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Feasible bias-corrected OLS, within-groups, and first-diferences estimators for typical micro and macro AR(1) panel data models

Joaquim J. S. Ramalho*

Universidade de Évora, Departamento de Economia

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UNIVERSIDADE DE ÉVORA
DEPARTAMENTO DE ECONOMIA
Largo dos Colegiais, 2 – 7000-803 Évora – Portugal
Tel.: +351 266 740 894 Fax: +351 266 742 494
www.decon.uevora.pt wp.economia@uevora.pt

Resumo/ Abstract:

Dynamic panel data (DPD) models are usually estimated by the generalized method of moments. However, it is well documented in the DPD literature that this estimator suffers from considerable finite sample bias, especially when the time series is highly persistent. Application of the asymptotically equivalent continuous updating method eliminates this problem but the resultant estimator exhibits too much variability in small samples. Thus, other estimation methods are considered in this paper. Focussing in the AR(1) case with no exogenous regressors, we analyze several alternative ways of correcting the bias of the traditional estimators utilized in non-dynamic settings, showing how to construct feasible bias-adjusted ordinary least squares, within-groups, and first-differences estimators. We obtain very promising results for some of these estimators in a Monte Carlo simulation study involving data with the qualities normally encountered by both microeconomists and macroeconomists.

Palavras-chave/Keyword dynamic panel data, bias-corrections, withingroups, first-diferences, GMM, continuous-updating **.**

Classificação JEL/JEL Classification: C13, C23

1 Introduction

In the last twenty years or so, the utilization of panel data in the empirical analysis of both microeconomic and macroeconomic problems has become increasingly popular. Error components models are generally employed, which, typically, assume that the stochastic error term has two components: a time-invariant individual-specific effect, which captures the unobservable individual heterogeneity, and the usual random noise term. In linear regression models, as some explanatory variables are likely to be correlated with the individual effects, ordinary least squares (OLS) estimation cannot be applied in general. Thus, the traditional solution is to first remove the individual effects, applying some transformation to the regression equation, and then estimate the parameters of interest by OLS. Examples of such methodology are the popular within-groups (WG) and first-differences (FD) estimators, both of which are unbiased and consistent provided that the transformed error term is not correlated with the transformed regressors. Unfortunately, regression models containing lagged dependent variables and a fixed time dimension do not meet this assumption. Hence, a specific methodology for linear dynamic panel data (DPD) models is required. In this paper we deal with the particular case of first-order autoregressive [AR(1)] panel data models with no exogenous regressors, an important baseline case in the DPD literature.

Traditionally, DPD models are estimated by the asymptotically efficient two-step generalized method of moments (GMM). Under a given set of assumptions, a set of orthogonality conditions is generated for the FD model, which is then used to retrieve consistent estimators for the autoregressive coefficient of the levels equation. However, these GMM estimators suffer from three important drawbacks. First, many applied researchers do not use them due to their complexity. Although sooner or later it will be possible to compute automatically these estimators using the most popular econometric packages, the large variety of GMM estimators available, each one implying different assumptions and small sample performances, will remain a barrier for their efficient application by practitioners. Second, simulation evidence indicates that, in this framework, many GMM estimators possess some bias in finite samples, particularly when the value of the autoregressive parameter is near the unity, in which case the moment conditions conventionally employed provide only weak identification of the parameters to be estimated; see *inter alia* Blundell and Bond (1998, 2000), Blundell, Bond and Windmeijer (2000), Bond and Windmeijer (2002) and Bond (2002). Third, the performance of GMM estimators is very sensitive to the ratio of variance of the individual-specific effects and the variance of the general disturbance term; see Kitazawa (2001).

As we show in the Monte Carlo study undertaken in section 5, those small sample issues can be eliminated if, instead of GMM, we use the asymptotically equivalent continuous-updating (CU) method proposed by Hansen, Heaton and Yaron (1996). However, the results obtained in section 5 also reveal that, in this context, the CU estimator displays much more dispersion than the GMM estimator. This was somewhat expected since previous simulation studies based on different settings had concluded that, in general, the CU estimator is effectively approximately median unbiased but has a finite sample distribution with very fat tails, exhibiting sometimes extreme outlier behaviour [e.g. Hansen, Heaton and Yaron (1996) and Stock and Wright (2000)].

Given the difficulty of obtaining GMM or CU estimators with good small sample properties in all DPD settings, in this paper we follow another route to achieve this objective. We abandon the moment condition formulation and consider the possibility of applying some analytical bias correction to the OLS, WG and FD estimators which allows the removal of their bias while preserving, as much as possible, their small variability. Thus, the first aim of this paper is the development and comparison of some feasible bias-corrected (FBC) OLS, WG, and FD estimators for the autoregressive parameter of AR(1) panel data models. These FBC estimators are obtained by subtracting from the standard estimator a consistent estimate of the respective asymptotic bias. While asymptotic expressions for the biases of all three estimators have already been derived under a variety of assumptions [see *inter alia* Nickell (1981), Sevestre and Trognon (1985), Hsiao (1986), Beggs and Nerlove (1998), Kiviet (1995, 1999), Blundell, Bond and Windmeijer (2000), Bun and Kiviet (2001) and Hahn and Kuersteiner (2002)], their utilization in the construction of FBC estimators has not been fully exploited yet. The problem is that, as the asymptotic bias depends, in all cases, on the unknown autoregressive parameter, an initial consistent estimate of this parameter is required, in general, to evaluate the bias of each estimator.

The first author to propose a FBC estimator was Kiviet (1995). He suggested a FBCWG estimator which uses a two-step GMM estimator to evaluate the bias correction. Possibly due to the poor performance of the GMM estimator in finite samples, his estimator did not perform very well in the Monte Carlo study he undertook. Recently, Hahn and Kuersteiner (2002) suggested another FBCWG estimator, which circumvents the need for a preliminary consistent estimator of the parameter of interest. However, simulation results reported by them indicate that their estimator, derived assuming a large time dimension (T), is not unbiased in panels of moderately large sizes (e.g. $T = 20$), especially when the time series is highly persistent. With regard to the OLS and FD cases, no FBC estimator has been developed.

In this paper, we suggest and compare several alternative ways of constructing and implementing FBC OLS, WG, and FD estimators. One of the FBCFD estimators that we propose does not require, similarly to Hahn and Kuersteiner's (2002) FBCWG estimator, an initial consistent estimate of the autoregressive parameter to evaluate the bias correction. Moreover, all FBC estimators suggested are simpler and quicker of computing than GMM estimators and some of them are independent on the relative strength of each error component. The main advantage of considering also FBCOLS estimators is that, in this case, it is possible to identify the parameters associated to any time-invariant explanatory variable.[1]

Usually, microeconomic datasets have a time dimension far smaller and a cross-sectional dimension far greater than typical macroeconomic panels. Therefore, the best estimation methodology in one case may not be the best in the other. Furthermore, when the time dimension gets large, the number of moment conditions and, hence, computational requirements, increases rapidly and GMM estimation using all available instruments may be infeasible or impractical to implement. However, existing panel estimation techniques are in general applied indistinctly to both types of datasets but tested only with the typical dimensions of microeconomic panels in mind.[2] Thus, the second main aim of this paper is to provide directions for choosing the most appropriate techniques for estimating AR(1) panel data models of various time and cross-sectional dimensions. Hence, in the Monte Carlo investigation of the small sample properties of the several FBC estimators proposed, we generate data with the qualities normally encountered by both macroeconomists and microeconomists.

The layout of this paper is as follows. In section 2 we establish the framework of the paper, both in terms of the theoretical model considered and the experimental design that will be used in the other sections to examine the finite sample properties of several estimators. In section 3 we briefly review and illustrate through a Monte Carlo study the main characteristics of standard OLS, WG, and FD estimators and the corresponding unfeasible bias-corrected versions. In section 4 we propose a simple FBCFD estimator and analyze some other possible ways of constructing FBC estimators. In section 5 we investigate the small sample performance of various FBC estimators and

[1] However, we do not explore this issue in this paper since our focus is univariate AR(1) panel data models.

[2] For example, Arellano and Bover (1995) performed Monte Carlo simulation studies considering only 3 time periods, Arellano and Bond (1991) 7, Kiviet (1995) 4 and 7, Blundell and Bond (1998) 4 and 11, Alonso-Borrego and Arellano (1999) 4 and 7, Blundell, Bond and Windmeijer (2000) 4 and 8 and Bond and Windmeijer (2002) 6. The only exception seems to be Judson and Owen (1999), who considered panels with 5, 10, 20 and 30 time periods. However, they compared only a limited set of estimators.

contrast it with those of GMM and CU estimators based on two distinct sets of moment conditions. In section 6 we provide concluding remarks and discuss possible extensions of the proposed FBC estimators.

2 Framework

2.1 The theoretical model

In this paper we consider estimation of the autoregressive parameter α in the AR(1) panel data model

$$y_{it} = \alpha y_{i,t-1} + u_{it}, \tag{1}$$

$$u_{it} = \eta_i + \upsilon_{it}, \tag{2}$$

for $i = 1, ..., N$ and $t = 2, ..., T$, where y_{it} denotes an observation on some series for individual i in period t and the error term u_{it} has the usual error components decomposition: η_i is an unobserved time-invariant individual-specific effect and υ_{it} is a random disturbance that varies both in the time and cross-sectional dimension.

All the estimators considered throughout this paper are based on subsets of the following assumptions concerning the error components structure and the initial conditions process. Regarding the former, we assume that

$$E(\eta_i) = E(\upsilon_{it}) = 0, \tag{3}$$

$$E(\eta_i \upsilon_{it}) = 0, \tag{4}$$

$$E(\upsilon_{it}\upsilon_{is}) = 0, \text{ for } t \neq s, \tag{5}$$

$$E\left(\upsilon_{it}^2\right) = \sigma_\upsilon^2 \tag{6}$$

and

$$E\left(\eta_i^2\right) = \sigma_\eta^2, \tag{7}$$

where $i = 1, ..., N$, $t = 2, ..., T$. With respect to the initial conditions, we assume that

$$E(y_{i1}\upsilon_{it}) = 0, \text{ for } t = 2, ..., T, \tag{8}$$

and

$$E\left[\left(y_{i1} - \frac{\eta_i}{1-\alpha}\right)\eta_i\right] = 0, \tag{9}$$

where $i = 1, ..., N$. We assume also that all observations are independent across individuals and that the model in (1) is dynamically stable, i.e. $|\alpha| < 1$.

The set of assumptions (3)-(5) and (8) is the most commonly adopted in the DPD literature; see Ahn and Schmidt (1995). The additional stationary mean assumption (9) on the initial conditions implies a set of instruments which allows a substantial reduction in the finite sample bias of GMM estimators; see Blundell and Bond (1998), Blundell, Bond and Windmeijer (2000), Bond and Windmeijer (2002), and section 5 of this paper for some Monte Carlo evidence. The FBC estimators that we consider in section 4 require assumptions (3)-(6) and, only for the FBCOLS estimator, (7). Note that assumption $E(\eta_i) = 0$ is not essential in any case since nonzero mean of η_i can be handled with an intercept.

2.2 The experimental design

All the experiments reported in sections 3 and 5 used the AR(1) panel data model described by equations (1) and (2) as data generating process, with $\eta_i \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ independent across $i = 1, ..., N$ and $\upsilon_{it} \sim \mathcal{N}\left(0, \sigma_\upsilon^2\right)$ independent across $i = 1, ..., N$ and $t = 1, ..., T$. We generate η_i and υ_{it} such that they are independent of each other. The initial observations for the dependent variable are obtained from

$$y_{i1} = \frac{\eta_i}{1-\alpha} + \frac{\upsilon_{i1}}{\sqrt{1-\alpha^2}}, \; i = 1, ..., N. \tag{10}$$

We consider $\{N, T\} = \{100, 4\}$, $\{100, 8\}$ and $\{500, 8\}$ for the microeconomic analysis and $\{N, T\} = \{25, 13\}$, $\{50, 13\}$ and $\{50, 26\}$ for the macroeconomic case. In both cases, we choose $\alpha \in \{0.05, 0.5, 0.95\}$ and run 1000 replications, with new values for all variables drawn in each replication. In order to control the relative strength of each error component on the variance of y_{it}, we choose σ_η^2 according to

$$\sigma_\eta^2 = \mu^2 \frac{1-\alpha}{1+\alpha} \sigma_\upsilon^2, \tag{11}$$

where $\mu^2 \equiv \frac{\frac{\sigma_\eta^2}{(1-\alpha)^2}}{\frac{\sigma_\upsilon^2}{1-\alpha^2}}$ is the ratio between the two variance components of y_{it}.[3] We set $\sigma_\upsilon^2 = 1$ and $\mu^2 = 1$, 5 or 10.

Results concerning the median bias, the standard deviation across replications and the root mean squared error (RMSE) are presented for each estimator examined.[4]

[3] Note that $Var(y_{it}) = \frac{\sigma_\eta^2}{(1-\alpha)^2} + \frac{\sigma_\upsilon^2}{1-\alpha^2}$.

[4] We decided to report for all estimators the median bias instead of the mean bias due to the following two reasons. First, in the setting considered in this paper, the CU estimator can be interpreted as a limited information maximum

3 Inconsistency of standard OLS, WG, and FD estimators

Even in non-dynamic contexts, error components models are, in general, inconsistently estimated by OLS, as some explanatory variables are likely to be correlated with the individual effects η_i in most cases. In dynamic models, this correlation is guaranteed since both y_{it} and $y_{i,t-1}$, the dependent variable and the right-hand regressor in (1), respectively, are function of η_i, as this error term is time-invariant. The traditional approach in a non-dynamic framework is to first eliminate this source of inconsistency by transforming the original equation to remove η_i and then estimate the parameters of interest by least squares. To this end, two popular methods are commonly employed: WG and FD. Using the DPD formulation given in (1)-(2) as example, in the former method the original observations are expressed as deviations from the individual means,

$$y_{it} - \bar{y}_i = \alpha \left(y_{i,t-1} - \bar{y}_{i,-1} \right) + \left(v_{it} - \bar{v}_i \right), \qquad (12)$$

where $\bar{y}_i = \frac{1}{T-1} \sum_{t=2}^{T} y_{it}$, $\bar{y}_{i,-1} = \frac{1}{T-1} \sum_{t=2}^{T} y_{i,t-1}$ and $\bar{v}_i = \frac{1}{T-1} \sum_{t=2}^{T} v_{it}$, while in the latter the original model is written in first-differences,

$$\Delta y_{it} = \alpha \Delta y_{i,t-1} + \Delta v_{it}, \text{ for } t = 3, ..., T, \qquad (13)$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\Delta y_{i,t-1} = y_{i,t-1} - y_{i,t-2}$ and $\Delta v_{it} = v_{it} - v_{i,t-1}$.

Although the individual effects η_i were eliminated from both (12) and (13), in the dynamic setting considered in this paper we cannot obtain consistent estimators for α estimating those equations by least squares. Indeed, both the transformations induced a non-negligible correlation between the transformed lagged dependent variable and the transformed error term. In the WG case, there are several sources of correlation between those transformed variables, namely those produced by the correlation between $-\frac{y_{it}}{T-1}$ and v_{it} (for $t = 2, ..., T-1$) and between $-\frac{v_{i,t-1}}{T-1}$ and $y_{i,t-1}$ ($t = 3, ..., T$), which dominate others such as that between $-\frac{v_{i,t-1}}{T-1}$ and $-\frac{y_{i,t-1}}{T-1}$ ($t = 2, ..., T-1$). On the other hand, in the FD case there is only one source of inconsistency, the correlation between $y_{i,t-1}$ and $v_{i,t-1}$, since first-differencing does not introduce all realizations of the disturbances v_{it} into the error term of the transformed equation for period t. In both cases, the cited correlations do not vanish as the number of individuals in the sample increases but, as the time dimension of the panel gets large, the bias of the WG estimator becomes less important, approaching zero as T approaches infinity.

likelihood estimator, which is known to have no finite moments; see *inter alia* Mariano (1982). Second, all the other estimators displayed very similar median and mean biases in the Monte Carlo experiments of section 5.

Analytical expressions for the bias of standard OLS, WG and FD estimators in AR(1) panel data models with no exogenous regressors have been deduced by several authors. A special attention has been devoted to the WG estimator, asymptotic bias expressions being derived by Nickell (1981), Sevestre and Trognon (1985), Beggs and Nerlove (1988), Kiviet (1995, 1999) and Hahn and Kuersteiner (2002) for a variety of situations. Sevestre and Trognon (1985) and Hsiao (1986) considered the OLS case, while Blundell, Bond and Windmeijer (2000) dealt, very briefly, with the FD estimator. The asymptotic ($N \to \infty$, T fixed) biases of these estimators for α are given by

$$b_{OLS}(\alpha) = (1-\alpha)\frac{\frac{\sigma_\eta^2}{\sigma_\upsilon^2}}{\frac{\sigma_\eta^2}{\sigma_\upsilon^2} + \frac{1-\alpha}{1+\alpha}}, \tag{14}$$

$$b_{WG}(\alpha) = -\frac{\frac{1+\alpha}{T-2}\left(1 - \frac{1}{T-1}\frac{1-\alpha^{T-1}}{1-\alpha}\right)}{1 - \frac{2\alpha}{(1-\alpha)(T-2)}\left(1 - \frac{1}{T-1}\frac{1-\alpha^{T-1}}{1-\alpha}\right)} \tag{15}$$

and

$$b_{FD}(\alpha) = -\frac{1+\alpha}{2}. \tag{16}$$

Notice that, for $\alpha > 0$, while the OLS estimator is biased upwards, with $\alpha < p\lim(\hat{\alpha}_{OLS}) < 1$, the WG and FD estimators are biased downwards, with $p\lim(\hat{\alpha}_{WG}) < \alpha$ and $p\lim(\hat{\alpha}_{FD}) = \frac{\alpha-1}{2} < 0$. Note also that the only bias that depends on T is that of the WG estimator, as could be anticipated from the exposition above, and that the bias of WG and FD estimators do not depend on the ratio of the variance of the individual specific effects and the variance of the general error term.

The next section discusses alternative procedures for evaluating these expressions in order to obtain FBC OLS, WG, and FD estimators. Before proceeding, to illustrate the accuracy of the bias approximations given, we report in Table 1 some results obtained using the experimental design described in section 2.2 for $N = 100$. We denote by UBCOLS, UBCWG, and UBCFD the unfeasible BC estimators obtained according to expressions (14), (15) and (16), respectively, which were evaluated at the unknown true values of α, σ_η^2 and σ_υ^2.

Table 1 about here

As expected, the standard OLS, WG, and FD estimators are clearly biased, the first upwards and increasing with μ^2, the others downwards. While the biases of the OLS and FD estimators are independent of the time dimension of the panel, the bias of the WG estimator decreases significantly as T gets large. However, even for $T = 26$ its bias is still between 10.9% ($\alpha = 0.95$) and 84% ($\alpha = 0.05$), which indicates that only for macroeconometric panels with an uncommon time dimension would this estimator be unbiased. On the other hand, all the unfeasible estimators are clearly unbiased in all cases simulated, which shows that the asymptotic bias expressions given above, even for samples with only 100 cross-sectional units, are pretty accurate.

4 Feasible bias-corrected estimators

The excellent performance of the unfeasible BC estimators in the Monte Carlo experiments reported in the previous section clearly evidences that the construction of FBC estimators based on expressions (14)-(16) should be an important research topic in the econometric literature of DPD models. However, to the best of our knowledge, only Kiviet (1995) and Hahn and Kuersteiner (2002) dedicated some attention to this issue, both of them suggesting FBCWG estimators. While Kiviet's (1995) estimator requires an initial consistent estimate of the autoregressive parameter, Hahn and Kuersteiner's (2002) proposal circumvents that problem in a very simple manner. In sub-section 4.1 we review Hahn and Kuersteiner's (2002) estimator and suggest a FBCFD estimator with similar characteristics. In sub-section 4.2 we discuss alternative FBC estimators which require preliminary consistent estimates of the unknown autoregressive parameter.

4.1 Estimators not requiring the use of preliminary consistent estimates

The analytical expression given in the previous section for the bias of the WG estimator, $b_{WG}(\alpha)$ of (15), was obtained assuming T fixed. As Hahn and Kuersteiner's (2002) demonstrated, (15) may be approximated by the much simpler expression

$$b_{WG}(\alpha) \simeq -\frac{1+\alpha}{T} \tag{17}$$

under large T, large N asymptotics. Using this result, they proposed the FBCWG estimator

$$\tilde{\alpha}_{WG} = \frac{T}{T-1}\hat{\alpha}_{WG} + \frac{1}{T-1}. \tag{18}$$

It is straightforward to see that $p\lim_{N\to\infty}(\tilde{\alpha}_{WG}) = \alpha$ for large T, since, from (17), $p\lim_{N\to\infty}$ $(\hat{\alpha}_{WG}) = \frac{T-1}{T}\alpha - \frac{1}{T}$. This estimator is very simple and has the important feature of not depending on unknown parameters. However, as it was derived assuming large T, it may present some small sample bias in short panels. Therefore, this estimator is expected to be useful only for macroeconomic panels with a considerable time dimension.

As we said before, no other papers were devoted to the study of FBC estimators. At least in which concerns the FD estimator, this situation is very surprising. Indeed, as the sources of inconsistency are much more complicated and numerous in the WG case assuming fixed T, compare expressions (15) and (16), it would seem more attractive to focus on the FD estimator when looking for FBC estimators for microeconomic panels.[5] As we show next, it is in fact very simple to construct a FBCFD estimator which, like Hahn and Kuersteiner's (2002) FBCWG estimator, does not require initial consistent estimates of α. From (16), we know that $p\lim_{N\to\infty}(\hat{\alpha}_{FD}) = \frac{\alpha-1}{2}$. Consider

$$\tilde{\alpha}_{FD} = 2\hat{\alpha}_{FD} + 1. \tag{19}$$

Clearly, $p\lim_{N\to\infty}(\tilde{\alpha}_{FD}) = \alpha$. This FBCFD estimator is as simple as $\tilde{\alpha}_{WG}$ of (18) but has the important advantage of being valid for any value of T, including, therefore, those common in micro panels. On the other hand, its standard deviation is twice that of $\hat{\alpha}_{FD}$, which may imply some significative variability for this estimator. Note that the standard deviation of $\tilde{\alpha}_{WG}$ for large T is approximately equal to that of $\hat{\alpha}_{WG}$ (which our simulation results in section 3 show to be similar to that of $\hat{\alpha}_{FD}$, which is half of that displayed by the unbiased $\tilde{\alpha}_{FD}$).

4.2 Other alternatives

The FBC estimators defined in (18) and (19) do not require the availability of a preliminary consistent estimator of the parameter of interest. If we consider this possibility, then other FBC estimators may be easily implemented. Denote by $\tilde{\alpha}$ a preliminary consistent estimate of α. FBC estimators can be obtained by subtracting any of the bias expressions given in (14)-(16), evaluated at $\tilde{\alpha}$, from the inconsistent estimators. For example,

$$\dot{\alpha}_{WG} = \hat{\alpha}_{WG} - b_{WG}(\tilde{\alpha}), \tag{20}$$

where $\dot{\alpha}_{WG}$ denotes a FBCWG estimator. In this framework, there are two relevant questions to be addressed: a) which $\tilde{\alpha}$ to use?; b) is it worth to calculate a FBC estimator according to an expression like (20) or it is better to make inferences using directly $\tilde{\alpha}$? The first question is discussed in the remaining of this section. The second question is answered in the next section, through a Monte Carlo simulation study.

We consider three alternative ways of obtaining a preliminary consistent estimator for α. The first consists in using any of the GMM estimators, $\tilde{\alpha}_{GMM}$, proposed in the econometric literature; see *inter alia* Anderson and Hsiao (1981, 1982), who developed instrumental variable estimators,

[5] In fact, this is the main reason why all GMM estimators proposed in the panel data literature are based on orthogonality conditions generated for the FD model.

and Arellano and Bond (1991), Arellano and Bover (1995), Ahn and Schmidt (1995) and Blundell and Bond (1998), who elaborated further and took them into the GMM framework.[6] All those estimators are consistent, although their finite sample performance is not uniform. Moreover, none seems to be unbiased in small samples when the value of the autoregressive parameter is near the unity or μ^2 is large. The other alternatives are the two FBC estimators discussed in the previous sub-section. Indeed, both $\tilde{\alpha}_{WG}$ and $\tilde{\alpha}_{FD}$ may be used to evaluate any of the bias expressions (14)-(16) and, hence, give rise to other FBC estimators. As we will see in the next section, this procedure offers some advantages in many cases.

In the Monte Carlo analysis undertaken in the next section we consider nine alternative FBC estimators for macro panels: FBCOLS1, FBCWG1 and FBCFD1, which are calculated according to an expression similar to (20) with the bias evaluated at a GMM estimator in all cases; FBCOLS2, FBCWG2 and FBCFD2, which use $\tilde{\alpha}_{FD}$ to evaluate the bias; and FBCOLS3, FBCWG3 and FBCFD3, which are based on $\tilde{\alpha}_{WG}$. For micro panels, for the reasons indicated before, only the two first sets of FBC estimators are examined. Note that $\dot{\alpha}_{FBCFD2} = \tilde{\alpha}_{FD}$, the FBC estimator proposed in (19), and $\dot{\alpha}_{FBCWG3} = \tilde{\alpha}_{WG}$, the estimator suggested by Hahn and Kuersteiner (2002) and given in (18).

In the case of the FBCOLS estimators, we need also consistent estimates of σ_η^2 and σ_v^2 to evaluate the bias expression (14). These may be obtained calculating

$$\tilde{\sigma}_v^2 = \frac{1}{N(T-1)-N-1} \sum_{i=1}^{N} \sum_{t=2}^{T} \left[(y_{it} - \bar{y}_i) - \tilde{\alpha} \left(y_{i,t-1} - \bar{y}_{i,-1} \right) \right]^2 \tag{21}$$

and

$$\tilde{\sigma}_\eta^2 = \tilde{\sigma}_u^2 - \tilde{\sigma}_v^2, \tag{22}$$

where

$$\tilde{\sigma}_u^2 = \frac{1}{N(T-1)-1} \sum_{i=1}^{N} \sum_{t=2}^{T} \left(y_{it} - \tilde{\alpha} y_{i,t-1} \right)^2 . \tag{23}$$

Note that (21) and (23) are based on consistent estimators of the residual sum of squares of equations (12) and (1), respectively.

[6] Basically, this is the procedure followed by Kiviet (1995). Note, however, that his bias correction is different from ours since he considers N finite in its derivation.

5 Monte Carlo results

In this section we assess the finite-sample behaviour of the FBC estimators proposed in the previous section and compare them with the most popular two-step GMM estimators of the DPD literature (and their CU counterparts). Namely, we consider Arellano and Bond's (1991) GMM-DIF and Arellano and Bover's (1995) GMM-SYS estimators. The former estimator is based on the validity of the $(T-1)(T-2)/2$ linear moment conditions

$$E\left[y_{i,t-s}\left(\Delta y_{it}-\alpha\Delta y_{i,t-1}\right)\right]=0, \text{ for } t=3,...,T \text{ and } s=2,...,t-1, \tag{24}$$

which requires assumptions (3)-(5) and (8), while the latter, by making also assumption (9), uses $(T-2)$ additional linear moment conditions,

$$E\left[\Delta y_{i,t-1}\left(y_{it}-\alpha y_{i,t-1}\right)\right]=0, \text{ for } t=3,...,T. \tag{25}$$

Moment conditions (24) are for the model in first-differences (13), utilizing appropriately lagged levels information as instruments, whereas conditions (25) are for the model in levels (1), employing lagged differences as instruments.[7]

Both the GMM and the CU estimators are based on minimization of the criterion

$$J_N=\left(\frac{1}{N}\sum_{i=1}^{N}q_i'Z_i\right)W_N\left(\frac{1}{N}\sum_{i=1}^{N}Z_i'q_i\right), \tag{26}$$

where

$$Z_i=Z_{di}=\begin{bmatrix} y_{i1} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & y_{i1} & y_{i2} & y_{i3} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{i,T-2} \end{bmatrix} \tag{27}$$

and $q_i=q_{di}=\begin{bmatrix}\Delta v_{i3} & \Delta v_{i4} & \Delta v_{i5} & \cdots & \Delta v_{iT}\end{bmatrix}'$, in the GMM-DIF / CU-DIF case, or

$$Z_i=\begin{bmatrix} Z_{di} & 0 & 0 & 0 & 0 \\ 0 & \Delta y_{i2} & 0 & 0 & 0 \\ 0 & 0 & \Delta y_{i3} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \Delta y_{i,T-1} \end{bmatrix} \tag{28}$$

[7] The DIF acronym refers to the use of instruments for the first *dif*ferenced equations given in (13), while the SYS acronym refers to the use of the *sys*tem of moment conditions formed by (24) and (25).

and $q_i = \begin{bmatrix} q_{di} & u_{i3} & u_{i4} & \cdots & u_{iT} \end{bmatrix}$, for the GMM-SYS / CU-SYS case, and

$$W_N = \left[\frac{1}{N} \sum_{i=1}^{N} \left(Z_i' q_i q_i' Z_i \right) \right]^{-1}, \tag{29}$$

which depends on the autoregressive parameter of interest α. While in the CU case α and the weight matrix W_N are estimated simultaneously, in the two-step case W_N is evaluated in a first step at a preliminary consistent estimator of α and then kept fixed during the minimization of (26). In the DPD setting, that initial consistent estimator is usually obtained by minimizing (26) using as weight matrix

$$W_{1N} = \left[\frac{1}{N} \sum_{i=1}^{N} \left(Z_i' H Z_i \right) \right], \tag{30}$$

where, in the GMM-DIF case, $H = H_d$ is a $(T-2)$ square matrix with 2's on the main diagonal, -1's on the first off-diagonals and zeros elsewhere, and, in the GMM-SYS case,

$$H = \begin{bmatrix} H_d & 0 \\ 0 & I_{T-2} \end{bmatrix}. \tag{31}$$

Notice that W_{1N} does not depend on any estimated parameters.[8]

The results reported below for the FBCOLS1, FBCWG1 and FBCFD1 estimators are based on the GMM-SYS estimator.

5.1 Micro datasets

Tables 2a and 2b report the results of the Monte Carlo experiments involving typical micro panels. They confirm recent findings by Blundell and Bond (1998), Blundell, Bond and Windmeijer (2000) and Bond and Windmeijer (2002) about the clearly superior performance of the GMM-SYS estimator relatively to the, still popular, GMM-DIF estimator. Indeed, the latter estimator exhibits a significant bias in all experiments, in particular for $\alpha = 0.95$ and $\mu^2 = 10$ (more than 20% even for $T = 8$ and $N = 500$), and its dispersion is always much larger than that of the GMM-SYS estimator. Note, however, that for $N = 100$ the performance of the GMM-SYS estimator is not the ideal: while for $\mu^2 = 1$ this estimator is approximately unbiased, higher values for μ^2 cause a deterioration on its performance, giving rise to some important biases (e.g. 86% for $N = 100$, $T = 4$, $\mu^2 = 10$ and $\alpha = 0.05$).

[8] For more details concerning the computation and a more thorough discussion of each one of the two-step GMM estimators considered in this paper see Blundell, Bond and Windmeijer (2000).

Tables 2a and 2b about here

Similar conclusions can be achieved for the CU estimator: it is clearly advantageous to estimate the SYS version whenever assumption (9) can be made. Moreover, both CU estimators are significantly better, in terms of bias, than their GMM counterparts: the CU-SYS estimator is approximately median unbiased in all cases, while the CU-DIF estimator displays some bias only for $N = 100$ and $\alpha = 0.95$. However, both CU estimators have an important drawback: their RMSE's are in many cases larger than those of the corresponding GMM estimators due to the higher variability exhibited by them. This issue is particularly serious when $\alpha = 0.95$ and only the DIF moment conditions are used, in which case, due to the weak identification of the model estimated, it was relatively common the production of extreme values for the CU estimators; see Ramalho (2002) for similar results obtained in poorly identified instrumental variable models.

The results found for the FBC estimators are very promising, especially those obtained for the FBC WG and FD estimators. On the one hand, their FBC1 version presents a small bias only for $N = 100$, being better than the GMM-SYS estimator that is used in their bias correction according to all criteria. On the other hand, their FBC2 version is approximately unbiased in all cases. Moreover, both versions display, in general, less dispersion than GMM and CU estimators, and do not change significantly (FBC1) or at all (FBC2) as μ^2 increases. Comparing the two FBC estimators, we see that the RMSE performance of the FBC WG estimators is clearly superior to that of their FD counterparts due to the lower dispersion displayed by them in all cases simulated.[9] With regard to the FBCOLS estimators, they did not perform so well as the other FBC estimators: the FBCOLS1 estimator exhibits very similar biases to the GMM-SYS estimator used in its evaluation and both versions are characterized by higher variability than the other FBC estimators, probably due to the necessity of estimating σ_η^2 and σ_υ^2. Finally, notice that, despite the large increment in their dispersion, which is a common feature of most FBC estimators, see *inter alia* MacKinnon and Smith (1998), most of the FBC estimators exhibit much less RMSE than the corresponding uncorrected estimators (the exceptions are the FBCOLS estimators for $\alpha = 0.95$; compare the two first parts of Tables 1 and 2b).

[9] As pointed out by a referee, the WG estimator is equal to generalized least squares on the FD model (13) (taking into account the moving average structure of the transformed disturbance term). As the FD estimator results from applying OLS on the FD model, it should have larger dispersion than the WG estimator. According to the results reported in Tables 1 and 2, in the dynamic setting analysed in this paper that conclusion holds only after correcting for the bias of each estimator.

The main recommendations from our Monte Carlo analysis for micro panels are summarized below.

Summary of recommendations for micro panels	
Criteria	Estimator
Bias	CU-SYS, FBCWG1, FBCFD1 or any FBC2
RMSE	any FBCWG

5.2 Macro datasets

With macro panels of substantial time dimension, GMM estimation based on all available instruments raises some computational issues, since the number of moment conditions grows rapidly as T increases. Actually, in panels with the time dimension simulated in this section there are available 66 or 77 ($T = 13$) and 300 or 324 ($T = 26$) instruments for GMM-DIF and GMM-SYS, respectively, which means that GMM estimation based on the full set of instruments is not practical to implement. Hence, in the simulation study that follows, we restricted the number of moment conditions utilized in the GMM estimation, as most empirical researchers would do. Namely, instead of (24), we used only the following $(T-2)$ moment conditions:

$$E\left[y_{i,t-2}\left(\Delta y_{it}-\alpha\Delta y_{i,t-1}\right)\right]=0,\text{ for }t=3,...,T. \tag{32}$$

Thus, the GMM-DIF estimator that we consider in this sub-section uses 11 ($T = 13$) or 24 ($T = 26$) instruments, while the GMM-SYS estimator based on the system of $2 \times (T-2)$ equations formed by (25) and (32) employs 22 ($T = 13$) or 48 ($T = 26$) moment conditions.

Tables 3a, 3b and 3c report the results obtained for macro panels. Now, the GMM-SYS estimator exhibits a more significant bias in all cases, particularly for high μ^2 and low α. Therefore, the necessity of having better alternatives available is even more important in this context.[10] The

[10] Note that the micro and macro experiments are not directly comparable as the total number of observations is different, so it is not possible to say that the increased bias displayed by the GMM-SYS estimator is a direct consequence of the necessary reduction of the number of instruments used in estimation. In order to assess the sensitivity of this estimator to the number of instruments employed in its estimation, we performed further experiments involving larger subsets of the FD moment conditions given in (24). Namely, we considered two additional GMM-SYS estimators which exploit all available instruments with a lag length less than or equal to q for each FD equation (24), with $q = 3$ or $q = 4$. The inclusion of the additional instruments produced a steadily reduction in the dispersion of the GMM-SYS estimator in all cases but, in terms of bias, the results were very similar. These results are available from the author upon request.

CU-SYS estimator and the FBC estimators based on $\tilde{\alpha}_{FD}$ of (19) display again the best bias performance, being clearly unbiased in all cases. Among them, the FBCWG2 exhibits again the least RMSE. Despite the poor performance of the GMM-SYS estimator, which is the reason for the deceiving results obtained for the FBCOLS1 and FBCFD1 estimators, the bias of the FBCWG1 is relatively small. Moreover, its RMSE is similar (for $\alpha \in \{0.05, 0.50\}$) or clearly inferior (for $\alpha = 0.95$) to that of FBCWG2. Notice also that the FBC estimators based on $\tilde{\alpha}_{WG}$ of (18) present the least dispersion of the three versions analyzed for each case, as was expected from the discussion below (19). However, only for very large panels can they be seen as serious competitors of the other FBC estimators, since they present important biases for $\alpha = 0.95$. Thus, clearly, even in macro panels, some of the new FBC estimators suggested in this paper possess better finite sample properties than Hahn and Kuersteiner's (2002) FBCWG3 estimator. The dispersion of the CU-SYS estimator is still incomparably larger than that of any other estimator.

Tables 3a, 3b and 3c about here

The main conclusions achieved by this Monte Carlo analysis for macro panels are thus very similar to those for micro panels (the only relevant difference is that is now less secure to use the GMM-SYS estimator in bias evaluation) and can be summarized as follows.

Summary of recommendations for macro panels	
Criteria	Estimator
Bias	CU-SYS, FBCWG1 or any FBC2
RMSE	FBCWG1

6 Concluding remarks and extensions

In this paper we suggested alternative procedures for constructing FBC methods for estimating the autoregressive parameter of AR(1) panel data models with no exogenous regressors. These estimators are simpler and quicker of calculating than the GMM estimators usually employed in this setting. Moreover, as our Monte Carlo simulation study illustrates, their finite sample properties are clearly superior, since most of the FBC estimators suggested are unbiased, even when the time series is highly persistent, present less RMSE, and are not affected by the relative magnitude of the variances for the individual effect and the idiosyncratic error. They seem to be also better than the CU-SYS estimator due to the large dispersion displayed by this estimator. Overall, the best

FBC estimators, both in micro and macro panels, seem to be FBCWG estimators, namely those based on the GMM-SYS estimator (in terms of RMSE) and on the $\tilde{\alpha}_{FD}$ suggested in this paper (in terms of bias). Both are superior to Hahn and Kuersteiner's (2002) FBCWG3 estimator, which suffers from some small sample bias, and to direct consideration of $\tilde{\alpha}_{FD}$, which exhibits too much variability.

The first important avenue for future research is the adaptation of these FBC estimators to the more interesting and general case of DPD models with exogenous regressors. This will require the derivation of analytical expressions for the asymptotic bias of the OLS and FD estimators for such circumstances since, as far as we know, only for the WG estimator has this expression already been derived; see, for example, Nickell (1981, p. 1424). As those derivations are straitghforward in both cases, it is very simple to construct FBC1-type estimators for the OLS, WG and FD cases also in models with exogenous regressors. However, it seems less trivial to obtain FBC estimators without resorting to outside initial consistent estimates since the asymptotic bias expressions are much more complex in this case.

The FBC estimators developed in this paper were derived under the assumption of homoscedasticity of the general disturbances υ_{it}. In order to check their robustness to the violation of that assumption, we performed further Monte Carlo experiments allowing for heteroscedasticity. The simulation results reported in the Appendix reveal that the FBC estimators behave reasonably well under that new scenario: on the one hand, cross-sectional heteroscedasticity do not seem to affect significantly the unbiasedness of most FBC estimators; on the other hand, under the presence of heteroscedasticity over time, FBCWG estimators, although no longer unbiased in small samples, still display less RMSE than GMM-SYS estimators. Nevertheless, a second important extension of our paper is the development of FBC estimators robust to general heteroscedasticity patterns.

Finally, in this paper we do not provide any analytical expressions for the asymptotic variances of the FBC estimators since those derivations for finite T are not trivial at all. However, although those expressions might be useful in order to make inference in empirical work, they are not the only alternative. Actually, we recommend the utilization of bootstrap procedures as those suggested by Bun and Kiviet (2001, pp. 15-16) to estimate the variance of our FBC estimators. To the best of our knowledge, Bun and Kiviet (2001) are the only authors who derived an analytical expression for the asymptotic variance of a particular FBC estimator for finite T. However, they found expressions too complex to be widely used by practitioners and their Monte Carlo simulation showed that bootstrap variance estimators are relatively accurate for a wider range of cases than the analytical ones.

Appendix

In this section we present further Monte Carlo results, which consider designs with heteroscedastic disturbances. Relative to the previous experiments, we focus on the case with $\mu^2 = 1$ and $\{N, T\} = \{100, 8\}$ (micro analysis) or $\{N, T\} = \{50, 26\}$ (macro panel) and change the data generation process of the general disturbance υ_{it}. We investigate two distinct heteroscedasticity patterns. In order to allow for cross-sectional heteroscedasticity, we generate $\upsilon_{it} \sim \mathcal{N}\left(0, \sigma^2_{\upsilon,i}\right)$, with $\sigma^2_{\upsilon,i} \sim \chi^2_1$. Heteroscedastic errors over time are generated according to $\upsilon_{it} \sim \mathcal{N}\left(0, \sigma^2_{\upsilon,t}\right)$, with $\sigma^2_{\upsilon,t} = a + 0.15\,(t-1)$ and $a = 0.475$ for $T = 8$ or $a = 0.1$ for $T = 13$. Note that in both cases υ_{it} has variance one across the whole sample, as in the homoscedastic case. Tables 4 and 5 report the results obtained for each case.

Tables 4 and 5 about here

As expected, cross-sectional heteroscedasticity considerably increase the variance of all estimators. However, this form of heteroscedasticity seems to have a small impact on the bias of each estimator. This is a somewhat surprising result since FBC estimators were derived under assumption (6). In opposition, in the case of heteroscedastic errors over time the increment in the variability of the estimators is less significant but the FBC estimators are no longer unbiased. In terms of bias, GMM or CU estimators based on the moment conditions "SYS" are now clearly the best, since they are robust to any form of heteroscedasticity. However, as in most cases the bias of the FBC estimators is not substantial (apart from the FBCFD2 estimator), they are still the best in terms of RMSE, namely the FBCWG estimators.

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Table 1: Monte Carlo simulation results for inconsistent and unfeasible bias-corrected estimators

T	α	OLS			WG	FD	UBCOLS			UBCWG	UBCFD
	μ^2	1	5	10	1/5/10	1/5/10	1	5	10	1/5/10	1/5/10
4	0.05	0.472	0.790	0.863	-0.354	-0.525	-0.003	-0.001	-0.001	-0.001	0.000
		(0.055)	(0.029)	(0.019)	(0.056)	(0.055)	(0.055)	(0.029)	(0.019)	(0.056)	(0.055)
		[0.473]	[0.789]	[0.862]	[0.357]	[0.526]	[0.056]	[0.029]	[0.019]	[0.056]	[0.055]
	0.50	0.248	0.416	0.454	-0.536	-0.750	-0.002	-0.001	-0.001	-0.000	-0.000
		(0.038)	(0.021)	(0.015)	(0.066)	(0.061)	(0.038)	(0.021)	(0.015)	(0.066)	(0.061)
		[0.250]	[0.416]	[0.454]	[0.541]	[0.751]	[0.039]	[0.021]	[0.015]	[0.066]	[0.061]
	0.95	0.025	0.042	0.045	-0.731	-0.974	-0.000	-0.000	-0.000	-0.002	0.001
		(0.013)	(0.007)	(0.005)	(0.073)	(0.070)	(0.013)	(0.007)	(0.005)	(0.073)	(0.070)
		[0.028]	[0.042]	[0.046]	[0.734]	[0.978]	[0.013]	[0.007]	[0.005]	[0.073]	[0.070]
8	0.05	0.472	0.790	0.863	-0.152	-0.526	-0.003	-0.002	-0.001	-0.001	-0.001
		(0.044)	(0.022)	(0.013)	(0.037)	(0.030)	(0.044)	(0.022)	(0.013)	(0.037)	(0.030)
		[0.473]	[0.789]	[0.862]	[0.156]	[0.526]	[0.044]	[0.022]	[0.014]	[0.037]	[0.030]
	0.50	0.249	0.416	0.454	-0.237	-0.750	-0.001	-0.001	-0.000	-0.002	-0.000
		(0.028)	(0.013)	(0.009)	(0.039)	(0.035)	(0.028)	(0.013)	(0.009)	(0.039)	(0.035)
		[0.249]	[0.416]	[0.454]	[0.240]	[0.752]	[0.028]	[0.014]	[0.009]	[0.039]	[0.035]
	0.95	0.025	0.042	0.045	-0.360	-0.977	-0.000	-0.000	-0.000	-0.002	-0.002
		(0.008)	(0.005)	(0.003)	(0.039)	(0.040)	(0.008)	(0.005)	(0.003)	(0.039)	(0.040)
		[0.026]	[0.042]	[0.046]	[0.363]	[0.977]	[0.008]	[0.005]	[0.003]	[0.039]	[0.040]
13	0.05	0.472	0.791	0.863	-0.087	-0.524	-0.003	-0.001	-0.001	0.001	0.001
		(0.040)	(0.021)	(0.013)	(0.029)	(0.022)	(0.040)	(0.021)	(0.013)	(0.029)	(0.022)
		[0.474]	[0.789]	[0.862]	[0.093]	[0.525]	[0.040]	[0.021]	[0.013]	[0.029]	[0.022]
	0.50	0.249	0.416	0.454	-0.134	-0.749	-0.001	-0.001	-0.000	-0.000	0.001
		(0.024)	(0.012)	(0.007)	(0.029)	(0.026)	(0.024)	(0.012)	(0.007)	(0.029)	(0.026)
		[0.249]	[0.415]	[0.454]	[0.138]	[0.750]	[0.024]	[0.012]	[0.007]	[0.029]	[0.026]
	0.95	0.025	0.042	0.045	-0.217	-0.974	-0.000	-0.000	-0.000	-0.001	0.001
		(0.006)	(0.003)	(0.002)	(0.024)	(0.030)	(0.006)	(0.003)	(0.002)	(0.024)	(0.030)
		[0.025]	[0.042]	[0.045]	[0.218]	[0.975]	[0.006]	[0.003]	[0.002]	[0.024]	[0.030]
26	0.05	0.475	0.792	0.864	-0.042	-0.525	0.000	0.000	0.000	-0.000	-0.000
		(0.037)	(0.020)	(0.012)	(0.019)	(0.015)	(0.037)	(0.020)	(0.012)	(0.019)	(0.015)
		[0.475]	[0.790]	[0.863]	[0.046]	[0.525]	[0.037]	[0.020]	[0.012]	[0.019]	[0.015]
	0.50	0.250	0.417	0.455	-0.062	-0.751	0.000	0.000	0.000	-0.000	-0.001
		(0.021)	(0.011)	(0.006)	(0.018)	(0.017)	(0.021)	(0.011)	(0.006)	(0.018)	(0.017)
		[0.250]	[0.416]	[0.454]	[0.065]	[0.750]	[0.021]	[0.011]	[0.006]	[0.018]	[0.017]
	0.95	0.025	0.042	0.045	-0.104	-0.975	-0.000	-0.000	0.000	-0.001	-0.000
		(0.004)	(0.002)	(0.002)	(0.013)	(0.020)	(0.004)	(0.002)	(0.002)	(0.013)	(0.020)
		[0.025]	[0.042]	[0.045]	[0.105]	[0.975]	[0.004]	[0.002]	[0.002]	[0.013]	[0.020]

Notes: (i) Results are based on 2000 draws, N = 100 and $\sigma_{\upsilon}^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 2a: Monte Carlo simulation results for microeconomic panels: GMM and CU estimators

N	T	α	μ^2	GMM-DIF 1	GMM-DIF 5	GMM-DIF 10	CU-DIF 1	CU-DIF 5	CU-DIF 10	GMM-SYS 1	GMM-SYS 5	GMM-SYS 10	CU-SYS 1	CU-SYS 5	CU-SYS 10
100	4	0.05		-0.013	-0.030	-0.032	0.000	-0.006	-0.004	0.004	0.024	0.043	0.000	-0.002	-0.002
				(0.129)	(0.174)	(0.192)	(0.136)	(0.192)	(0.217)	(0.101)	(0.126)	(0.154)	(0.106)	(0.117)	(0.120)
				[0.129]	[0.175]	[0.192]	[0.136]	[0.192]	[0.218]	[0.101]	[0.130]	[0.166]	[0.107]	[0.117]	[0.120]
		0.50		-0.028	-0.080	-0.110	0.005	0.001	-0.010	-0.000	0.029	0.064	0.004	0.002	0.003
				(0.198)	(0.305)	(0.360)	(0.222)	(0.508)	(1.551)	(0.121)	(0.142)	(0.159)	(0.132)	(0.151)	(0.158)
				[0.199]	[0.310]	[0.368]	[0.223]	[0.513]	[1.554]	[0.121]	[0.144]	[0.169]	[0.132]	[0.151]	[0.159]
		0.95		-0.355	-0.630	-0.725	-0.119	-0.309	-0.406	-0.011	0.012	0.026	0.012	0.010	0.003
				(0.736)	(0.898)	(0.909)	(17.582)	(8.557)	(22.501)	(0.157)	(0.162)	(0.152)	(0.166)	(0.177)	(0.172)
				[0.836]	[1.122]	[1.185]	[17.582]	[8.561]	[22.508]	[0.160]	[0.162]	[0.152]	[0.166]	[0.177]	[0.173]
	8	0.05		-0.016	-0.018	-0.019	-0.001	-0.002	-0.003	0.000	0.018	0.042	0.000	0.001	0.001
				(0.063)	(0.068)	(0.070)	(0.074)	(0.081)	(0.083)	(0.053)	(0.058)	(0.071)	(0.062)	(0.062)	(0.062)
				[0.065]	[0.071]	[0.073]	[0.074]	[0.081]	[0.083]	[0.053]	[0.062]	[0.086]	[0.062]	[0.062]	[0.062]
		0.50		-0.032	-0.047	-0.052	0.003	-0.001	-0.001	-0.003	0.019	0.047	-0.001	0.002	0.003
				(0.080)	(0.095)	(0.100)	(0.096)	(0.122)	(0.131)	(0.059)	(0.067)	(0.076)	(0.072)	(0.076)	(0.077)
				[0.087]	[0.107]	[0.114]	[0.096]	[0.122]	[0.131]	[0.059]	[0.070]	[0.092]	[0.072]	[0.076]	[0.077]
		0.95		-0.223	-0.392	-0.458	-0.006	-0.046	-0.070	-0.016	0.006	0.021	0.007	0.006	0.005
				(0.222)	(0.289)	(0.302)	(5.452)	(5.400)	(5.114)	(0.063)	(0.055)	(0.048)	(0.079)	(0.083)	(0.084)
				[0.341]	[0.521]	[0.581]	[5.456]	[5.401]	[5.117]	[0.067]	[0.055]	[0.050]	[0.079]	[0.083]	[0.085]
500	8	0.05		-0.004	-0.005	-0.006	-0.000	-0.001	-0.001	-0.001	-0.000	0.002	-0.001	-0.000	-0.000
				(0.026)	(0.029)	(0.029)	(0.027)	(0.029)	(0.030)	(0.021)	(0.021)	(0.022)	(0.021)	(0.021)	(0.021)
				[0.026]	[0.029]	[0.029]	[0.027]	[0.029]	[0.030]	[0.021]	[0.021]	[0.022]	[0.021]	[0.021]	[0.021]
		0.50		-0.007	-0.010	-0.012	0.000	-0.000	-0.000	0.000	0.002	0.004	0.001	0.001	0.001
				(0.033)	(0.040)	(0.042)	(0.034)	(0.041)	(0.044)	(0.024)	(0.026)	(0.027)	(0.024)	(0.025)	(0.026)
				[0.034]	[0.042]	[0.044]	[0.034]	[0.041]	[0.044]	[0.024]	[0.026]	[0.027]	[0.024]	[0.025]	[0.026]
		0.95		-0.052	-0.131	-0.191	0.006	0.002	-0.000	-0.002	0.000	0.006	0.002	0.002	0.002
				(0.082)	(0.150)	(0.196)	(0.092)	(0.355)	(3.619)	(0.026)	(0.031)	(0.031)	(0.026)	(0.033)	(0.036)
				[0.099]	[0.212]	[0.297]	[0.092]	[0.356]	[3.619]	[0.027]	[0.031]	[0.031]	[0.026]	[0.033]	[0.036]

Notes: (i) Results are based on 2000 draws and $\sigma_v^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 2b: Monte Carlo simulation results for microeconomic panels: feasible bias-corrected estimators

N	T	α	μ^2	FBCOLS1 1	FBCOLS1 5	FBCOLS1 10	FBCWG1 1	FBCWG1 5	FBCWG1 10	FBCFD1 1	FBCFD1 5	FBCFD1 10	FBCOLS2 1	FBCOLS2 5	FBCOLS2 10	FBCWG2 1/5/10	FBCFD2 1/5/10
100	4	0.05		0.004	0.023	0.043	0.002	0.011	0.020	0.003	0.012	0.026	0.005	0.003	0.002	-0.002	0.001
				(0.104)	(0.129)	(0.157)	(0.090)	(0.097)	(0.103)	(0.096)	(0.106)	(0.116)	(0.102)	(0.106)	(0.108)	(0.094)	(0.110)
				[0.105]	[0.133]	[0.169]	[0.090]	[0.098]	[0.107]	[0.097]	[0.108]	[0.121]	[0.102]	[0.106]	[0.108]	[0.094]	[0.110]
		0.50		0.005	0.032	0.065	-0.001	0.010	0.024	0.000	0.016	0.033	0.000	0.000	-0.001	-0.002	-0.001
				(0.133)	(0.150)	(0.168)	(0.107)	(0.112)	(0.116)	(0.111)	(0.119)	(0.125)	(0.162)	(0.138)	(0.124)	(0.111)	(0.122)
				[0.133]	[0.153]	[0.178]	[0.107]	[0.113]	[0.118]	[0.111]	[0.120]	[0.128]	[0.162]	[0.138]	[0.124]	[0.111]	[0.122]
		0.95		-0.019	0.011	0.025	-0.014	-0.004	0.006	-0.009	0.001	0.012	-0.011	-0.001	0.002	0.001	0.003
				(0.354)	(0.174)	(0.153)	(0.123)	(0.115)	(0.108)	(0.129)	(0.122)	(0.114)	(0.853)	(0.145)	(0.141)	(0.129)	(0.140)
				[0.356]	[0.175]	[0.153]	[0.124]	[0.115]	[0.108]	[0.130]	[0.122]	[0.114]	[0.854]	[0.145]	[0.141]	[0.129]	[0.140]
100	8	0.05		-0.000	0.017	0.039	-0.001	0.002	0.007	-0.000	0.008	0.022	-0.002	-0.003	-0.002	-0.001	-0.002
				(0.049)	(0.057)	(0.070)	(0.045)	(0.045)	(0.045)	(0.053)	(0.055)	(0.059)	(0.052)	(0.058)	(0.059)	(0.045)	(0.061)
				[0.049]	[0.060]	[0.084]	[0.045]	[0.045]	[0.046]	[0.053]	[0.056]	[0.063]	[0.052]	[0.058]	[0.059]	[0.045]	[0.061]
		0.50		-0.001	0.019	0.048	-0.002	0.003	0.010	-0.002	0.009	0.024	0.000	-0.002	-0.002	-0.002	-0.001
				(0.059)	(0.068)	(0.077)	(0.050)	(0.050)	(0.051)	(0.059)	(0.063)	(0.065)	(0.067)	(0.069)	(0.070)	(0.050)	(0.070)
				[0.059]	[0.071]	[0.092]	[0.050]	[0.051]	[0.052]	[0.059]	[0.063]	[0.070]	[0.067]	[0.069]	[0.070]	[0.050]	[0.070]
		0.95		-0.022	0.006	0.019	-0.009	-0.001	0.004	-0.011	0.001	0.008	-0.013	-0.006	-0.004	-0.003	-0.003
				(3.309)	(0.100)	(0.060)	(0.052)	(0.048)	(0.045)	(0.064)	(0.058)	(0.053)	(0.888)	(0.105)	(0.093)	(0.059)	(0.080)
				[3.310]	[0.100]	[0.061]	[0.053]	[0.048]	[0.045]	[0.065]	[0.058]	[0.053]	[0.889]	[0.105]	[0.093]	[0.059]	[0.080]
500	8	0.05		-0.001	-0.000	0.001	-0.001	-0.000	0.000	-0.002	0.001	0.000	-0.001	-0.001	-0.001	-0.001	-0.001
				(0.021)	(0.021)	(0.022)	(0.020)	(0.020)	(0.020)	(0.022)	(0.022)	(0.023)	(0.023)	(0.025)	(0.026)	(0.020)	(0.027)
				[0.021]	[0.021]	[0.022]	[0.020]	[0.020]	[0.020]	[0.022]	[0.022]	[0.023]	[0.023]	[0.025]	[0.026]	[0.020]	[0.027]
		0.50		0.001	0.002	0.004	-0.000	0.000	0.001	-0.001	-0.000	0.001	-0.001	-0.002	-0.002	-0.000	-0.002
				(0.025)	(0.026)	(0.027)	(0.022)	(0.023)	(0.023)	(0.025)	(0.026)	(0.026)	(0.029)	(0.031)	(0.031)	(0.022)	(0.031)
				[0.025]	[0.026]	[0.028]	[0.022]	[0.023]	[0.023]	[0.025]	[0.026]	[0.026]	[0.029]	[0.031]	[0.031]	[0.022]	[0.031]
		0.95		-0.003	-0.000	0.007	-0.001	-0.001	0.001	-0.002	-0.001	0.002	-0.008	-0.003	-0.002	-0.000	-0.002
				(0.163)	(2.395)	(0.042)	(0.024)	(0.024)	(0.024)	(0.028)	(0.030)	(0.029)	(0.159)	(0.050)	(0.106)	(0.026)	(0.035)
				[0.163]	[2.395]	[0.042]	[0.024]	[0.024]	[0.024]	[0.028]	[0.030]	[0.029]	[0.159]	[0.050]	[0.106]	[0.026]	[0.035]

Notes: (i) Results are based on 2000 draws and $\sigma_\upsilon^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 3a: Monte Carlo simulation results for macroeconomic panels: GMM and CU estimators

N	T	α	GMM-DIF			CU-DIF			GMM-SYS			CU-SYS		
		μ^2	1	5	10	1	5	10	1	5	10	1	5	10
25	13	0.05	-0.016	-0.019	-0.024	0.019	0.013	0.009	0.028	0.150	0.275	-0.001	0.002	0.005
			(0.097)	(0.106)	(0.113)	(0.147)	(0.151)	(0.158)	(0.088)	(0.110)	(0.133)	(0.235)	(0.329)	(0.256)
			[0.098]	[0.107]	[0.114]	[0.150]	[0.152]	[0.158]	[0.092]	[0.189]	[0.304]	[0.235]	[0.329]	[0.257]
		0.50	-0.036	-0.045	-0.057	0.007	-0.000	-0.009	0.019	0.125	0.213	-0.017	-0.004	-0.005
			(0.119)	(0.135)	(0.153)	(0.170)	(0.188)	(0.213)	(0.097)	(0.099)	(0.097)	(0.233)	(0.232)	(0.240)
			[0.125]	[0.144]	[0.165]	[0.171]	[0.188]	[0.213]	[0.099]	[0.157]	[0.229]	[0.233]	[0.232]	[0.240]
		0.95	-0.154	-0.227	-0.270	-0.062	-0.063	-0.072	-0.006	0.024	0.035	0.003	-0.001	0.001
			(0.242)	(0.314)	(0.322)	(1.372)	(10.398)	(8.804)	(0.064)	(0.044)	(0.035)	(0.210)	(0.202)	(0.214)
			[0.307]	[0.431]	[0.462]	[1.372]	[10.399]	[8.807]	[0.064]	[0.050]	[0.049]	[0.210]	[0.202]	[0.214]
50	13	0.05	-0.006	-0.010	-0.013	0.015	0.006	0.003	0.016	0.070	0.140	0.005	0.003	0.002
			(0.068)	(0.073)	(0.077)	(0.076)	(0.080)	(0.085)	(0.067)	(0.078)	(0.099)	(0.098)	(0.096)	(0.096)
			[0.068]	[0.073]	[0.078]	[0.077]	[0.081]	[0.085]	[0.069]	[0.106]	[0.175]	[0.098]	[0.096]	[0.096]
		0.50	-0.020	-0.028	-0.032	0.005	-0.004	-0.006	0.008	0.064	0.125	-0.005	-0.011	-0.016
			(0.085)	(0.094)	(0.105)	(0.093)	(0.103)	(0.113)	(0.078)	(0.081)	(0.086)	(0.123)	(0.125)	(0.125)
			[0.087]	[0.098]	[0.111]	[0.093]	[0.103]	[0.114]	[0.078]	[0.102]	[0.149]	[0.123]	[0.125]	[0.125]
		0.95	-0.099	-0.176	-0.226	-0.037	-0.054	-0.056	-0.008	0.022	0.032	-0.005	-0.002	-0.004
			(0.186)	(0.300)	(0.338)	(3.962)	(5.535)	(5.294)	(0.068)	(0.052)	(0.042)	(0.138)	(0.132)	(0.131)
			[0.223]	[0.388]	[0.461]	[3.962]	[5.539]	[5.297]	[0.069]	[0.055]	[0.053]	[0.139]	[0.132]	[0.131]
50	26	0.05	-0.006	-0.006	-0.007	0.015	0.012	0.008	0.015	0.085	0.162	-0.001	-0.001	0.003
			(0.047)	(0.048)	(0.049)	(0.067)	(0.067)	(0.068)	(0.043)	(0.052)	(0.066)	(0.156)	(0.161)	(0.157)
			[0.047]	[0.049]	[0.050]	[0.069]	[0.069]	[0.069]	[0.046]	[0.101]	[0.177]	[0.156]	[0.161]	[0.158]
		0.50	-0.014	-0.015	-0.017	0.006	0.004	0.001	0.012	0.077	0.140	0.001	-0.004	-0.007
			(0.057)	(0.059)	(0.062)	(0.077)	(0.080)	(0.082)	(0.049)	(0.050)	(0.053)	(0.154)	(0.163)	(0.159)
			[0.059]	[0.062]	[0.064]	[0.078]	[0.080]	[0.082]	[0.050]	[0.092]	[0.149]	[0.154]	[0.163]	[0.159]
		0.95	-0.052	-0.066	-0.083	-0.026	-0.026	-0.026	0.002	0.026	0.035	0.001	0.002	0.001
			(0.076)	(0.111)	(0.144)	(0.112)	(0.324)	(0.760)	(0.037)	(0.025)	(0.019)	(0.140)	(0.126)	(0.127)
			[0.095]	[0.140]	[0.183]	[0.117]	[0.326]	[0.761]	[0.037]	[0.035]	[0.039]	[0.140]	[0.126]	[0.127]

Notes: (i) Results are based on 2000 draws and $\sigma_\upsilon^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 3b: Monte Carlo simulation results for macroeconomic panels: feasible bias-corrected type 1 estimators

N	T	α		FBCOLS1			FBCWG1			FBCFD1		
			μ^2	1	5	10	1	5	10	1	5	10
25	13	0.05		0.019	0.135	0.264	0.003	0.015	0.028	0.015	0.076	0.138
				(0.075)	(0.107)	(0.133)	(0.063)	(0.064)	(0.064)	(0.085)	(0.090)	(0.095)
				[0.077]	[0.176]	[0.295]	[0.063]	[0.065]	[0.069]	[0.086]	[0.119]	[0.168]
		0.50		0.014	0.121	0.211	0.001	0.016	0.029	0.010	0.064	0.107
				(0.090)	(0.101)	(0.102)	(0.064)	(0.063)	(0.063)	(0.097)	(0.093)	(0.088)
				[0.091]	[0.156]	[0.230]	[0.064]	[0.065]	[0.069]	[0.098]	[0.112]	[0.137]
		0.95		-0.014	0.020	0.033	-0.003	0.005	0.007	-0.003	0.013	0.019
				(0.423)	(0.698)	(0.675)	(0.056)	(0.052)	(0.050)	(0.082)	(0.069)	(0.065)
				[0.423]	[0.698]	[0.676]	[0.056]	[0.052]	[0.051]	[0.082]	[0.070]	[0.067]
50	13	0.05		0.012	0.064	0.133	0.002	0.006	0.014	0.009	0.035	0.071
				(0.055)	(0.074)	(0.098)	(0.046)	(0.046)	(0.047)	(0.062)	(0.065)	(0.071)
				[0.057]	[0.098]	[0.168]	[0.046]	[0.047]	[0.049]	[0.063]	[0.075]	[0.102]
		0.50		0.008	0.062	0.123	0.000	0.008	0.016	0.005	0.032	0.061
				(0.070)	(0.080)	(0.086)	(0.047)	(0.047)	(0.047)	(0.073)	(0.072)	(0.071)
				[0.070]	[0.100]	[0.148]	[0.047]	[0.047]	[0.050]	[0.073]	[0.078]	[0.094]
		0.95		-0.016	0.018	0.031	-0.003	0.004	0.007	-0.004	0.010	0.017
				(0.885)	(0.335)	(0.318)	(0.043)	(0.039)	(0.037)	(0.069)	(0.058)	(0.052)
				[0.885]	[0.335]	[0.318]	[0.043]	[0.040]	[0.038]	[0.069]	[0.059]	[0.055]
50	26	0.05		0.009	0.074	0.152	0.002	0.005	0.008	0.008	0.042	0.080
				(0.035)	(0.049)	(0.065)	(0.029)	(0.029)	(0.029)	(0.042)	(0.044)	(0.048)
				[0.036]	[0.091]	[0.168]	[0.029]	[0.029]	[0.030]	[0.043]	[0.062]	[0.095]
		0.50		0.009	0.073	0.137	0.000	0.004	0.008	0.005	0.038	0.069
				(0.041)	(0.049)	(0.053)	(0.027)	(0.027)	(0.027)	(0.049)	(0.048)	(0.047)
				[0.042]	[0.088]	[0.146]	[0.027]	[0.027]	[0.028]	[0.049]	[0.061]	[0.084]
		0.95		-0.003	0.024	0.034	-0.001	0.004	0.006	0.001	0.013	0.018
				(0.114)	(0.100)	(0.079)	(0.022)	(0.020)	(0.019)	(0.044)	(0.036)	(0.033)
				[0.114]	[0.103]	[0.087]	[0.022]	[0.021]	[0.020]	[0.044]	[0.038]	[0.037]

Notes: (i) Results are based on 2000 draws and $\sigma_v^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 3c: Monte Carlo simulation results for macroeconomic panels: feasible bias-corrected type 2 estimators

N	T	α	μ^2	FBCOLS2			FBCWG2	FBCFD2	FBCOLS3			FBCWG3	FBCFD3
				1	5	10	1/5/10	1/5/10	1	5	10	1/5/10	1/5/10
25	13	0.05		0.002	0.001	0.002	-0.000	0.001	-0.005	-0.006	-0.007	-0.008	-0.002
				(0.073)	(0.082)	(0.084)	(0.064)	(0.087)	(0.063)	(0.062)	(0.062)	(0.062)	(0.070)
				[0.073]	[0.082]	[0.084]	[0.064]	[0.087]	[0.063]	[0.063]	[0.063]	[0.063]	[0.070]
		0.50		0.003	0.002	0.003	-0.001	0.004	-0.018	-0.021	-0.021	-0.021	-0.010
				(0.093)	(0.101)	(0.102)	(0.064)	(0.104)	(0.062)	(0.061)	(0.061)	(0.060)	(0.074)
				[0.093]	[0.101]	[0.102]	[0.064]	[0.104]	[0.065]	[0.065]	[0.065]	[0.064]	[0.075]
		0.95		-0.015	-0.003	-0.002	-0.003	0.001	-0.075	-0.075	-0.074	-0.073	-0.037
				(0.372)	(1.014)	(0.122)	(0.070)	(0.118)	(0.065)	(0.052)	(0.052)	(0.052)	(0.075)
				[0.372]	[1.014]	[0.122]	[0.070]	[0.118]	[0.101]	[0.093]	[0.093]	[0.093]	[0.084]
50	13	0.05		0.001	0.001	0.001	0.000	0.001	-0.004	-0.007	-0.007	-0.008	-0.004
				(0.053)	(0.059)	(0.061)	(0.046)	(0.063)	(0.046)	(0.046)	(0.045)	(0.045)	(0.050)
				[0.053]	[0.059]	[0.061]	[0.046]	[0.063]	[0.046]	[0.046]	[0.046]	[0.046]	[0.050]
		0.50		-0.001	0.001	0.000	-0.001	-0.000	-0.017	-0.020	-0.020	-0.021	-0.010
				(0.067)	(0.072)	(0.073)	(0.047)	(0.074)	(0.046)	(0.045)	(0.044)	(0.044)	(0.053)
				[0.067]	[0.072]	[0.073]	[0.047]	[0.074]	[0.049]	[0.049]	[0.049]	[0.049]	[0.054]
		0.95		-0.008	0.000	0.002	-0.001	0.003	-0.074	-0.074	-0.074	-0.074	-0.036
				(0.745)	(0.136)	(0.090)	(0.049)	(0.084)	(0.036)	(0.036)	(0.036)	(0.036)	(0.053)
				[0.745]	[0.136]	[0.090]	[0.049]	[0.084]	[0.084]	[0.083]	[0.083]	[0.083]	[0.065]
50	26	0.05		0.001	0.001	0.001	0.001	0.000	0.000	-0.001	-0.001	-0.001	0.000
				(0.034)	(0.039)	(0.041)	(0.029)	(0.042)	(0.029)	(0.029)	(0.029)	(0.029)	(0.033)
				[0.034]	[0.039]	[0.041]	[0.029]	[0.043]	[0.029]	[0.029]	[0.029]	[0.029]	[0.033]
		0.50		0.000	-0.000	-0.000	-0.000	-0.000	-0.004	-0.005	-0.005	-0.005	-0.002
				(0.041)	(0.048)	(0.049)	(0.027)	(0.050)	(0.027)	(0.027)	(0.027)	(0.027)	(0.034)
				[0.041]	[0.048]	[0.049]	[0.027]	[0.050]	[0.027]	[0.027]	[0.027]	[0.027]	[0.034]
		0.95		-0.008	-0.002	-0.001	-0.001	0.000	-0.031	-0.031	-0.031	-0.031	-0.016
				(0.531)	(1.440)	(0.061)	(0.025)	(0.056)	(0.019)	(0.019)	(0.019)	(0.019)	(0.032)
				[0.532]	[1.440]	[0.061]	[0.025]	[0.056]	[0.037]	[0.037]	[0.037]	[0.037]	[0.036]

Notes: (i) Results are based on 2000 draws and $\sigma_{\upsilon}^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 4: <u>Monte Carlo simulation results for cross-sectional heteroscedastic panels</u>

N	T	α	GMM-DIF	CU-DIF	GMM-SYS	CU-SYS	FBCOLS1	FBCWG1	FBCFD1	FBCOLS2	FBCWG2	FBCFD2	FBCOLS3	FBCWG3	FBCFD3
100	8	0.05	-0.028	-0.001	-0.005	-0.001	-0.003	0.001	-0.001	0.004	0.002	0.004			
			(0.085)	(0.129)	(0.079)	(0.114)	(0.077)	(0.074)	(0.087)	(0.089)	(0.077)	(0.103)	---	---	---
			[0.089]	[0.129]	[0.079]	[0.114]	[0.077]	[0.074]	[0.087]	[0.089]	[0.077]	[0.103]			
		0.50	-0.060	0.005	-0.014	0.005	-0.012	-0.005	-0.006	0.004	-0.002	0.004			
			(0.102)	(0.172)	(0.088)	(0.128)	(0.090)	(0.080)	(0.098)	(0.154)	(0.085)	(0.119)	---	---	---
			[0.120]	[0.172]	[0.090]	[0.128]	[0.091]	[0.080]	[0.099]	[0.155]	[0.085]	[0.119]			
		0.95	-0.325	-0.026	-0.041	0.015	-0.047	-0.022	-0.021	-0.012	-0.002	0.002			
			(0.237)	(14.800)	(0.086)	(0.123)	(1.152)	(0.080)	(0.098)	(1.316)	(0.098)	(0.135)	---	---	---
			[0.423]	[14.800]	[0.101]	[0.123]	[1.163]	[0.083]	[0.101]	[1.317]	[0.098]	[0.135]			
50	13	0.05	-0.012	0.021	0.019	0.003	0.014	0.002	0.012	0.006	0.002	0.008	-0.005	-0.008	0.001
			(0.099)	(0.215)	(0.096)	(0.195)	(0.084)	(0.075)	(0.100)	(0.089)	(0.076)	(0.108)	(0.075)	(0.074)	(0.085)
			[0.099]	[0.216]	[0.098]	[0.196]	[0.085]	[0.075]	[0.101]	[0.089]	[0.076]	[0.108]	[0.075]	[0.075]	[0.085]
		0.50	-0.032	0.005	0.011	-0.010	0.008	-0.002	0.009	0.006	-0.002	0.006	-0.018	-0.023	-0.007
			(0.120)	(0.174)	(0.108)	(0.206)	(0.100)	(0.075)	(0.114)	(0.121)	(0.077)	(0.126)	(0.075)	(0.072)	(0.089)
			[0.123]	[0.174]	[0.108]	[0.206]	[0.100]	[0.076]	[0.114]	[0.121]	[0.077]	[0.126]	[0.077]	[0.076]	[0.090]
		0.95	-0.139	-0.052	-0.008	-0.004	-0.017	-0.009	-0.003	-0.011	-0.006	0.001	-0.081	-0.080	-0.039
			(0.247)	(2.777)	(0.076)	(0.203)	(0.436)	(0.066)	(0.099)	(0.439)	(0.084)	(0.142)	(0.105)	(0.061)	(0.090)
			[0.304]	[2.777]	[0.077]	[0.203]	[0.437]	[0.067]	[0.100]	[0.439]	[0.084]	[0.142]	[0.134]	[0.100]	[0.098]

Notes: (i) Results are based on 2000 draws, $\sigma_\upsilon^2 = 1$ and $\mu^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.

Table 5: Monte Carlo simulation results for panels with heteroscedastic errors over time

N	T	α	GMM-DIF	CU-DIF	GMM-SYS	CU-SYS	FBCOLS1	FBCWG1	FBCFD1	FBCOLS2	FBCWG2	FBCFD2	FBCOLS3	FBCWG3	FBCFD3
100	8	0.05	-0.019	-0.000	-0.001	0.001	0.035	-0.013	-0.041	-0.014	-0.025	-0.082			
			(0.071)	(0.084)	(0.057)	(0.066)	(0.051)	(0.050)	(0.058)	(0.054)	(0.050)	(0.067)	---	---	---
			[0.074]	[0.084]	[0.057]	[0.066]	[0.063]	[0.051]	[0.071]	[0.055]	[0.056]	[0.105]			
		0.50	-0.044	0.003	-0.003	0.002	0.022	-0.010	-0.039	-0.038	-0.026	-0.074			
			(0.096)	(0.118)	(0.061)	(0.073)	(0.060)	(0.054)	(0.064)	(0.070)	(0.055)	(0.077)	---	---	---
			[0.107]	[0.118]	[0.061]	[0.073]	[0.063]	[0.055]	[0.076]	[0.080]	[0.061]	[0.108]			
		0.95	-0.388	-0.041	-0.009	0.008	-0.005	0.027	-0.018	-0.025	0.024	-0.026			
			(0.305)	(7.301)	(0.055)	(0.068)	(0.639)	(0.055)	(0.067)	(1.934)	(0.065)	(0.088)	---	---	---
			[0.526]	[7.302]	[0.057]	[0.068]	[0.639]	[0.061]	[0.069]	[1.934]	[0.069]	[0.091]			
50	13	0.05	-0.008	0.018	0.014	-0.004	0.045	-0.006	-0.032	-0.008	-0.014	-0.079	0.026	-0.015	-0.047
			(0.080)	(0.090)	(0.072)	(0.096)	(0.060)	(0.054)	(0.070)	(0.058)	(0.055)	(0.075)	(0.053)	(0.054)	(0.060)
			[0.081]	[0.091]	[0.073]	[0.096]	[0.075]	[0.055]	[0.077]	[0.058]	[0.057]	[0.108]	[0.059]	[0.056]	[0.076]
		0.50	-0.026	0.009	0.004	-0.014	0.027	-0.006	-0.035	-0.035	-0.015	-0.076	0.004	-0.027	-0.052
			(0.107)	(0.118)	(0.076)	(0.107)	(0.068)	(0.054)	(0.078)	(0.075)	(0.055)	(0.087)	(0.052)	(0.051)	(0.064)
			[0.110]	[0.118]	[0.076]	[0.108]	[0.073]	[0.054]	[0.087]	[0.082]	[0.057]	[0.115]	[0.052]	[0.058]	[0.082]
		0.95	-0.317	-0.044	-0.001	0.001	0.005	0.036	-0.018	-0.027	0.029	-0.034	-0.019	-0.032	-0.033
			(0.406)	(2.838)	(0.057)	(0.104)	(0.259)	(0.047)	(0.073)	(0.697)	(0.056)	(0.097)	(0.055)	(0.040)	(0.062)
			[0.567]	[2.846]	[0.057]	[0.105]	[0.259]	[0.057]	[0.076]	[0.698]	[0.062]	[0.103]	[0.060]	[0.053]	[0.070]

Notes: (i) Results are based on 2000 draws, $\sigma_v^2 = 1$ and $\mu^2 = 1$. (ii) The first value for each estimator is its Monte Carlo median bias. (iii) The value in parenthesis is the Monte Carlo standard deviation of each estimate. (iv) The value in each bracket below the parenthesis is the RMSE of each estimator.