006563511072 for 1 variables and 31 observations
```

```
Modelling FRate26 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate26_1        0.97502    0.0075382  129.345  0.0000  0.9982

R^2 = 0.99821  \sigma = 0.00531515  DW = 2.17
* R^2 does NOT allow for the mean *
RSS = 0.0008475246732 for 1 variables and 31 observations
```

```
Modelling FRate27 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0093202    0.0046531    2.003  0.0546  0.1215
FRate27_1        0.90334     0.038419   23.513  0.0000  0.9502

R^2 = 0.95016  F(1,29) = 552.86 [0.0000]  \sigma = 0.00402936  DW = 2.32
RSS = 0.000470837209 for 2 variables and 31 observations
```

```
Modelling FRate28 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant        0.012506    0.0055232    2.264  0.0312  0.1502
FRate28_1        0.87613     0.048195   18.179  0.0000  0.9193

R^2 = 0.919327  F(1,29) = 330.48 [0.0000]  \sigma = 0.00408627  DW = 2.45
RSS = 0.0004842314549 for 2 variables and 31 observations
```

```
Modelling FRate29 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant        0.013052    0.0067936    1.921  0.0646  0.1129
FRate29_1        0.86813     0.063711   13.626  0.0000  0.8649

R^2 = 0.864909  F(1,29) = 185.67 [0.0000]  \sigma = 0.00459869  DW = 2.24
RSS = 0.0006132898117 for 2 variables and 31 observations
```

```
Modelling FRate30 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant        0.011214    0.0050319    2.229  0.0338  0.1462
FRate30_1        0.87694     0.050646   17.315  0.0000  0.9118

R^2 = 0.911804  F(1,29) = 299.81 [0.0000]  \sigma = 0.00396265  DW = 1.38
RSS = 0.0004553748441 for 2 variables and 31 observations
```

```
Modelling FRate31 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant        0.012874    0.0047697    2.699  0.0115  0.2008
FRate31_1        0.84064     0.054214   15.506  0.0000  0.8924

R^2 = 0.892365  F(1,29) = 240.43 [0.0000]  \sigma = 0.00432299  DW = 1.93
RSS = 0.0005419582693 for 2 variables and 31 observations
```

```
Modelling FRate32 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant        0.011311    0.0031302    3.614  0.0011  0.3105
FRate32_1        0.83681     0.039754   21.050  0.0000  0.9386

R^2 = 0.938571  F(1,29) = 443.09 [0.0000]  \sigma = 0.00361017  DW = 1.91
RSS = 0.000377967473 for 2 variables and 31 observations
```

```
Modelling FRate33 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0072273    0.0024296    2.975  0.0061  0.2468
FRate33_1         1.3109      0.16253    8.066  0.0000  0.7067
FRate33_2       -0.43015      0.14427   -2.982  0.0060  0.2477

R^2 = 0.957193  F(2,27) = 301.87 [0.0000]  \sigma = 0.00277491  DW = 2.12
RSS = 0.0002079027157 for 3 variables and 30 observations
```

```
Modelling FRate34 by OLS  (using FRateP.in7)
```

```
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0067915    0.0027061    2.510  0.0184  0.1892
FRate34_1         1.1602      0.17949    6.464  0.0000  0.6074
FRate34_2       -0.29223      0.16135   -1.811  0.0813  0.1083

R^2 = 0.928574  F(2,27) = 175.51 [0.0000]  \sigma = 0.00366903  DW = 2.15
RSS = 0.0003634686018 for 3 variables and 30 observations
```

```
Modelling FRate35 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0039377    0.0017653    2.231  0.0342  0.1556
FRate35_1         1.3978      0.16565    8.438  0.0000  0.7251
FRate35_2       -0.48754      0.14477   -3.368  0.0023  0.2958

R^2 = 0.963378  F(2,27) = 355.14 [0.0000]  \sigma = 0.00259955  DW = 1.79
RSS = 0.0001824570579 for 3 variables and 30 observations
```

```
Modelling FRate36 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0031776    0.0012816    2.479  0.0197  0.1855
FRate36_1         1.4182      0.15821    8.964  0.0000  0.7485
FRate36_2       -0.50707      0.14105   -3.595  0.0013  0.3237

R^2 = 0.973091  F(2,27) = 488.19 [0.0000]  \sigma = 0.0022242  DW = 2.13
RSS = 0.0001335706009 for 3 variables and 30 observations
```

```
Modelling FRate37 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0023320    0.0010165    2.294  0.0298  0.1631
FRate37_1         1.4416      0.15080    9.560  0.0000  0.7719
FRate37_2       -0.52455      0.13687   -3.832  0.0007  0.3523

R^2 = 0.975655  F(2,27) = 541.02 [0.0000]  \sigma = 0.00206258  DW = 2.00
RSS = 0.0001148639809 for 3 variables and 30 observations
```

```
Modelling FRate38 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0014628   0.00085497    1.711  0.0985  0.0978
FRate38_1         1.2715      0.17872    7.115  0.0000  0.6522
FRate38_2       -0.34696      0.15944   -2.176  0.0385  0.1492

R^2 = 0.978561  F(2,27) = 616.18 [0.0000]  \sigma = 0.00193219  DW = 1.68
RSS = 0.0001008007301 for 3 variables and 30 observations
```

```
Modelling FRate39 by OLS  (using FRateP.in7)
The present sample is:  1974 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
Constant       0.0011314   0.00061483    1.840  0.0772  0.1152
FRate39_1         1.1505      0.11749    9.793  0.0000  0.7867
FRate39_3       -0.22299     0.094614   -2.357  0.0262  0.1760

R^2 = 0.983634  F(2,26) = 781.35 [0.0000]  \sigma = 0.00145043  DW = 1.77
RSS = 5.469763919e-005 for 3 variables and 29 observations
```

```
Modelling FRate40 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate40_1        0.93324     0.010625   87.835  0.0000  0.9961

R^2 = 0.996127  \sigma = 0.0014413  DW = 1.55
* R^2 does NOT allow for the mean *
RSS = 6.232038232e-005 for 1 variables and 31 observations
```

```
Modelling FRate41 by OLS  (using FRateP.in7)
The present sample is:  1973 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate41_1         1.2355      0.14898    8.293  0.0000  0.7106
FRate41_2       -0.29484      0.13898   -2.121  0.0429  0.1385

R^2 = 0.997429  \sigma = 0.000861822  DW = 2.03
* R^2 does NOT allow for the mean *
RSS = 2.07966641e-005 for 2 variables and 30 observations
```

```
Modelling FRate42 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate42_1        0.91851     0.012049   76.230  0.0000  0.9949

R^2 = 0.994864  \sigma = 0.000996912  DW = 1.84
* R^2 does NOT allow for the mean *
RSS = 2.981498392e-005 for 1 variables and 31 observations
```

```
Modelling FRate43 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate43_1        0.91171     0.010222   89.194  0.0000  0.9962

R^2 = 0.996243  \sigma = 0.000611274  DW = 2.50
* R^2 does NOT allow for the mean *
RSS = 1.120966389e-005 for 1 variables and 31 observations
```

```
Modelling FRate44 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate44_1        0.91461     0.015444   59.221  0.0000  0.9915

R^2 = 0.991519  \sigma = 0.000601667  DW = 2.71
* R^2 does NOT allow for the mean *
RSS = 1.086009814e-005 for 1 variables and 31 observations
```

```
Modelling FRate45 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate45_1        0.92829     0.013195   70.351  0.0000  0.9940

R^2 = 0.993975  \sigma = 0.000287366  DW = 2.27
* R^2 does NOT allow for the mean *
RSS = 2.477376318e-006 for 1 variables and 31 observations
```

```
Modelling FRate46 by OLS  (using FRateP.in7)
```

```
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate46_1        0.93168     0.026759   34.818  0.0000  0.9759

R^2 = 0.975851  \sigma = 0.000318659  DW = 2.56
* R^2 does NOT allow for the mean *
RSS = 3.046307982e-006 for 1 variables and 31 observations
```

```
Modelling FRate47 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate47_1        0.94589     0.027044   34.976  0.0000  0.9761

R^2 = 0.976064  \sigma = 0.000148167  DW = 2.22
* R^2 does NOT allow for the mean *
RSS = 6.586068406e-007 for 1 variables and 31 observations
```

```
Modelling FRate48 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate48_1        0.86906     0.051147   16.991  0.0000  0.9059

R^2 = 0.905869  \sigma = 0.000149692  DW = 3.22
* R^2 does NOT allow for the mean *
RSS = 6.722350332e-007 for 1 variables and 31 observations
```

```
Modelling FRate49 by OLS  (using FRateP.in7)
The present sample is:  1972 to 2002

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2
FRate49_1        0.89247     0.072951   12.234  0.0000  0.8330

R^2 = 0.833023  \sigma = 0.000248229  DW = 2.55
* R^2 does NOT allow for the mean *
RSS = 1.848524407e-006 for 1 variables and 31 observations
```

Appendix 2 – Mean reversion in the Portuguese fertility rates

Table 1: The values for the γ statistic

	λ = 7	λ = 8	λ = 9	λ = 10	λ = 11	λ = 12	λ = 13
16 years old	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355
17 years old	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871
18 years old	0.580645	0.580645	0.580645	0.580645	0.580645	0.580645	0.580645
19 years old	0.580645	0.580645	0.580645	0.580645	0.580645	0.580645	0.580645
20 years old	0.451613	0.451613	0.451613	0.451613	0.451613	0.516129	0.516129
21 years old	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129
22 years old	0.645161	0.709677	0.709677	0.645161	0.645161	0.645161	0.645161
23 years old	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161
24 years old	0.677419	0.677419	0.677419	0.741935	0.741935	0.741935	0.741935
25 years old	0.580645	0.580645	0.580645	0.645161	0.709677	0.709677	0.709677
26 years old	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129
27 years old	0.612903	0.612903	0.612903	0.612903	0.645161	0.645161	0.645161
28 years old	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387
29 years old	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387
30 years old	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161
31 years old	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161	0.645161
32 years old	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129	0.580645
33 years old	0.548387	0.548387	0.612903	0.612903	0.612903	0.612903	0.612903
34 years old	0.419355	0.419355	0.419355	0.483871	0.483871	0.483871	0.483871
35 years old	0.548387	0.612903	0.612903	0.612903	0.612903	0.612903	0.677419
36 years old	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871	0.548387
37 years old	0.612903	0.612903	0.612903	0.612903	0.677419	0.677419	0.677419
38 years old	0.580645	0.580645	0.580645	0.580645	0.580645	0.612903	0.677419
39 years old	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871
40 years old	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129	0.516129
41 years old	0.548387	0.548387	0.548387	0.548387	0.612903	0.612903	0.612903
42 years old	0.387097	0.387097	0.387097	0.387097	0.451613	0.483871	0.483871
43 years old	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871	0.483871
44 years old	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355
45 years old	0.387097	0.387097	0.387097	0.387097	0.387097	0.387097	0.387097
46 years old	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355	0.419355
47 years old	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387	0.548387
48 years old	0.354839	0.354839	0.354839	0.354839	0.354839	0.354839	0.354839

The $\lambda = 10$ case

N.B. – In all the figures, the top panel displays the fertility rates (in red) and the mean of the fertility rates (in blue) whereas the bottom panel displays the deviations from the mean

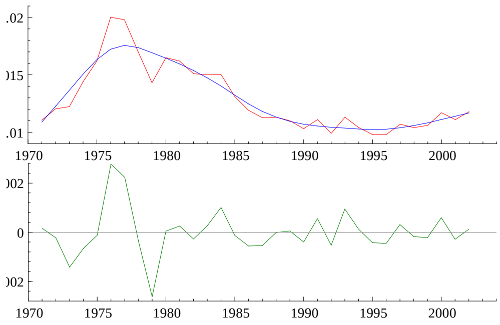

Figure 3 -- The 16 years old case

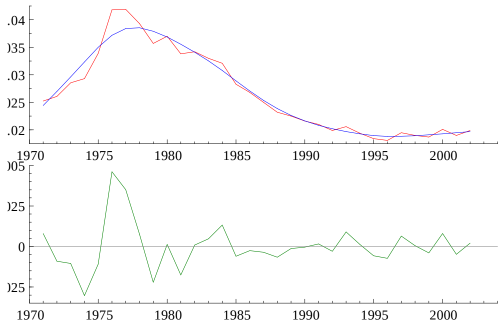

Figure 4 -- The 17 years old case

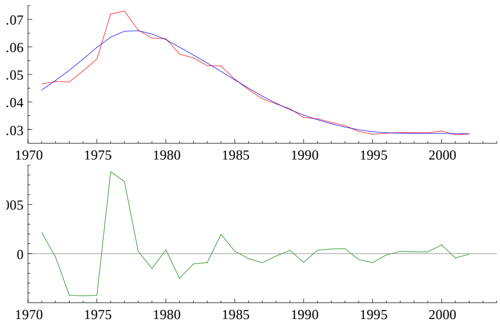

Figure 5 -- The 18 years old case

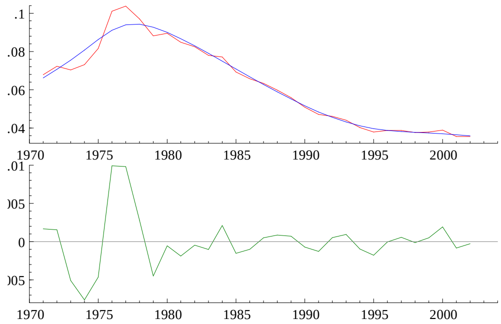

Figure 6 -- The 19 years old case

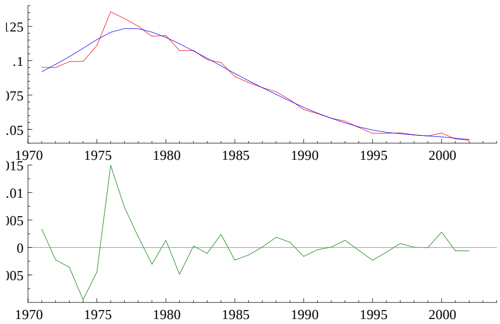

Figure 7 -- The 20 years old case

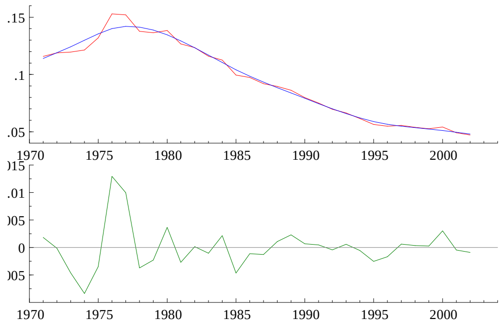

Figure 8 -- The 21 years old case

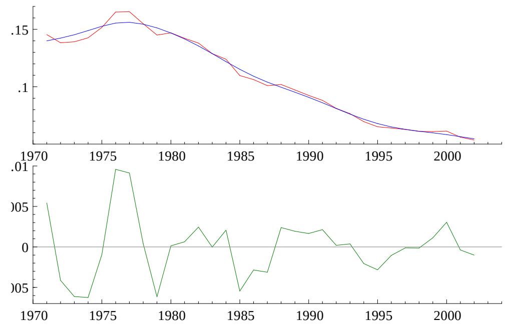

Figure 9 -- The 22 years old case

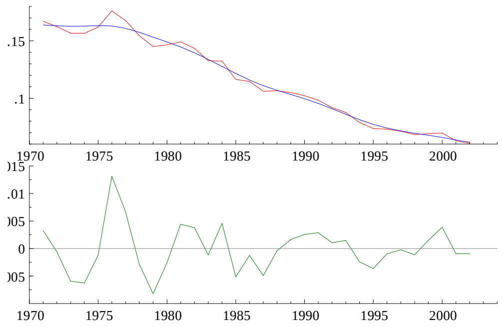

Figure 10 -- The 23 years old case

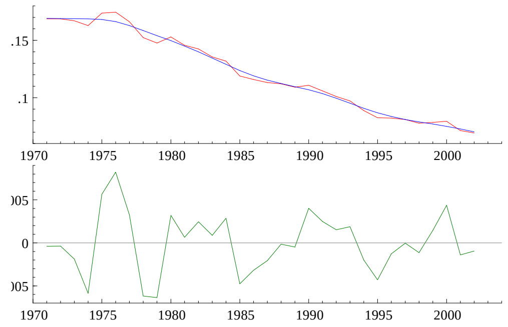

Figure 11 -- The 24 years old case

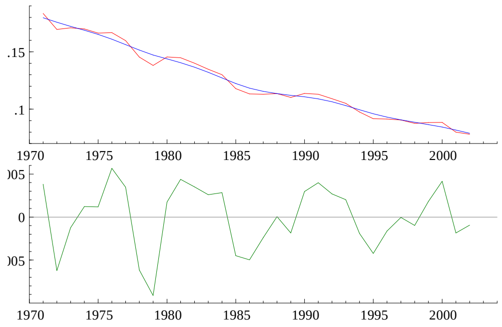

Figure 12 -- The 25 years old case

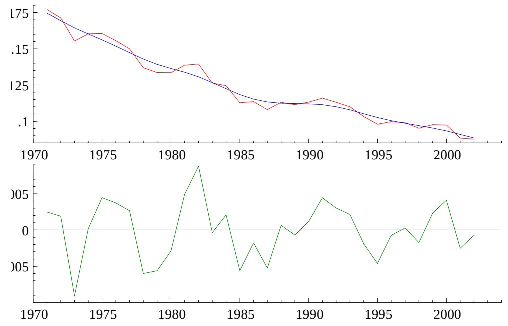

Figure 13 -- The 26 years old case

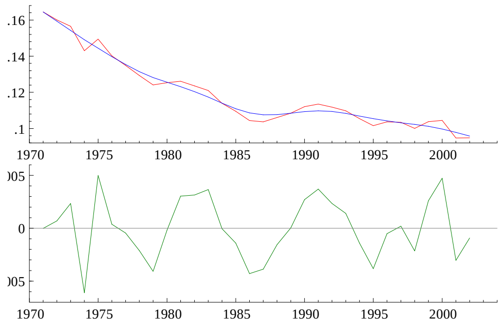

Figure 14 -- The 27 years old case

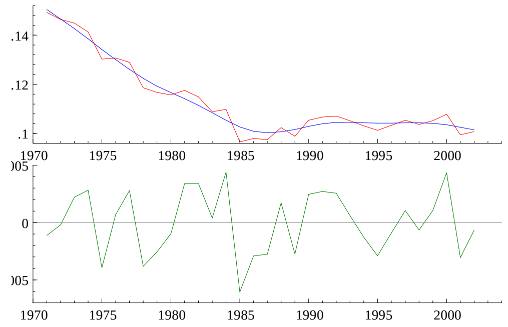

Figure 15 -- The 28 years old case

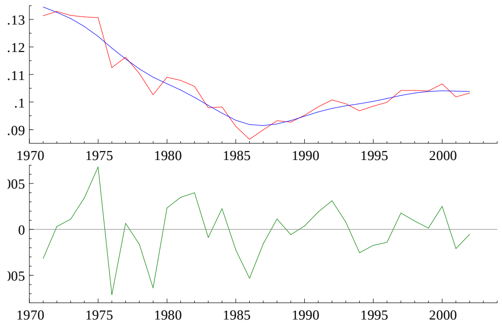

Figure 16 -- The 29 years old case

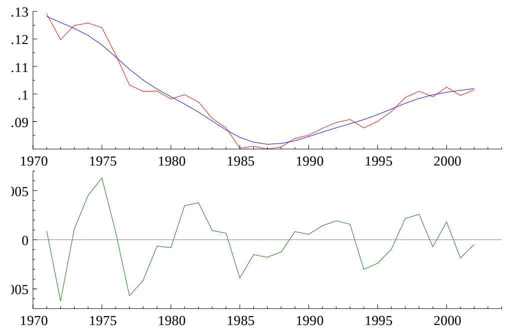

Figure 17 -- The 30 years old case

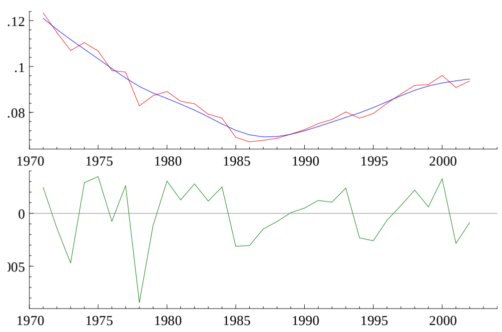

Figure 18 -- The 31 years old case

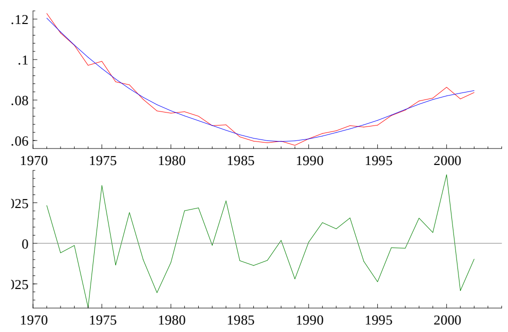

Figure 19 -- The 32 years old case

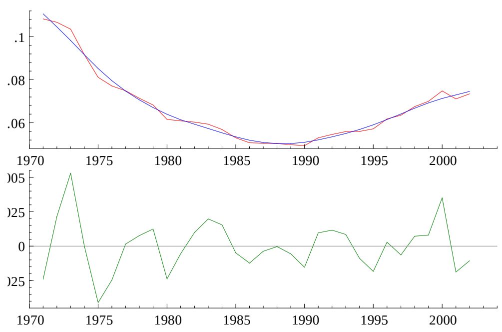

Figure 20 -- The 33 years old case

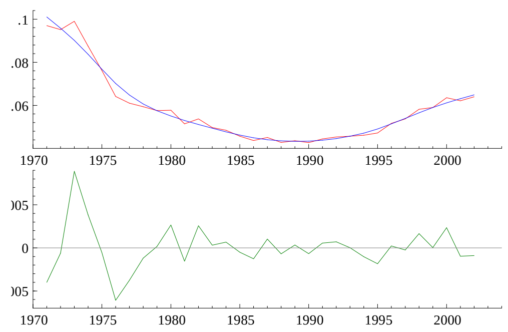

Figure 21 -- The 34 years old case

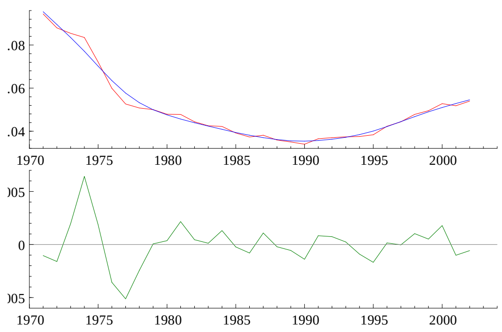

Figure 22 -- The 35 years old case

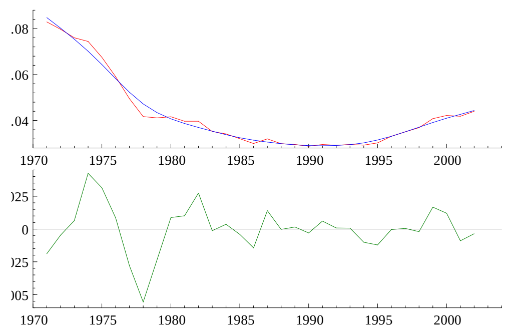

Figure 23 -- The 36 years old case

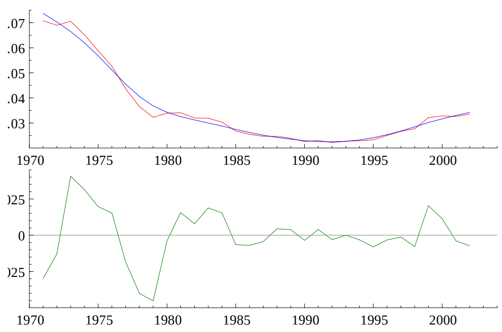

Figure 24 -- The 37 years old case

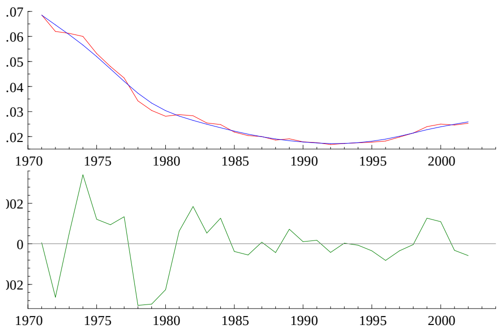

Figure 25 -- The 38 years old case

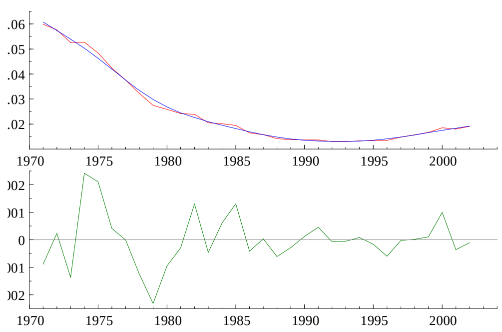

Figure 26 -- The 39 years old case

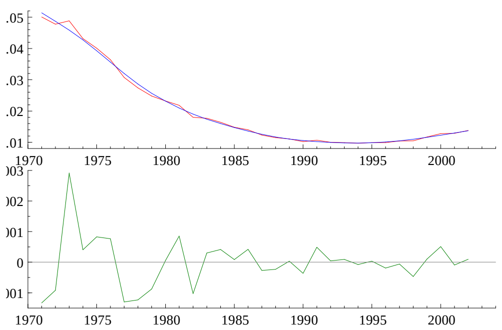

Figure 27 -- The 40 years old case

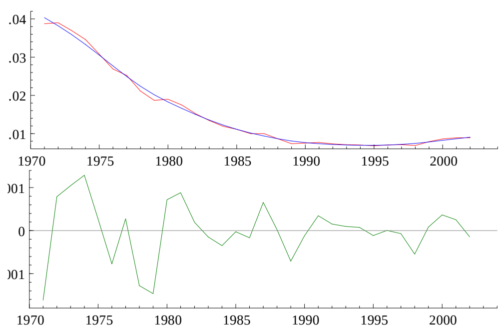

Figure 28 -- The 41 years old case

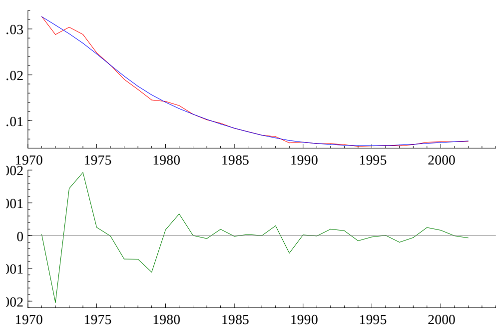

Figure 29 -- The 42 years old case

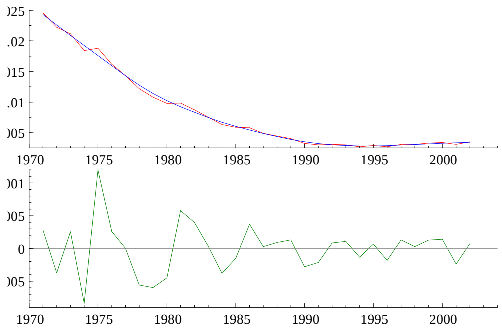

Figure 30 -- The 43 years old case

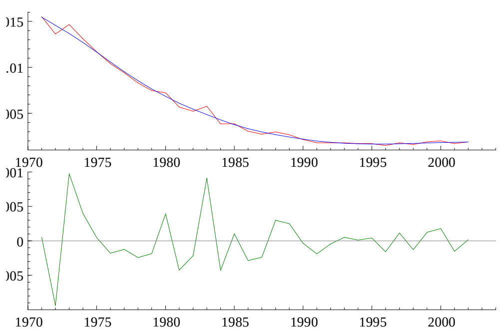

Figure 31 -- The 44 years old case

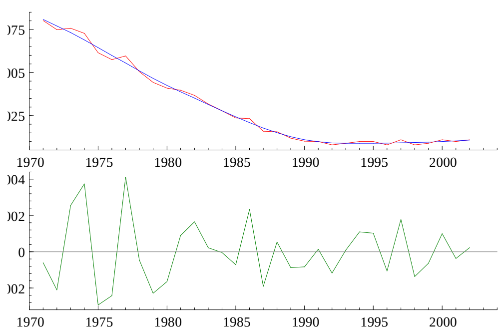

Figure 32 -- The 45 years old case

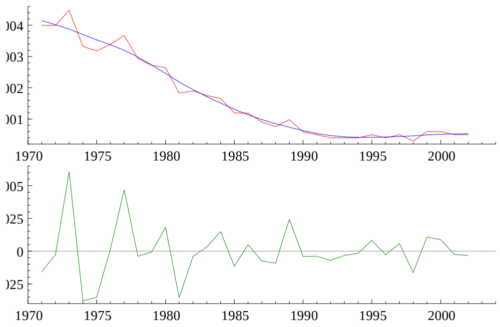

Figure 33 -- The 46 years old case

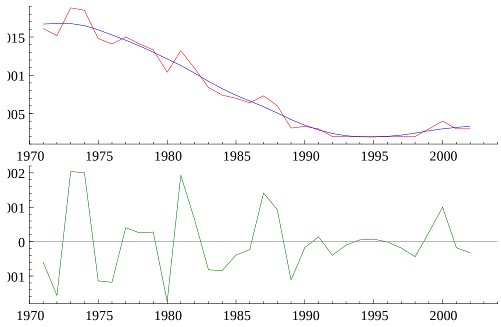

Figure 34 -- The 47 years old case

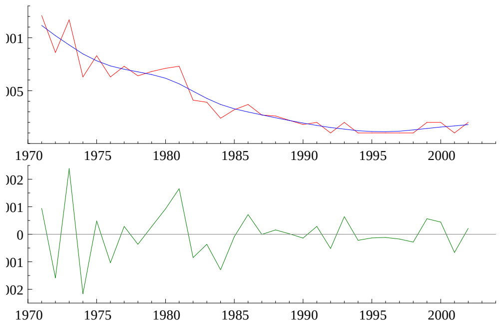

Figure 35 -- The 48 years old case