Simple model of finite resource exploitation

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Abstract

Here we present a simple model of finite resource exploitation that is parameterized by initial *cost ratio*, a technologic and management index m accounting for the rise of the exploitation cost with decrease of resource amount, plus another index ε accounting for the avidity of the market to the resource. The analysis showed that the model captures essential features of finite resource exploitation, namely those concerning to oil exploitation. Scenarios for future oil exploitation based on parameters estimated from historical data are analyzed and discussed, as well some other scenarios of generic resource exploitation.

1. Introduction

Most resources of economic value are finite, and the "window of opportunity" for its exploitation is also a finite period in History. Though they cannot fully predict the future, models of resource exploitation can help to put up prospective scenarios of resource availability. The most paradigmatic of such models is due to the pioneering work of M. K. Hubert in the 1950s (see Hubbert, 1962) who proposed that oil production would follow a "bell shaped" symmetric curve. Based on oil data exploitation Hubbert was able to successfully predict 1970 as the peak year for oil production in the USA. Hubbert's curve is now applied to prospective studies on world's oil production (e. g. Campbell and Laherrère, 1998; Rosa, 2003, 2006; Bardi, 2009), as well to exploitation of coal and minerals (Bardi, 2007, 2009). Attempts have been made to establish the theoretical grounds of Hubert's curve by using either systems dynamics (Naill, 1973), or stochastic modeling (Bardi, 2005), or else economics (Holland, 2008). In a recent paper Bardi (2009) develops an explanation of Hubert's curve based on a "predator and prey" model.

In this paper we try a different approach based on two main drivers of the rate at which resources are exploited: the level of economic demand, and the level of the technology available for resource exploitation.

2. The model

Differently from renewable resources, the global amount of finite resources is either constant or continuously it decreases due to irreversible degradation as they are used in human and economic activities Everyday examples of the first case are minerals, while oil gas and coal stand for the second. In this way, a characteristic common to every finite resource is that there is a window of opportunity in History to explore them. Some of them, such as minerals, remain relatively stable as they are used, while others suffer irreversible transformations (e.g. coal, gas, oil) and once they become depleted will never be used in the global economy.

No resource is exploited without a cost associated to its exploitation. The simplest measure of this cost is the faction of a resource unit whose market value equals the cost of exploitation of a unit of resource. We will call this non-dimensional measure, the *cost ratio*. If we speak of oil, the *cost ratio* is the ratio of the exergy spent in the extraction of some oil amount to the useful exergy that it is able to deliver in the average conditions of its use in society. With respect to measuring exploitation cost of oil, gas and coal a measure commonly used is EROI which means Energy Return On Investment (Hall, 1981). Thought EROI has various definitions (Bardi, 2009) the most suitable to the present case is *societal* EROI, which is defined as the ratio of the energy content of some amount of fuel to the energy lost in its exploitation. EROI does not take into account the quality of energy, i.e. does not differentiate between heat and power, fact that assigns it some ambiguity as a measure of exploitation cost. Such ambiguity would vanish if EROI was defined as the ratio of the *exergy* content of some amount of fuel to the *exergy* lost in its exploitation. In this case, EROI would come close to the inverse of *cost ratio*, because it may be shown that exergy can be taken as an appropriate measure of market value (Reis, 2006).

2.1 Cost ratio

Let us analyze first the case of fossil fuels and consider that at a time t some fossil fuel (finite), whose *known* global (planetary) magnitude amounts to X_0, is exploited at the rate $\dot{X}$ with a *cost ratio* ϕ. Therefore, the "net fuel" (i.e. the part that is available for the other economic sectors) is put into the market at a rate $\dot{X}_u$ that is given by

$$\dot{X}_u = \dot{X}(1-\phi) \tag{1}$$

We assume that the *cost ratio* increases in time as the reservoirs become depleted of the resource, or new reservoirs are discovered at a greater depth. In this way, we also assume that such increase may be modelled as a power law of the degree of global shortage of that fossil fuel in the natural reservoirs, i.e.

$$\phi = A(1 - X/X_0)^{-m} \tag{2}$$

where X is total amount exploited up to time *t, A* stands for the cost ratio at the beginning of the exploitation (X ~0), and *m* is an exponent accounting for the degree of efficiency of the exploitation process. When *m*>1, relatively high *m* means low efficiency, while low *m* means high efficiency (high developed exploitation technologies together with good management of exploitation). If *m*<1the *cost ratio* decreases in time, which means that the degree of exploitation efficiency is high enough to overshadow the negative effect of reservoir shortage.

By integrating equation (1) with the help of equation (2) one obtains:

$$x_u = x + \frac{A}{1-m}(1-x)^{1-m} \tag{3}$$

where x_u and x stand for X_u/X_0 and X/X_0, respectively.

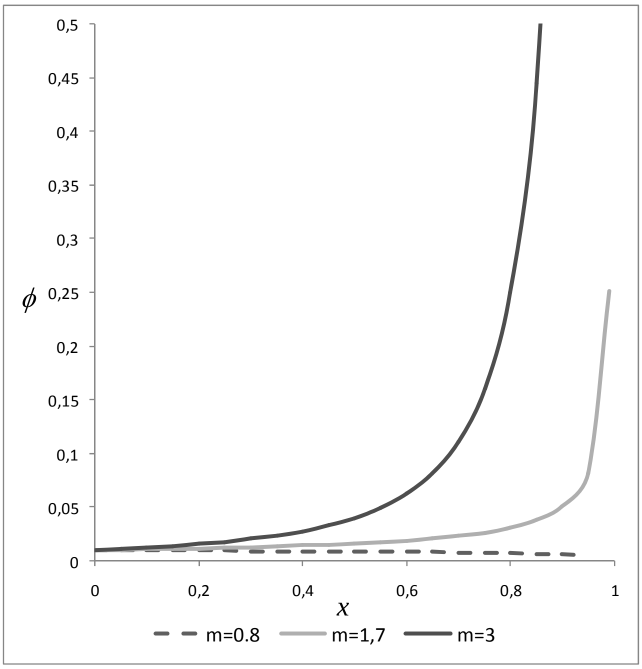

Fig. 1. Cost ratio as function of fraction of resource extracted in various technologic scenarios

Equation (3) describes the total (historic) amount of “net fuel” put into the market in relation to total fuel extracted from the natural reservoirs.

2.2 Market demand

The exploitation of finite natural resources is driven by the market demand. Demand in the case of fossil fuels is driven by all sectors in society that increasingly use energy, the most of it with origin in fossil fuel consumption. In a mental picture we can imagine the growth of use of fossil fuels in economy and in society as a diffusive process in which energy of fossil origin is used in an increasingly higher number of activities. Every simple diffusive process scales with $t^{1/2}$ (where t stands for time), and therefore the number N of activities that use fossil fuel energy scales accordingly with

$$N \sim Dt^{1/2} \tag{4}$$

where D is a constant that accounts for the *diffusibility* of fossil fuel energy in society. Additionally, we assume that the number n of units in each activity a (industrial unit, house, vehicle, etc.) grows in time according to a power law

$$\sum_{i,a} n_i \sim b_i t^{\varepsilon} \tag{5}$$

where b_i is a constant, and the exponent ε accounts for the growth of sector i powered by the economic development, and namely by the availability of energy of fossil fuel origin. Therefore, the total fossil fuel energy consumption rate in each activity (sector) is given by:

$$\dot{X}_a = \sum_i n_i \dot{X}_{i,a} \sim \sum_i b_i \dot{X}_{i,a} t^{\varepsilon} = B_a t^{\varepsilon} \tag{6}$$

where $\dot{X}_{i,a}$ is the consumption rate of unit i in sector a. In this way, by using equation (6) and summing for all activities (sectors) we find an estimate of the rate of global energy demand as:

$$\dot{X}_u \sim Bt^{1/2+\varepsilon} \tag{7}$$

In equation (7) $\varepsilon = 0$ corresponds to pure physical driven diffusion of resource X, and means indifference of the market with respect to resource X, while $\varepsilon > 0$ represents avidity for that resource, and $\varepsilon < 0$ stands for the cases when X is combated by the society (e.g., pollution, hallucinogenic drugs).

2.3 Exploitation curve

By combining equations (1) and (7) and integrating the resulting equation one obtains:

$$x+\frac{A}{1-m}(1-x)^{1-m}-\hat{t}^{3/2+\varepsilon}=0 \qquad (8)$$

In equation (8) the constant B has been eliminated through an appropriate choice of the time scale. Therefore here $\hat{t}$ represents a time scale in which $\hat{t}=1$ corresponds to the period at the end of which resource *X* is completely depleted $(x=1)$.

The exploitation curve represented by equation (8) is parameterized by:

(i) *A*, the *cost ratio* at the beginning of the exploitation (X ~0);

(ii) *m*, the exponent accounting for the degree of efficiency of the exploitation process (technology and management);

(iii) ε , the exponent that accounts for the market avidity for energy of fossil fuel origin.

These parameters may be estimated through the data available from resource exploitation, therefore enabling us to construct prospective scenarios of resource availability.

3. Analysis of finite resource exploitation curve

3.1 The case of oil production

For the case of world oil exploitation, the relation $\dot{x}\sim t^{\alpha}$ with the exponent $\alpha=3.1$ fits pretty well the curve of annual production in the period 1930-1980 (see Fig.2). On the other hand the average value of *A* would be close to 0.01 (Gagnon et al, 2009). These basic data enable us to draw some future scenarios for oil production. These scenarios will not include production from shale oil reservoirs because the parameters were estimated from data of oil production from normal crude oil reservoirs. With this purpose, we assume that α must be close to $1/2+\varepsilon$ (see equation 7), because for low values of x both the curves $\dot{x}(t)$ and $\dot{x}_u(t)$ practically coincide (see Figs. 3-5). Therefore, by taking $\varepsilon=2{,}6$ and $A=0.01$ we use equation (8) to find out the production curve that in its first part is described by the same exponent $\alpha=3.1$. Such curve, which is represented in Fig. 3, is parameterized by $m=1.7$. In this way, the curve in Fig.3 is likely to stand for a liable scenario of future oil production. On the other hand, as discussed above, the exponent $m=1.7$ indicates a moderate level of exploitation technology and exploitation management.

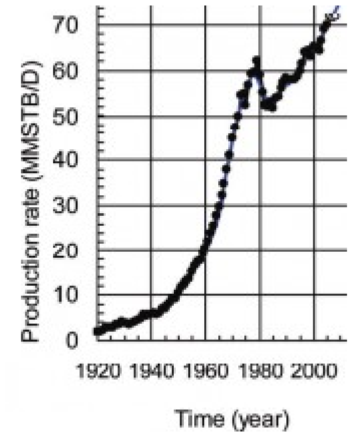

Fig. 2. Evolution of crude oil production rate

Several interesting comments can be drawn from Fig. 3. One first comment respects to *cost ratio*. Though it is not usual to find estimates of this variable in the literature, it can be estimated indirectly because its value is very close to the inverse of the EROI. In a recent paper Gagnon et al. (2009) have published recent estimates of EROI for crude oil in the period 1950-2005, which indicate that EROI in the period 1992-1999 was relatively stable close to 38, and has decreased to about 20 from 1999 to 2005. A very recent estimate indicates that EROI might be close to 11 in 2009 (Hall et al., 2009). By coming back to Fig. 3, this kind of evolution fits the steepest part of the *cost ratio* curve, while the *cost ratio*=1/EROI=0.05 roughly corresponds to the peak of net production. Beyond this point the rate at which oil is put into the market decreases sharply. If the curve somehow represents a realistic scenario for world oil production, we must conclude that the peak of global *net oil production* is occurring at the present time. The claim that global oil production is reaching its peak is assumed by many people and international groups, namely the Association for the Study of Peak Oil (ASPO).

In the scenario depicted on Fig. 3 the peak would occur late in the period corresponding to the "window of opportunity" for oil exploitation, more precisely at time $\hat{t} \sim 0.91$. Based on Equation (8) we conclude that at $\hat{t} \sim 0.91$ about 79% of the initial oil amount should have been extracted from reservoirs. The remaining 21% would not be extracted due either to technologic reasons or to lack of economic interest.

Moreover, Fig. 3 indicates that the period mediating between the peak of production and the end of economic interest of oil exploitation is of order $\hat{t} \sim 0.09$.

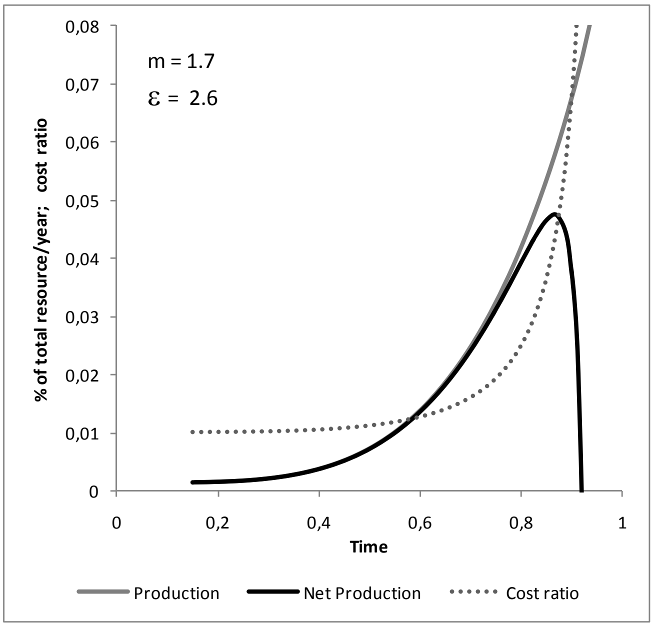

Fig. 3. Scenario for evolution crude oil production rate together with associated *cost ratio* with parameters estimated from actual data.

If we assign 200 years to the duration of the "window of opportunity" for oil extraction, it means that between the peak and the end of exploitation will mediate a period of 18 years. This result must be viewed with some caution because the exponent for oil demand $\alpha = 3.1$ in the period of globalization of the use of oil (1930-1980) was also used for establishing the scenario for the period after the peak production has occurred. Modeling of oil demand in this period must not be described by a single exponent only due to the fact that energy demand will move towards other energy sources, namely coal and the Renewables.

By contrast with Hubert's the present model does not predict a symmetric curve for oil production. The reason is that not only oil extraction technology is much more developed but also demand is global, and therefore huge tensions must be at stage by the end of oil production.

A scenario in which oil extraction technology is pushed to its limits by achieving reduction of *cost ratio* $(m = 0.5)$ in the context of increasing complexity of oil production is represented in Fig.4. Here we can see that more oil would be extracted, however the post-peak period would be shorter that in the more realistic case of Fig.3.

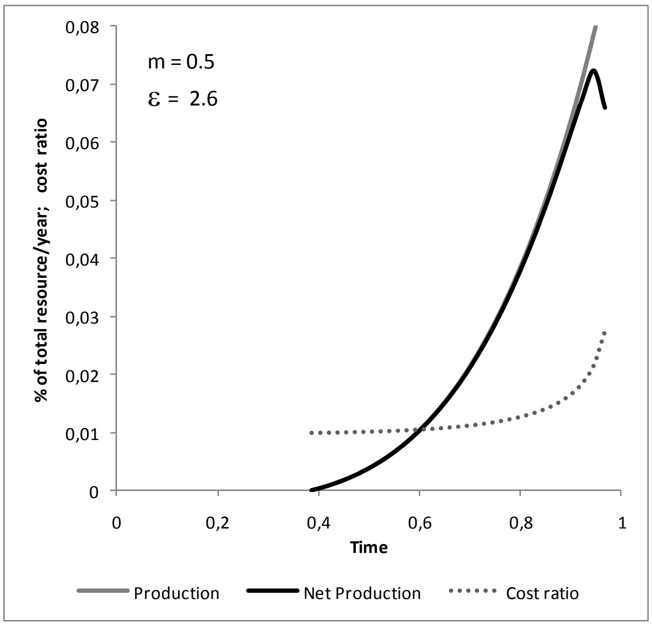

Fig. 4. Scenario for evolution crude oil production rate together with associated *cost ratio* with parameters corresponding to extremely developed exploitation technology.

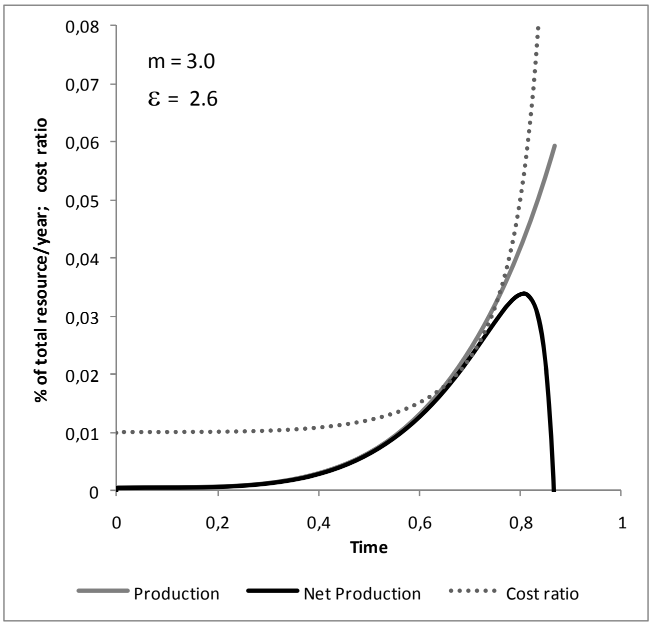

Fig. 5. Scenario for evolution crude oil production rate together with associated *cost ratio* with parameters corresponding to underdeveloped exploitation technology.

The scenario in which technology and extraction management would perform worse than that of Fig. 3 is represented in Fig. 5. In this case, not only much more oil would remain unexplored but also the cost ratio would start to increase earlier.

3.2 Some limiting cases for generic finite resources

The previous analysis may be extended to other finite resources with the appropriate adaptations.

The scenario represented in Fig. 6 corresponds to both poor demand $(\varepsilon = 0.5)$ and poor exploitation technology $(m = 3.0)$. In this scenario the *cost ratio* stands high and rises significantly since the beginning of the exploitation. Only about 59% of the resource would be extracted at the end of exploitation (see equation 8).

The scenario in Fig. 7 is intended to represent average conditions: moderate demand $(\varepsilon = 1)$ together with average technology development $(m = 2.0)$. The resource will be exploited up to 72% while the *cost ratio* will rise moderately.

Finally the scenario in Fig. 8 stands for a finite resource whose trade is combated by the society (e.g. uranium), the case in which $\varepsilon < 0$.

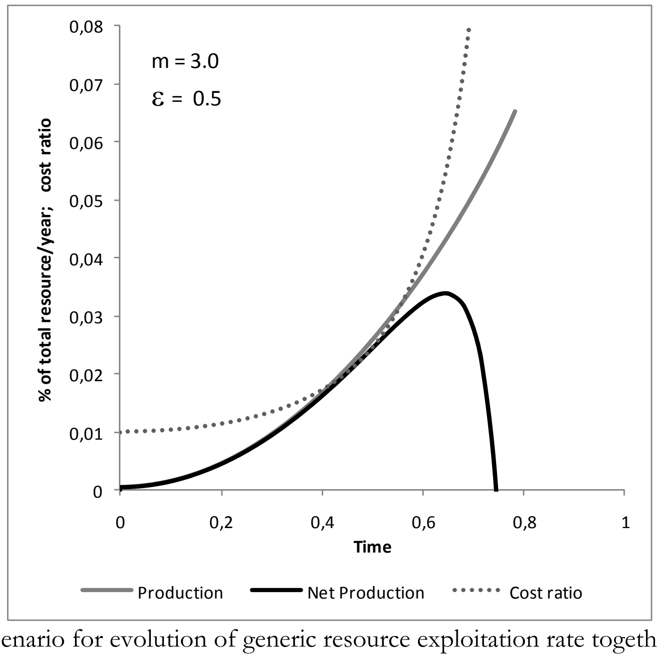

Fig. 6. Scenario for evolution of generic resource exploitation rate together with associated *cost ratio* with parameters corresponding to low market demand together with underdeveloped exploitation technology.

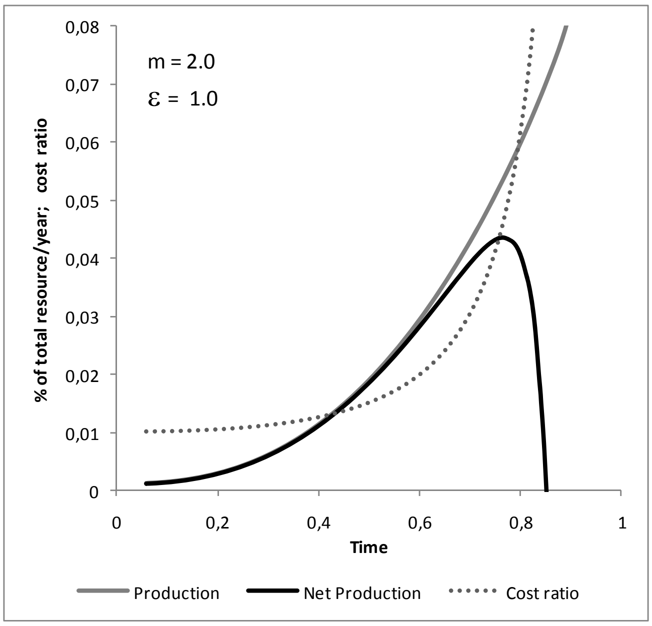

Fig. 7. Scenario for evolution of generic resource exploitation rate together with associated *cost ratio* with parameters corresponding to average market demand together with average exploitation technology.

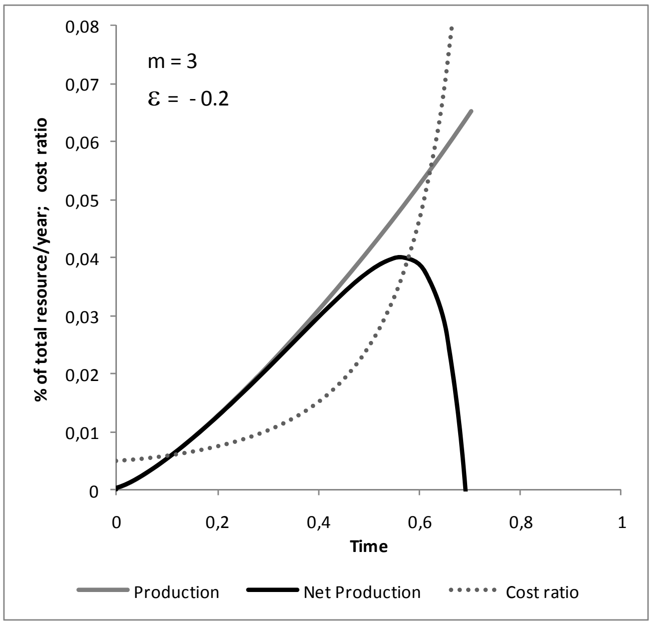

Fig. 8. Scenario for evolution of resource exploitation rate together with associated *cost ratio* and A=0.05 in negative market demand conditions (restrained exploitation).

For the case of uranium, the exponent m must be high due to the many technologic problems with its exploitation. In such a case, the resource would be explored for economic reasons only up to 66% of the global reservoir, while the cost ratio would rise significantly since the beginning of the exploration.

As a general comment one must stress that the model presented above is very simple and may be improved to better describe real cases, namely by allowing the exponents m and ε to be corrected for accommodating either the technologic breakthroughs or sudden discovery of new reservoirs, or else unexpected changes in global policies.

4. Conclusions

Despite its simplicity the model developed in this paper enables capture of basic features of resource exploitation. The inputs to the model, namely resource market demand index and initial *cost ratio* of exploitation, may be estimated from historic data, while the technologic index, which is a parameter associated to the level of exploitation technology may be also inferred from the application of the model to historic data. The case of oil production was considered in the analysis, and a possible future scenario of oil production rate was put up on the basis of the historic production rate. Some scenarios for generic resource exploitation were also considered and analyzed. The model allows future improvements namely by considering either unexpected discovery of new reservoirs or market demand transition for new energy sources.

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