# Constrained ALE-based discrete fracture in shells with quasi-brittle and ductile materials

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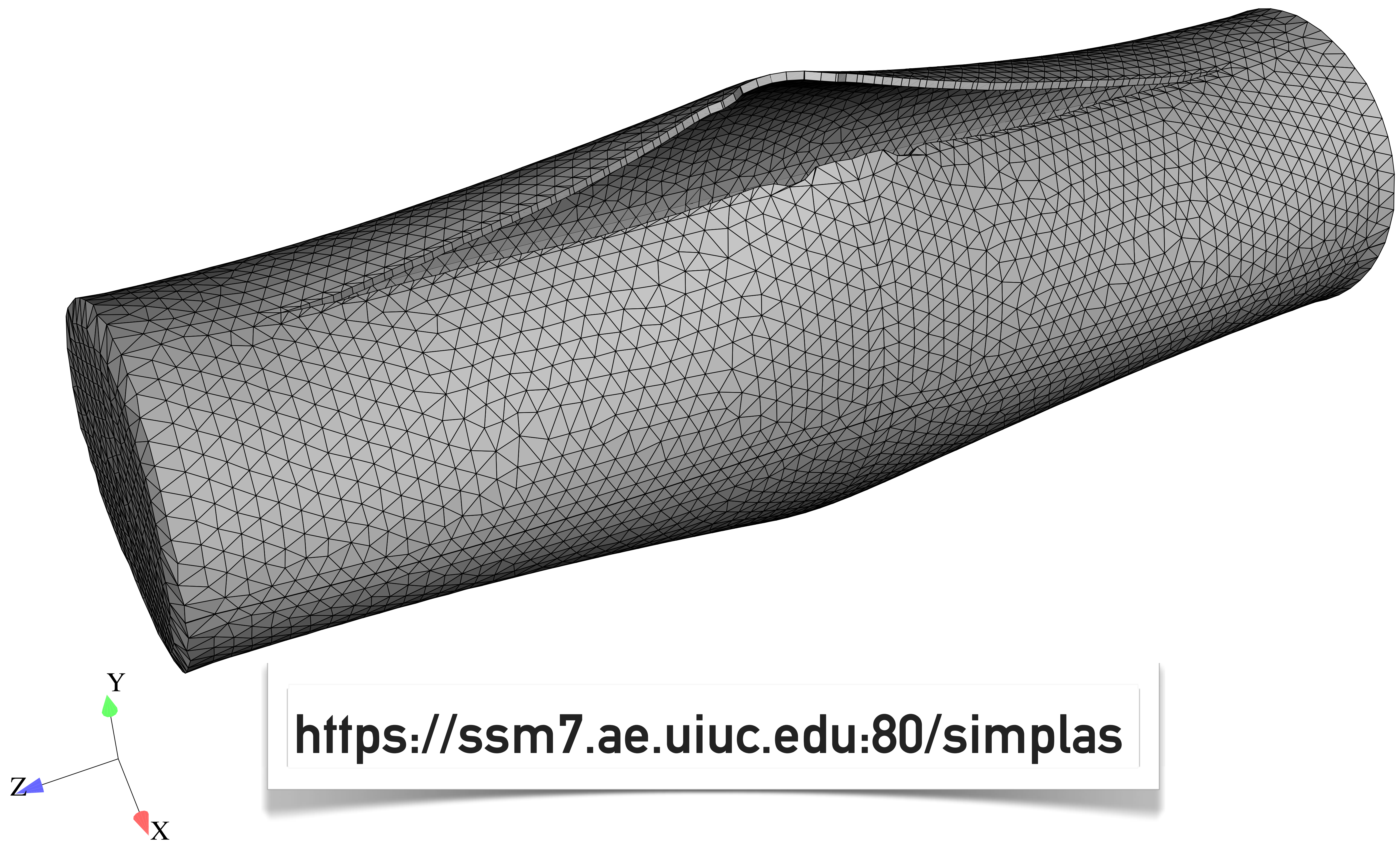



https://ssm7.ae.uiuc.edu:80/simplas

CFRAC 2011, 6-8 June, Barcelona, Spain

*in memory of JAC Martins*

# Goals, methodologies and tools

**Goals:**

- To produce a definite computational tool allowing a systematic reproduction of results for quasi-brittle and ductile fracture in finite strains+.
- Create a underlying framework where each physical law (Cauchy equilibrium, Maxwell's equations, heat transfer, etc) is *automatically* used with time-tested (and published) discretization technologies.
- Allow the testing and validation of new constitutive laws, thermal coupling, electro-magnetic coupling.
- Allow an automated incorporation of technical requirements such as:
  - Plane stress condition.
  - Non-local state variables.
- Introduce and test general heuristics and solution control.
- Incorporate new technologies in shell and beam elements prone to fracture.

**Methodologies and tools:**

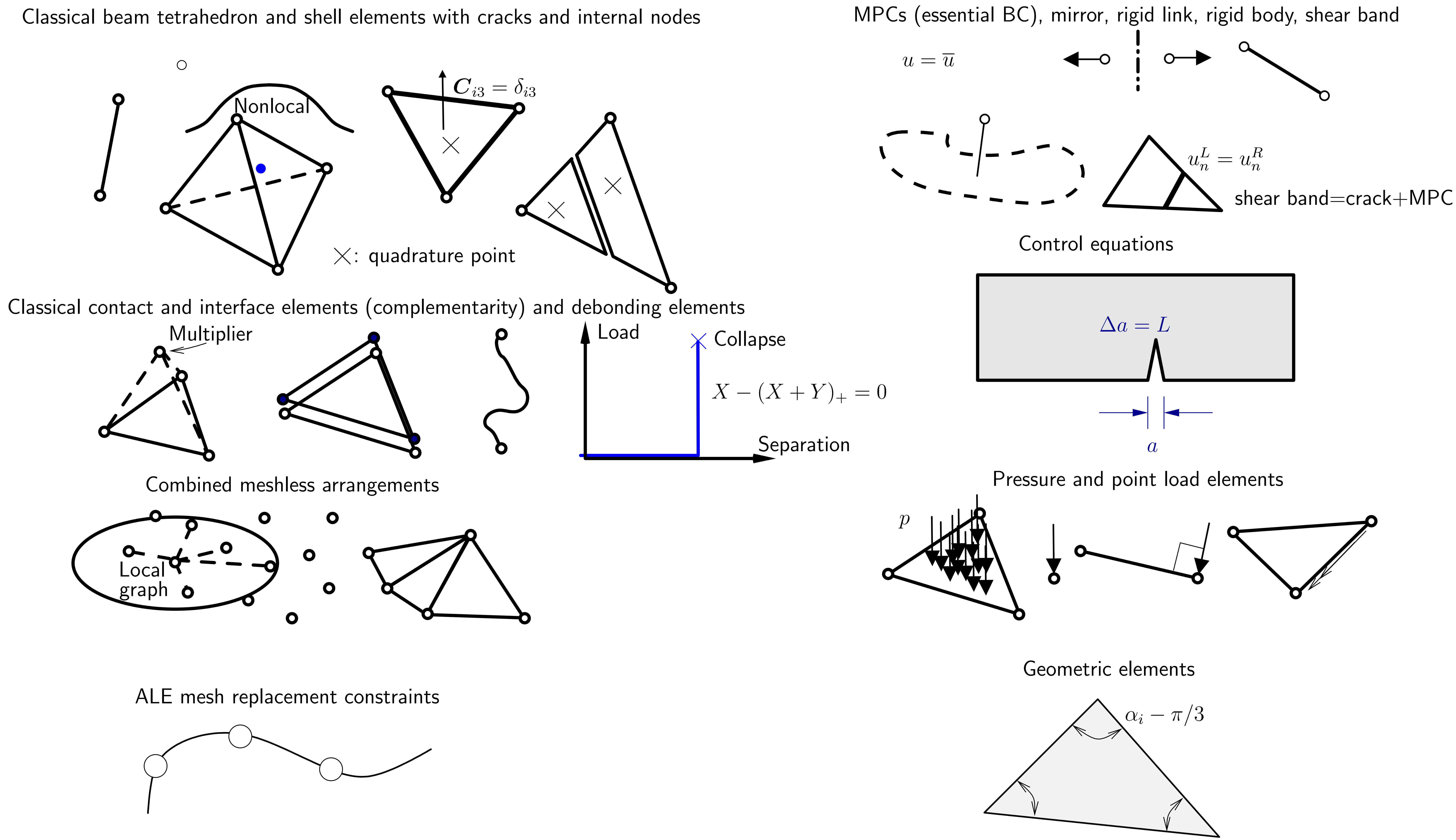
- Consistently linearize all equations and perform preliminary tests (isoerror maps, convergence radius, etc).
- Use Chen-Mangasarian replacement functions for complementarity conditions (elasto-plasticity, contact and friction, cohesive laws).
- Make extensive use of the ACEGEN add-on to Mathematica.
- Use of a in-house sparse library along with a graph database (also in-house).
- Continue to develop SIMPLAS wrapped in a C++ graph database.
- Use ALE and geometric elements.
- *Avoid enrichment or "enhancement" techniques*

+We find that ductile fracture is the most complex problem that can be dealt with Newton's method, hence the motivation

# Global perspective of our approach

*All components of a discrete "engineering" system are either additive (e.g. elements or cliques) or multiplicative (e.g. boundary conditions or multiple-point constraints).*

*Components may introduce non-smoothness to the system.*



# Fracture problems in finite strains

**Ingredients:**

- Element technology:
  - Plane stress with thickness field (<u>Comp. Mech</u>).
  - Plane strain and 3D with pressure unknowns (inf-sup verified) (<u>CMAME and IJNME</u>).
  - Fully finite strain exact shell (6 Dofs with physical drilling) (<u>Comp. Mech</u>).
- Geometrical element:
  - 2D (<u>Comp. Mech</u>).
  - Shell (to be submitted).
  - 3D (not yet implemented).
- Constitutive modeling:
  - Correct multiplicative plasticity with Chen-Mangasarian replacements (<u>IJNME</u> and to be submitted).
  - Multiple-surface approach for ductile damage (to be submitted).
- Solution control and multiple-point constraints.
  - Clique processor and sparse library

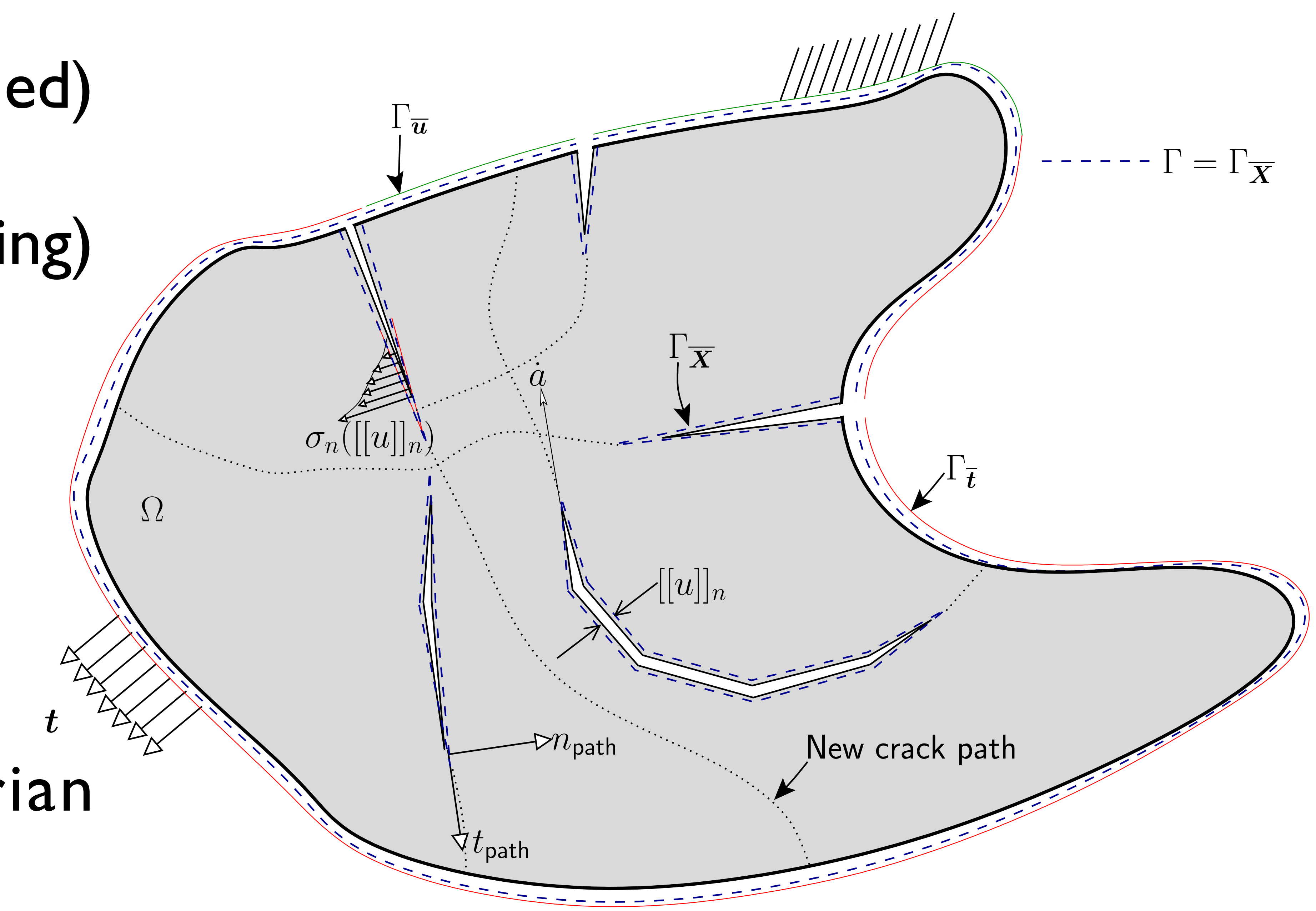
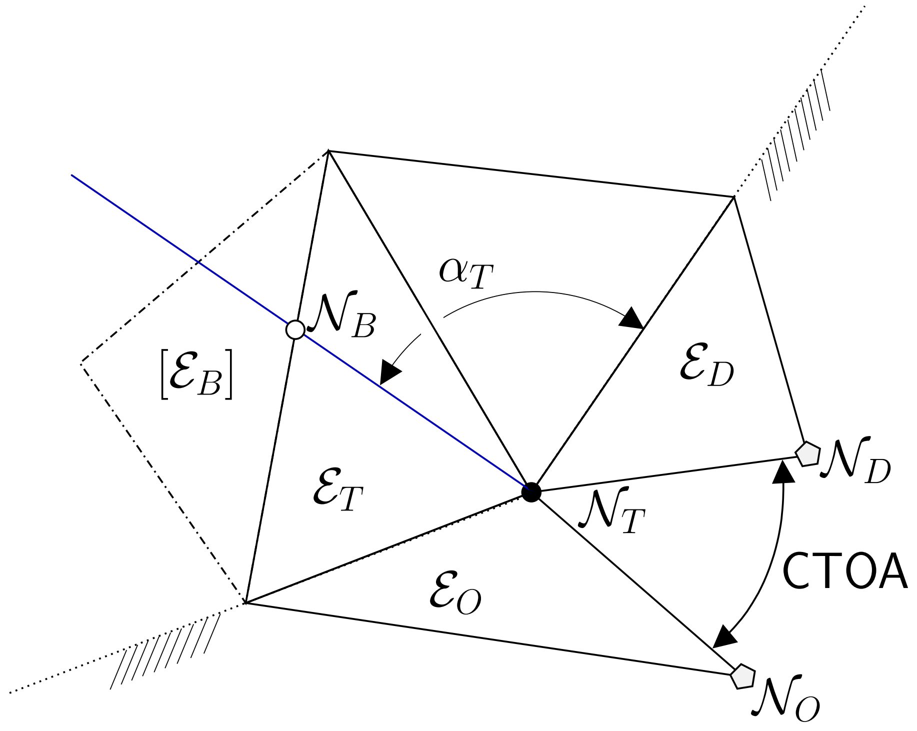
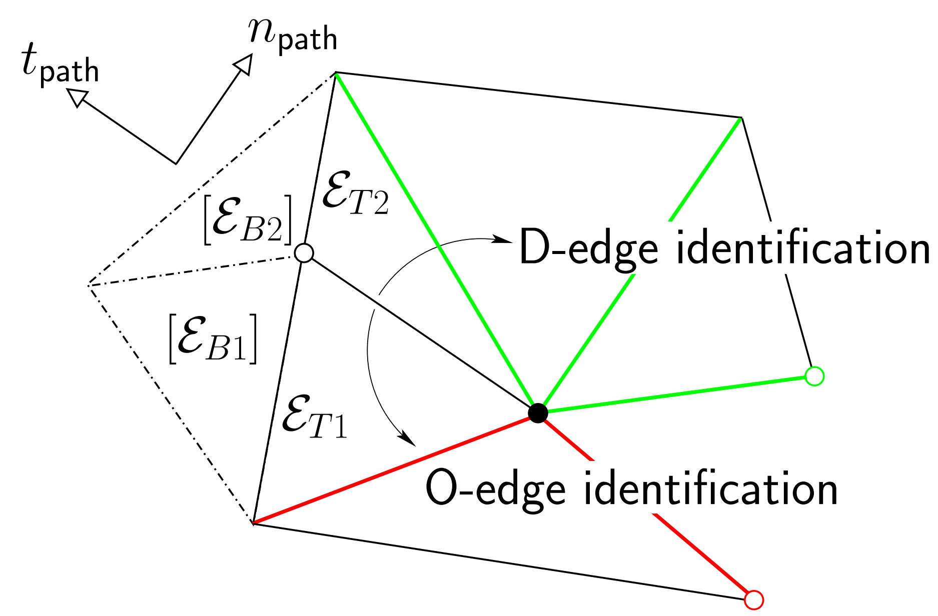



Fig. Relevant ingredients

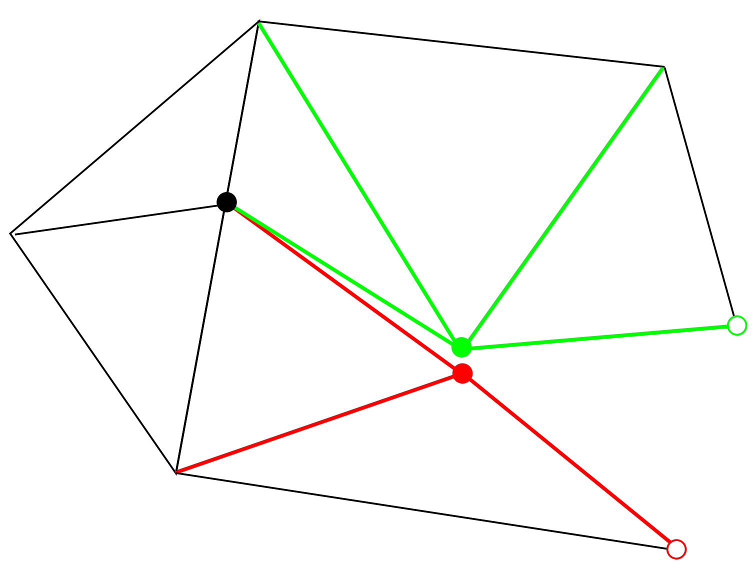
# Base technology



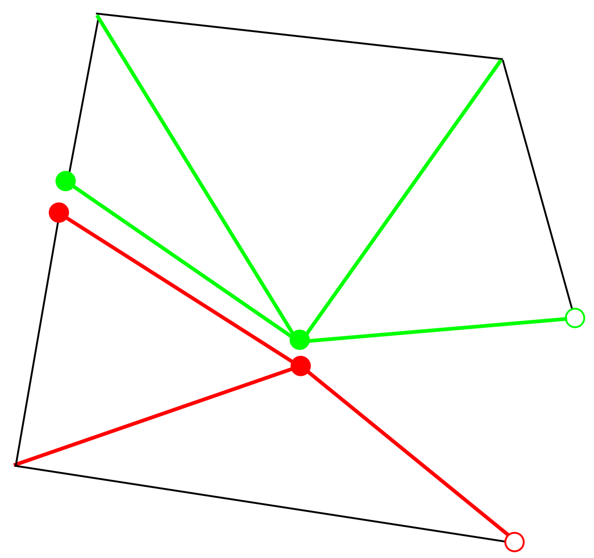

i) Tip segment appending




ii) New elements relative position



iii-a) Node splitting with subsequent element

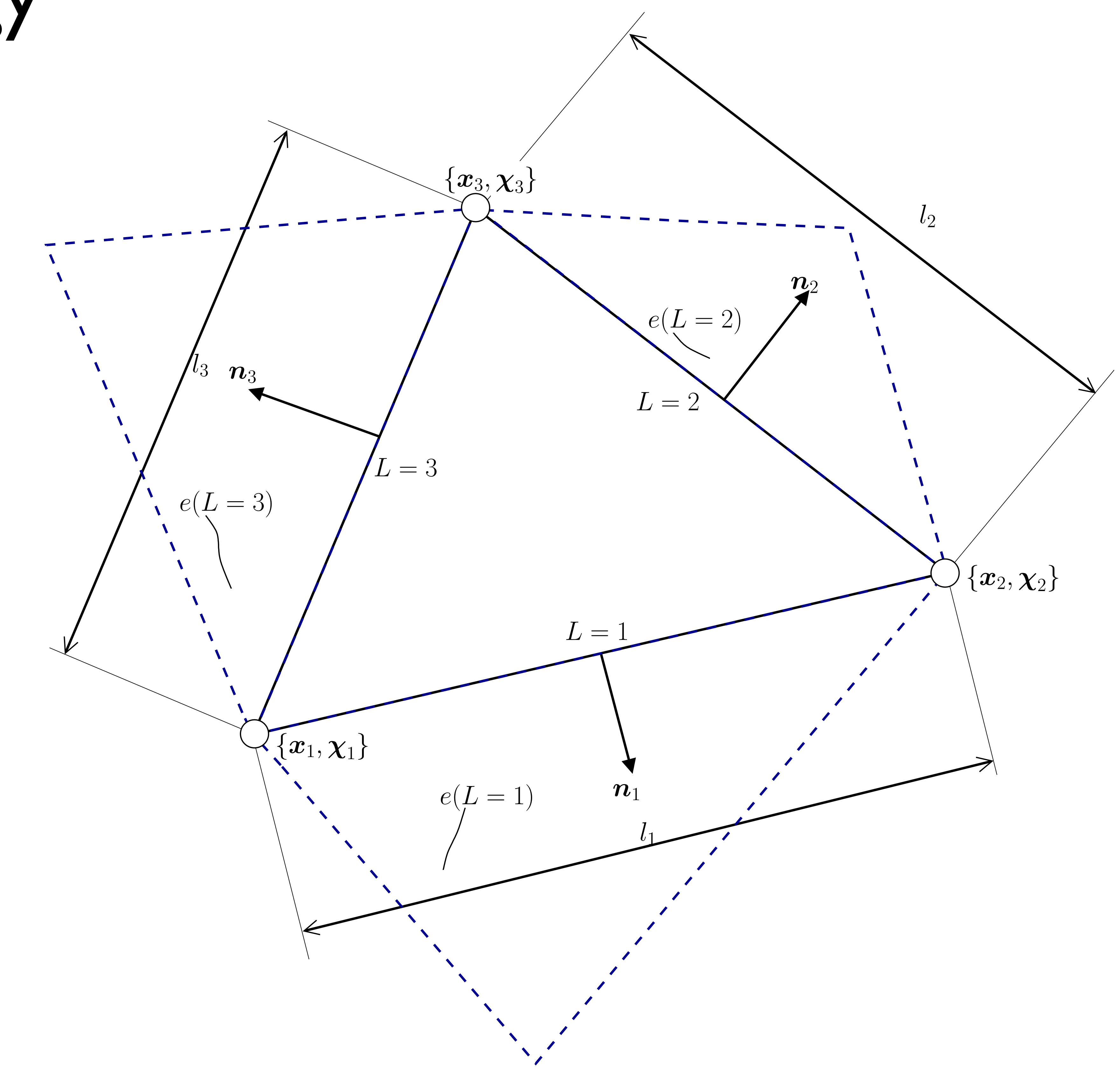


iii-b) Node splitting without subsequent element

$$\boldsymbol{f}^{\alpha} = \frac{\partial \Pi_{\text{angle}}}{\partial \boldsymbol{\chi}_v}$$

$$\boldsymbol{K}^{\alpha} = \frac{\partial^2 \Pi_{\text{angle}}}{\partial \boldsymbol{\chi}_v \partial \boldsymbol{\chi}_v}$$

Geometric element

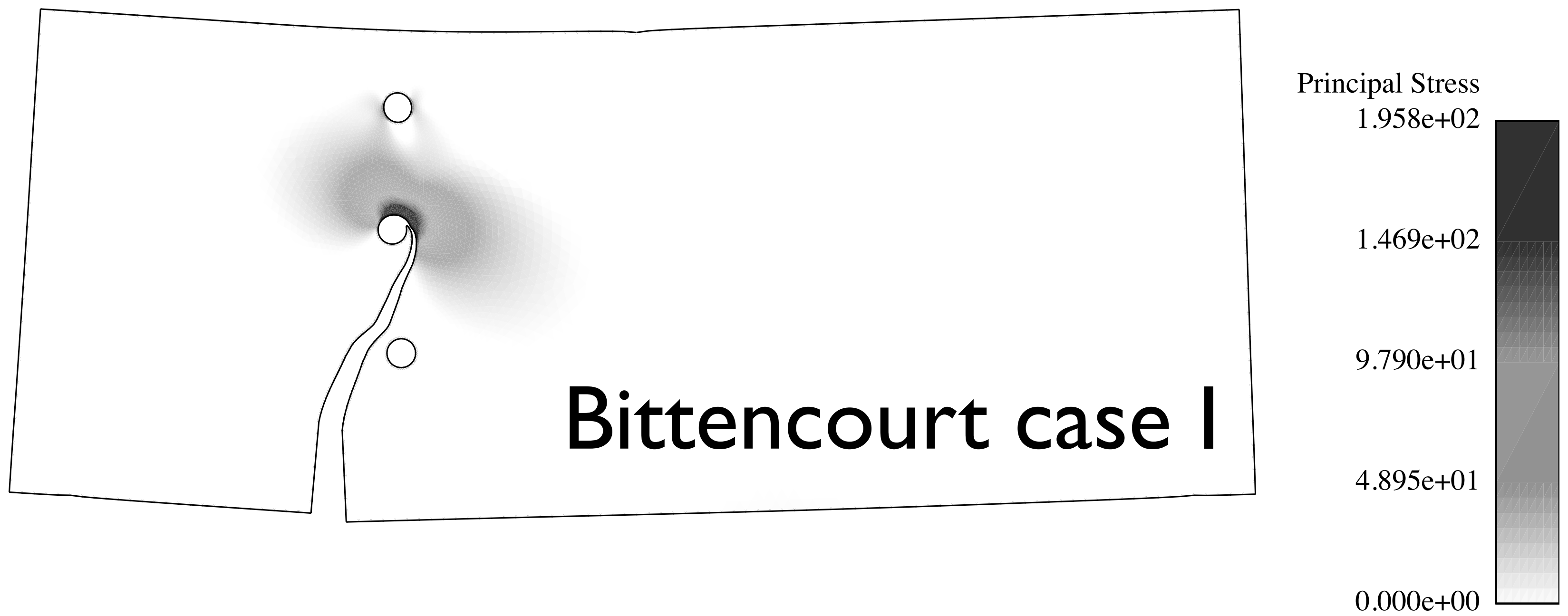


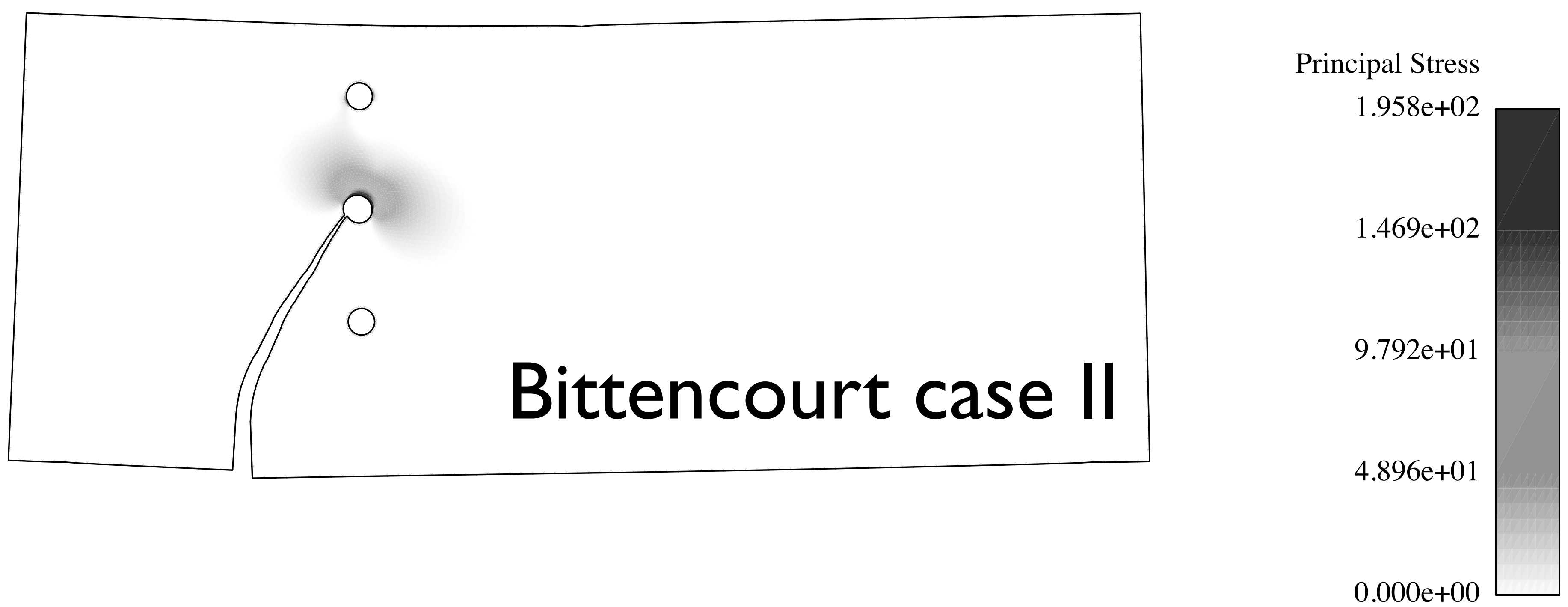

Relevant quantities in Godunov scheme

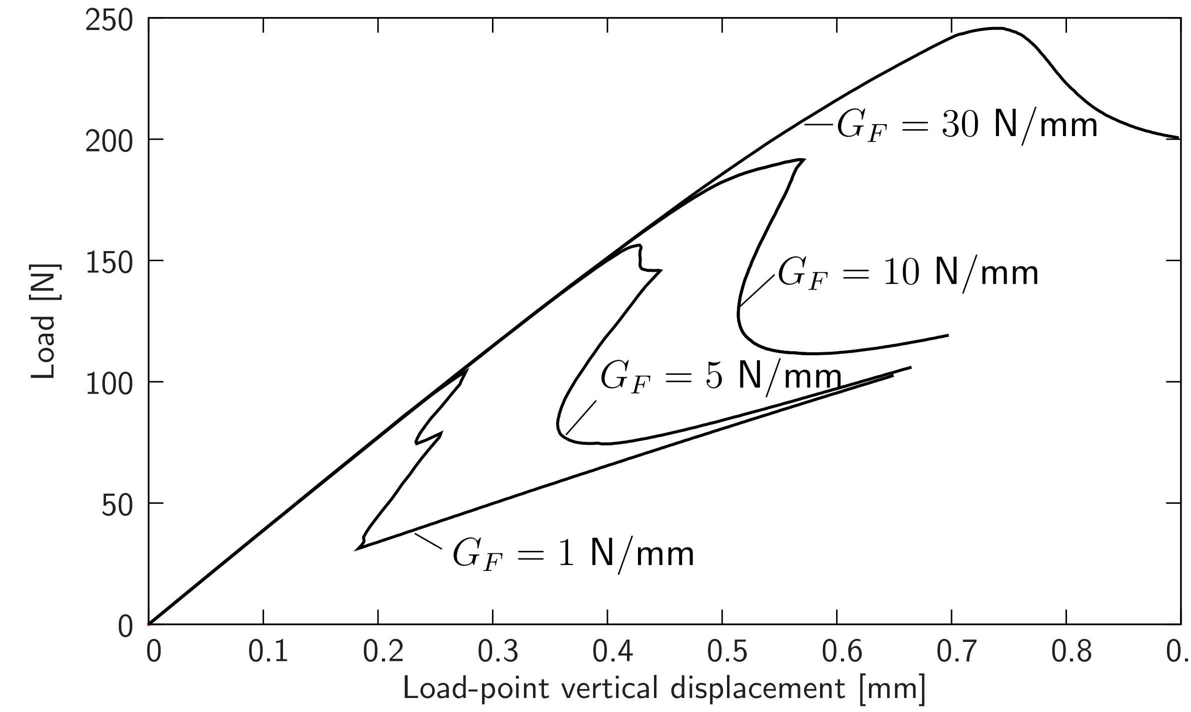
$$\boldsymbol{F}^e_o = \boldsymbol{F}^e + \frac{1}{2A_e}\sum_{L=1}^{3}\left\{\left[l_L \boldsymbol{n}_L \cdot \left(\Delta\boldsymbol{x}_{\overline{L}_3} + \Delta\boldsymbol{x}_{\overline{L+1}_3} - \Delta\boldsymbol{\chi}_{\overline{L}_3} - \Delta\boldsymbol{\chi}_{\overline{L+1}_3}\right)\right]_+ \left(\boldsymbol{F}^{e(L)} - \boldsymbol{F}^e\right)\right\}$$

Advection steps

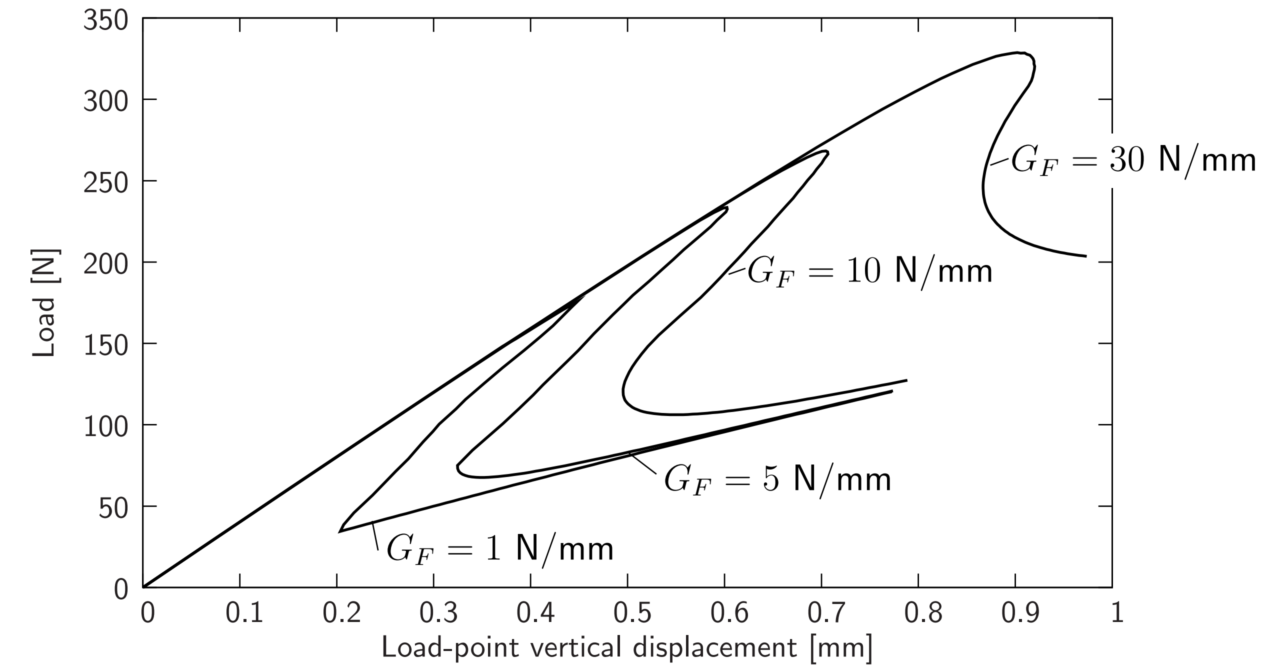
# Base technology - results (quasi-brittle)



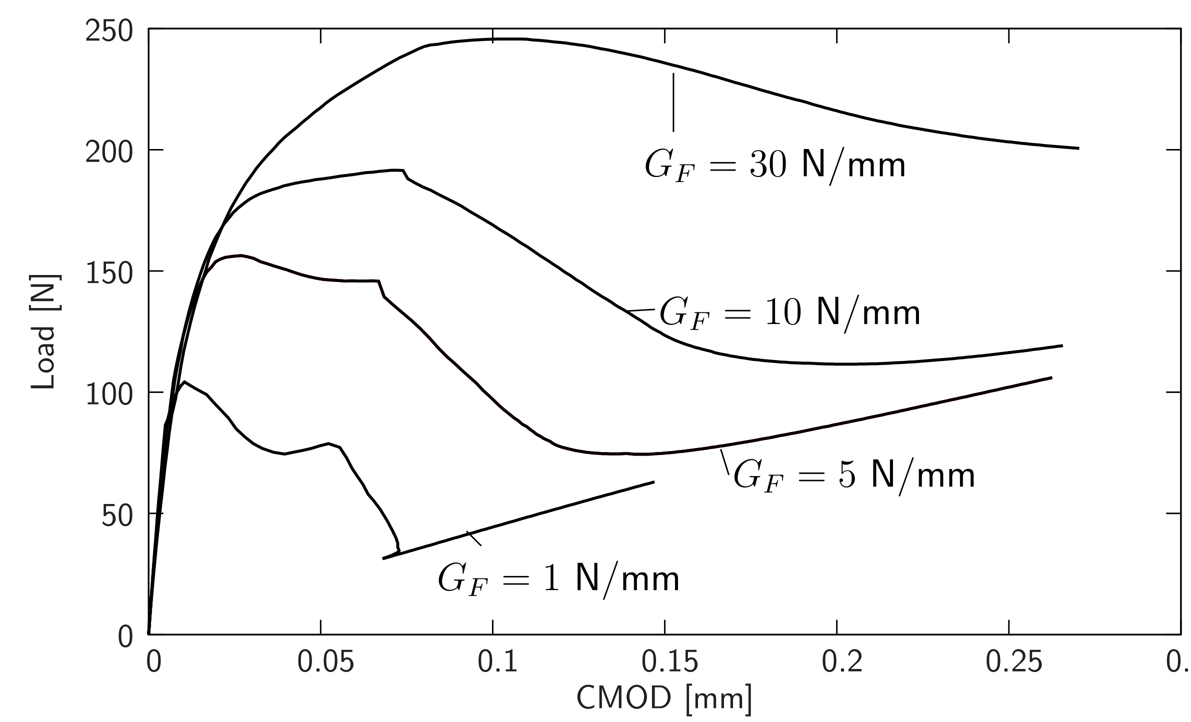



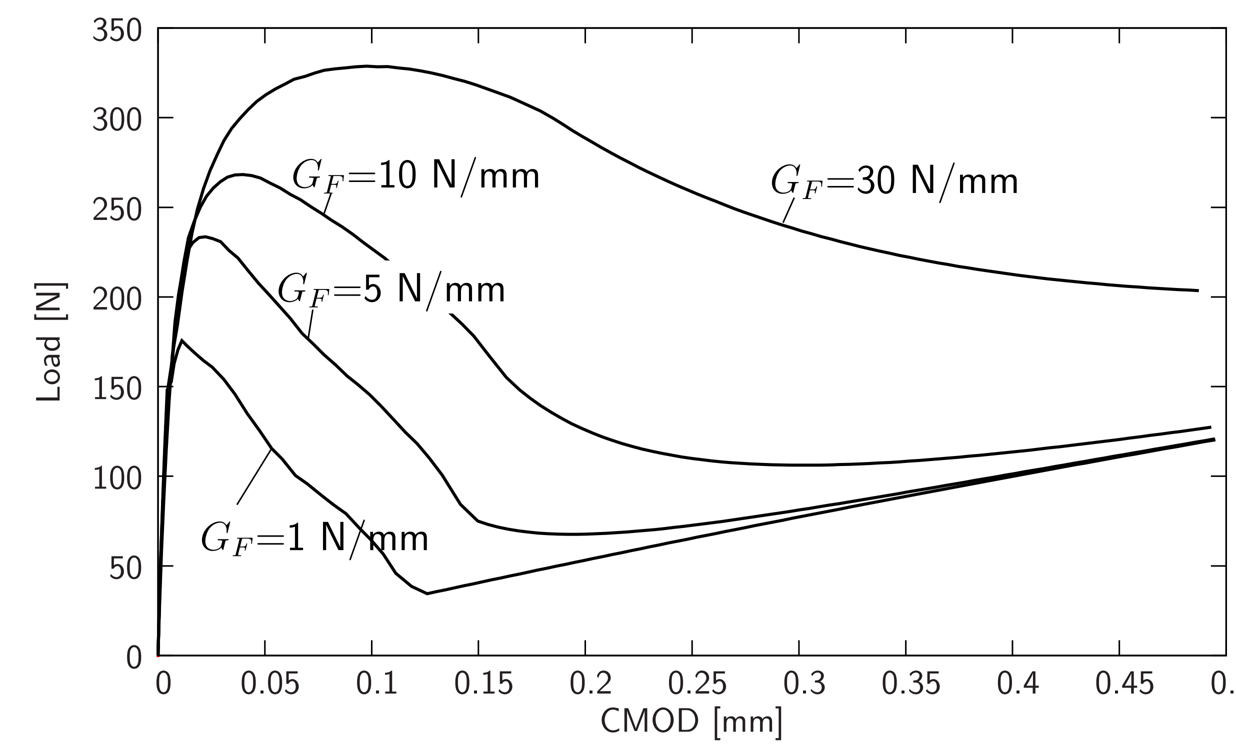

(a) Specimen #1, vertical displacement



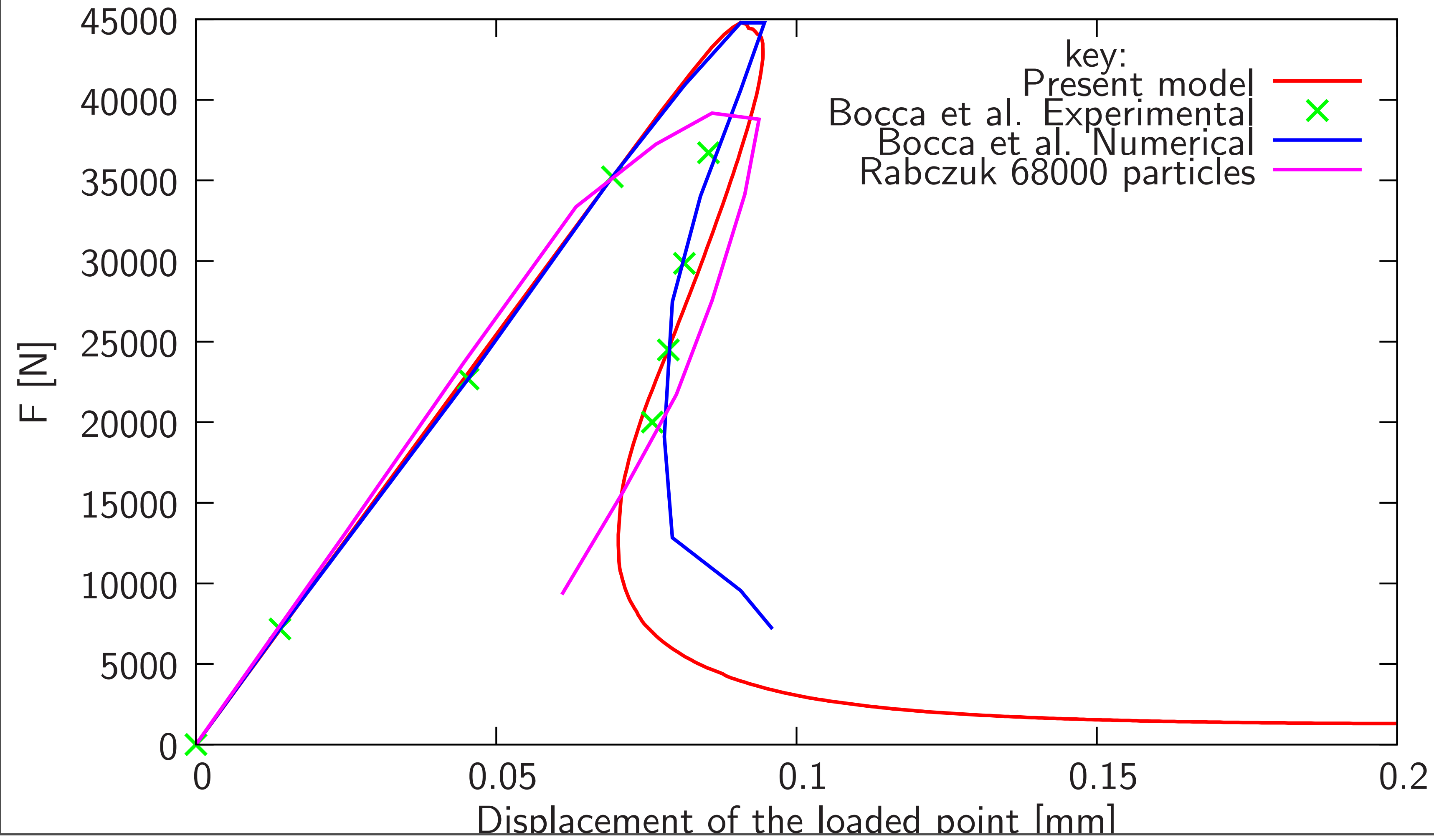

(a) Specimen #2, vertical displacement




(b) Specimen #1, CMOD

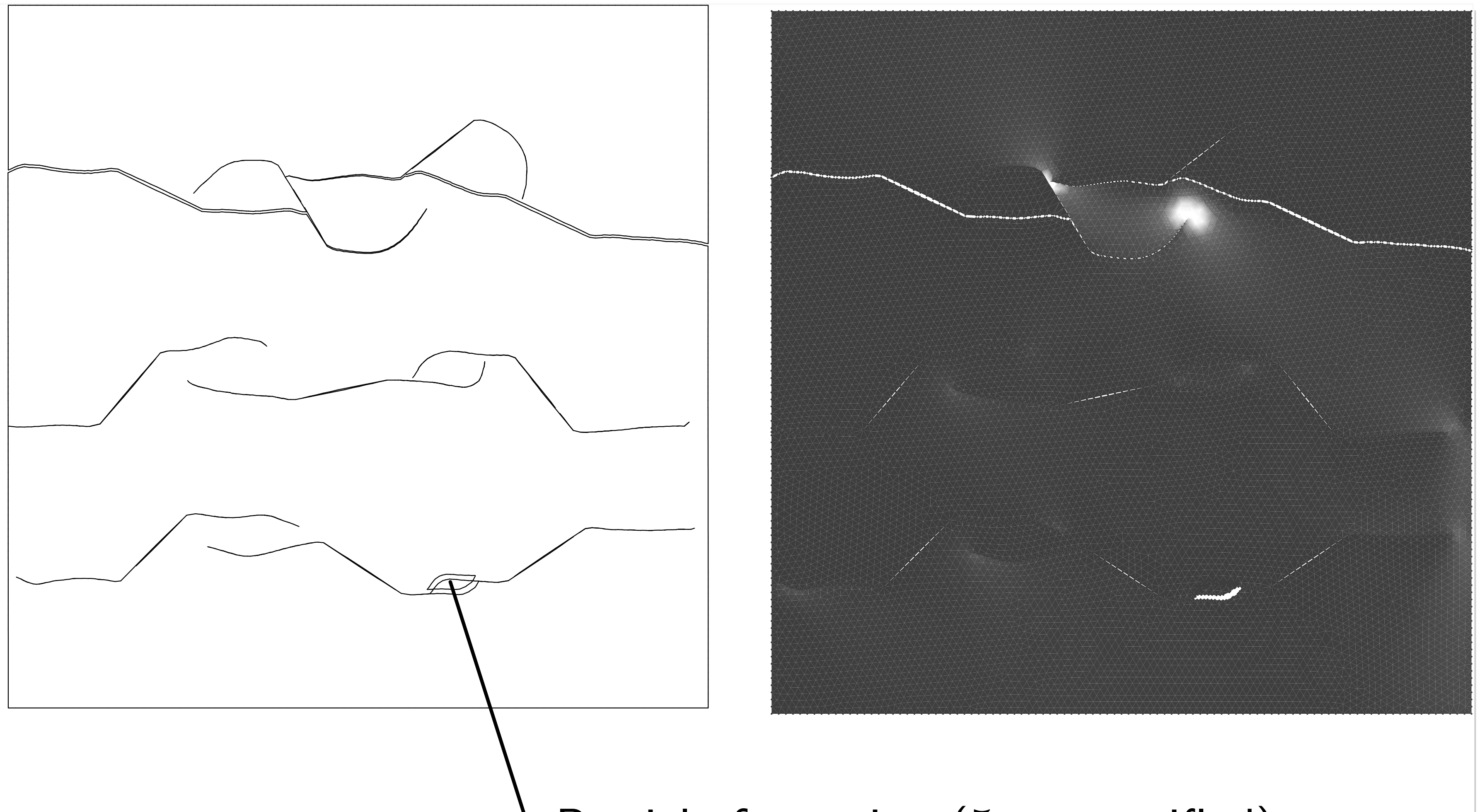



(b) Specimen #2, CMOD




<u>Many literature problems solved with success</u>

## Also with simultaneous crack growth



Particle formation ($5\times$ magnified)

## Any hyperelastic law along with any plasticity model.

| Yield criterion | Number of yield surfaces | Equivalent stresses |
|---|---|---|
| von-Mises | 1 | $\sigma_{\text{eq}_1} = \sqrt{I_1^2 - 3I_2}$ |
| Tresca | 6 | $\sigma_{\text{eq}_k} = \widetilde{\tau}_i - \widetilde{\tau}_j, \quad i \neq j$ |
| Ductile damage | 2 | $\sigma_{eq_1} = \frac{\sqrt{I_1^2-3I_2}-fc_1I_1}{1-f}$ <br> $\sigma_{eq_2} = \frac{\sqrt{I_1^2-3I_2}}{1-f}$ |

$I_1 = \text{tr}\boldsymbol{\tau}, \quad I_2 = \frac{1}{2}\left[(\text{tr}\boldsymbol{\tau})^2 - \text{tr}\boldsymbol{\tau}^2\right], \quad I_3 = \det \boldsymbol{\tau}$

## However... **much** tougher than quasi-brittle is *ductile* fracture

### Finite strain plasticity as we see it

$$\boldsymbol{\tau} = 2\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}\boldsymbol{b}_e = 2\boldsymbol{b}_e\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}$$

$$\frac{[\mathrm{d}\boldsymbol{b}]_{ij}}{[\mathrm{d}\boldsymbol{F}]_{mn}} = \delta_{im}[\boldsymbol{F}]_{jl} + \delta_{jm}[\boldsymbol{F}]_{in}$$

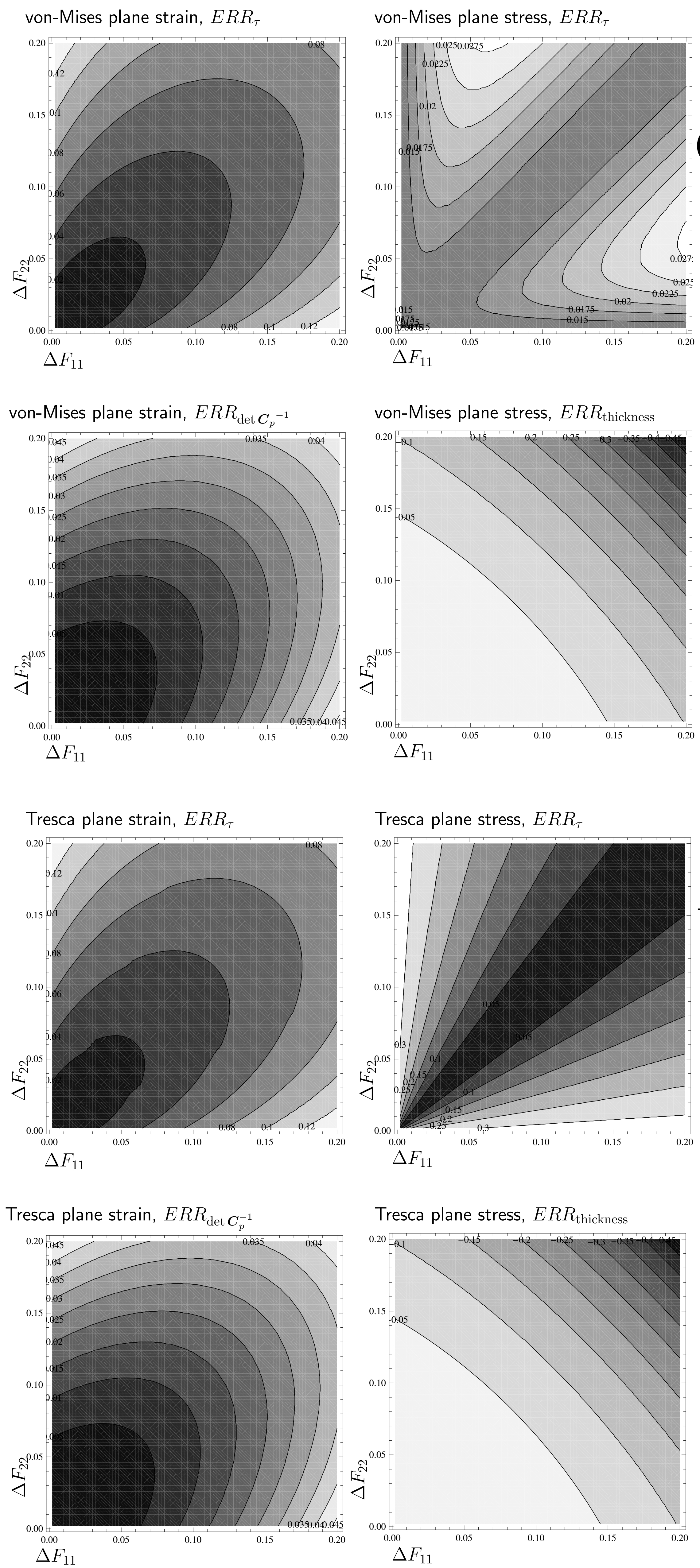
$$\begin{aligned}
\overset{\star}{\boldsymbol{b}}_{eV} &= -4\sum_{i=1}^{n_s}\dot{\gamma}_i\boldsymbol{A}^{-1}\boldsymbol{n}_i \\
\dot{\boldsymbol{v}} &= -\sum_{i=1}^{n_s}\dot{\gamma}_i\boldsymbol{\varphi}_i \\
\boldsymbol{\tau} &= 2\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}\boldsymbol{b}_e \\
\mu\dot{\gamma}_i - \langle\mu\dot{\gamma}_i + \phi_i\rangle &= 0
\end{aligned}$$

### What the books do not describe

$$\mathrm{d}\varepsilon^i_{p_{n+1}} = \frac{\boldsymbol{n}_i : \boldsymbol{\tau}}{\sigma_{eq_i}}\mathrm{d}\Delta\gamma_i + \left[\frac{\Delta\gamma_i}{\sigma_{eq_i}}\left(\boldsymbol{\tau} : \frac{\mathrm{d}\boldsymbol{n}_i}{\mathrm{d}\boldsymbol{\tau}} + \boldsymbol{n}_i\right) - \frac{\boldsymbol{n}_i : \boldsymbol{\tau}}{\sigma_{eq}}\right] : \frac{\partial\boldsymbol{\tau}}{\partial\boldsymbol{b}_e} : \mathrm{d}\boldsymbol{b}_e$$

$$4\boldsymbol{d}_{pV} = -\boldsymbol{A}\overset{\star}{\boldsymbol{b}}_{eV}$$

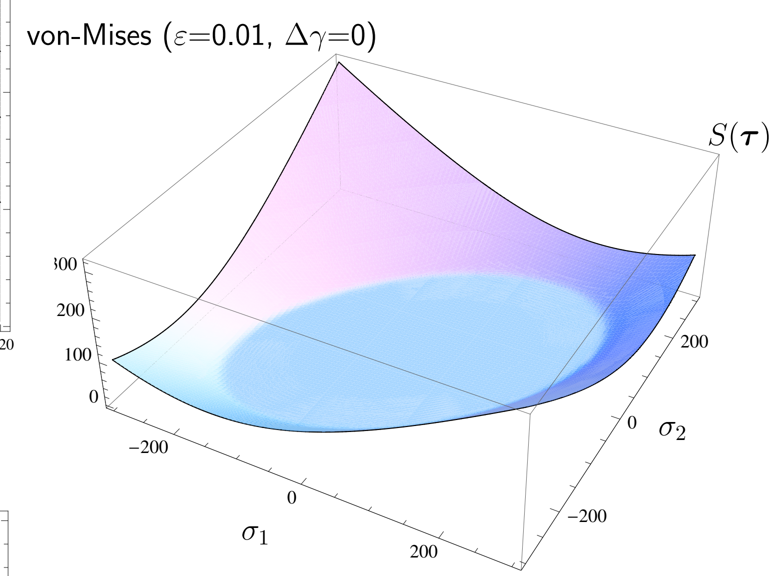
# No requirement for active set strategies, and no "return-mapping" and *much better accuracy than classical methods, including Simo's...*

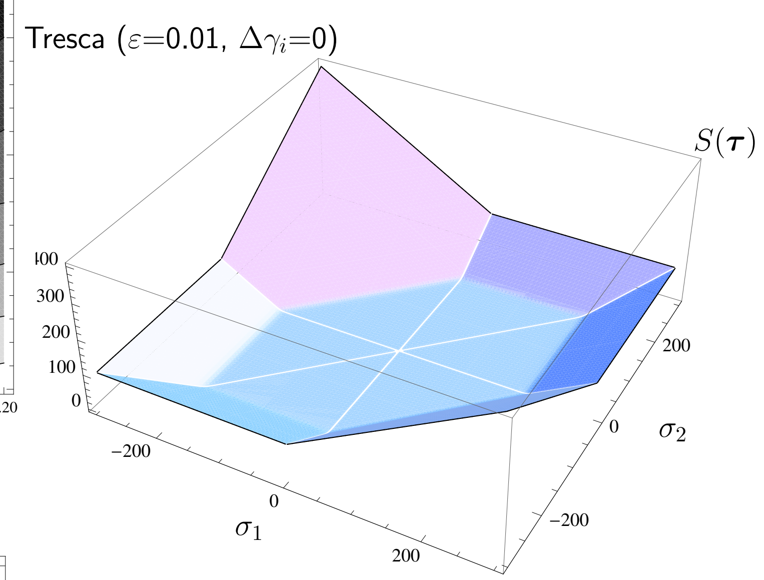


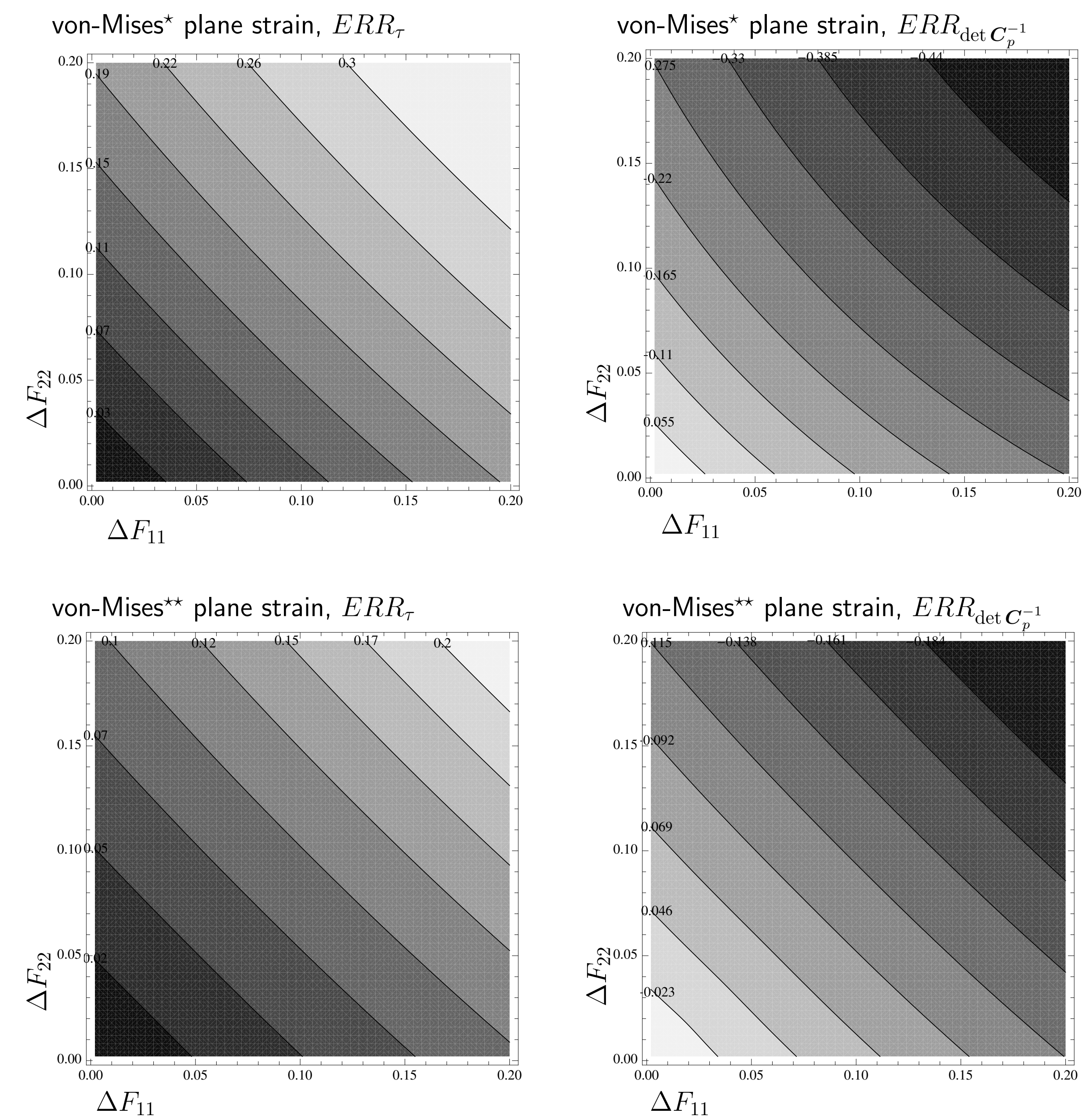

Ours

Material properties

$E = 200$ GPa
$\nu = 0.3$
$\sigma_1 = 200$ MPa
$\sigma_2 = 200$ MPa
$\sigma_y = 200$ MPa



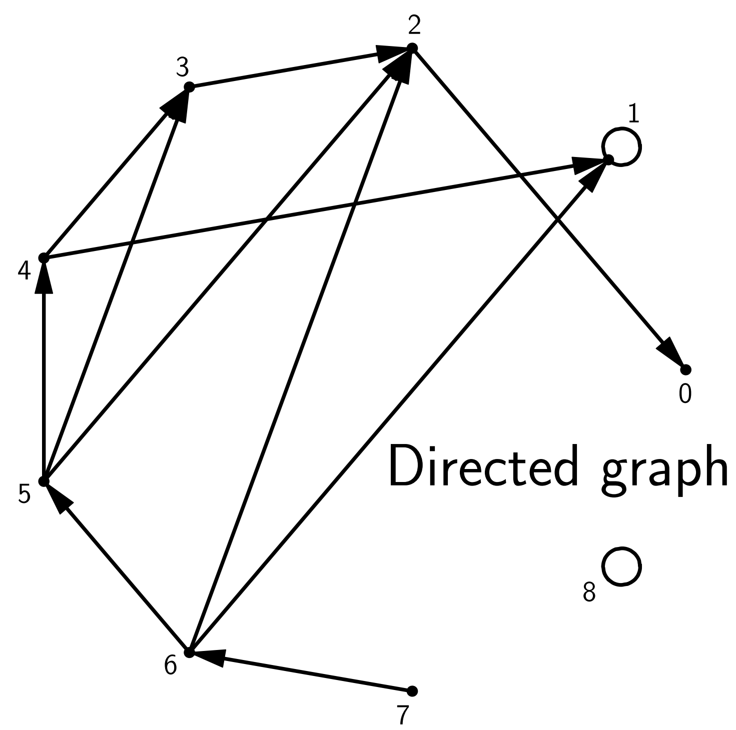



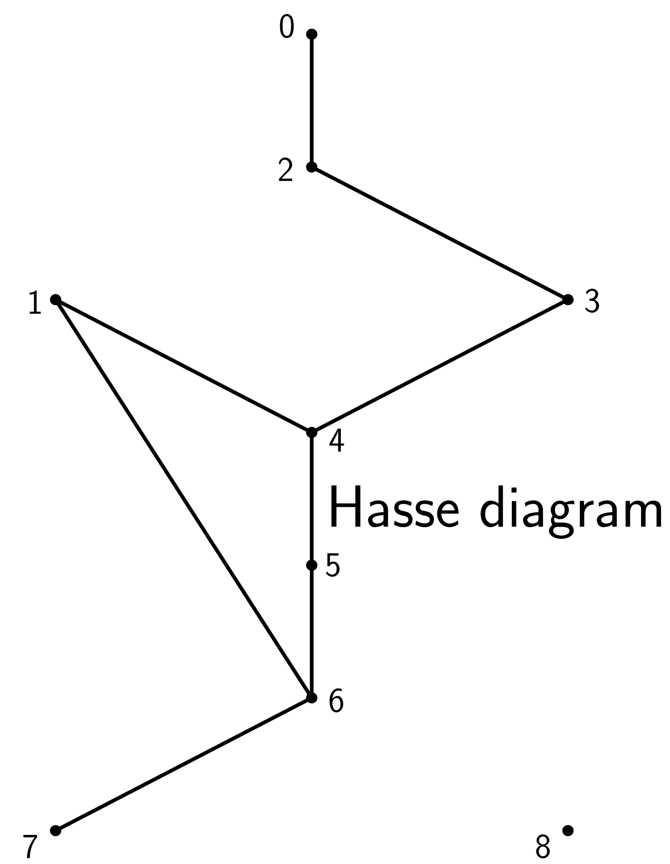

Simo 1988 (*) and 1992 (**)

# Multiple-point constraints (*control, ALE repositioning, ...*)

$$
\boldsymbol{T}_\star^T \left( \sum_{i=1}^{n_e} \boldsymbol{K}_i^e \right) \boldsymbol{T}_\star + \boldsymbol{T}_\star^T \left\{ - \sum_{j=1}^{m} \left[ \left( \sum_{k=1}^{n_e} \boldsymbol{f}_k^e \right)^T \left( \boldsymbol{c}_j \boldsymbol{g}_j'' \right) \right] \right\} \boldsymbol{T}_\star \mathrm{d}\boldsymbol{a}_r \quad = \\
-\boldsymbol{T}_\star^T \left( \sum_{j=1}^{n_e} \boldsymbol{f}_j^e \right) - \boldsymbol{T}_\star^T \left( \sum_{l=1}^{n_e} \boldsymbol{K_j^e} \right) \boldsymbol{b}_\star
$$

**Very** hard to implement efficiently - a combination of clique and sparse data structures




Acyclic test
Topological sort

| | Unroll of nested DOFs | Collapse (and sum) |
|---|---|---|
| $1:1$ | $1:\mathbf{1}$ | $1:1$ |
| $2:0$ | $2:\mathbf{0}$ | $2:0$ |
| $3:2$ | $3:2,\mathbf{0}$ | $3:0$ |
| $4:1,3$ | $4:1,\mathbf{1}\vert 3,\mathbf{0}$ | $4:1,0$ |
| $5:3,4,2$ | $5:3,\mathbf{0}\vert 4,\mathbf{1}\vert 4,\mathbf{0}\vert 2,\mathbf{0}$ | $5:0,1$ |
| $6:2,1,5$ | $6:2,\mathbf{0}\vert 1,\mathbf{1}\vert 5,\mathbf{0}\vert 5,\mathbf{1}\vert 5,\mathbf{0}\vert 5,\mathbf{0}$ | $6:0,1$ |
| $7:6$ | $7:6,\mathbf{0}\vert 6,\mathbf{1}\vert 6,\mathbf{0}\vert 6,\mathbf{1}\vert 6,\mathbf{0}\vert 6,\mathbf{0}$ | $7:0,1$ |
| $8:8$ | $8:\mathbf{8}$ | $8:8$ |

Degree-of-freedom:list of masters

# Mixed elements

Given $t \in \mathbb{R}_0^+$ $\boldsymbol{t} \in [L^2(\Gamma_{0t}^N)]^2$ and $\boldsymbol{b} \in [L^2(\Omega_{0t})]^2$, find $\boldsymbol{u} \in \left[H^1(\Omega_{0t})\right]^2$ (with non-homogeneous boundary conditions on $\Gamma_{0t}^D$) and $p \in L^2(\Omega_{0t})$ such that $\forall \widetilde{\boldsymbol{u}} \in \left[H^1(\Omega_{0t})\right]^2$ (with homogeneous boundary conditions on $\Gamma_{0t}^D$) and $\forall \widetilde{\theta} \in L^2(\Omega_{0t})$:

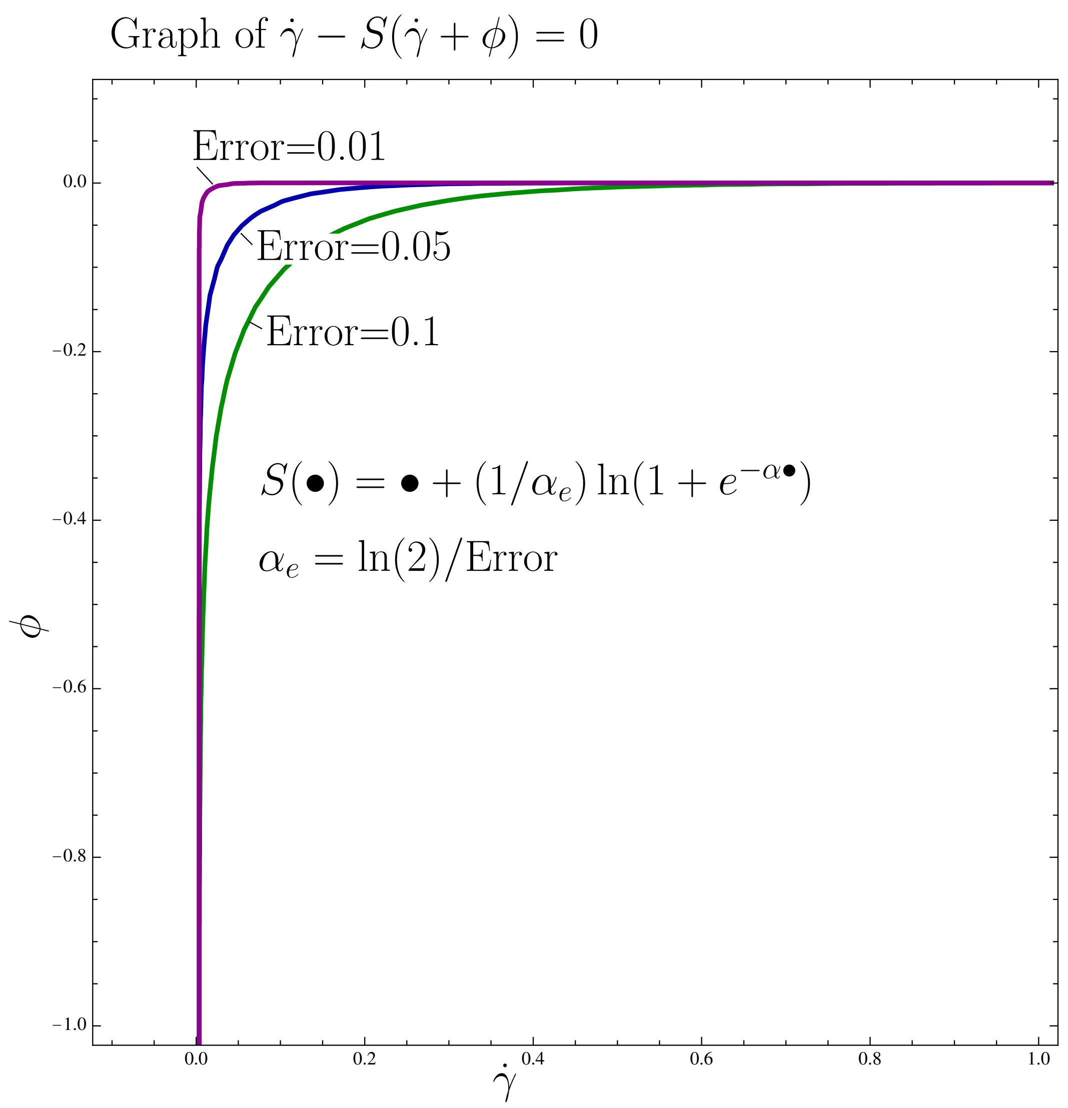
$$
\begin{aligned}
\int_{\Omega_{0t}} \{\mathscr{P} : \boldsymbol{\tau}_c\left[\boldsymbol{F}(\boldsymbol{u}), t\right] + p\boldsymbol{I}\} : \nabla \widetilde{\boldsymbol{u}} \, \mathrm{d}\Omega_{0t} &= \int_{\Gamma_{0t}^N} \boldsymbol{t} \cdot \widetilde{\boldsymbol{u}} \, \mathrm{d}\Gamma_{0t} + \int_{\Omega_{0t}} \boldsymbol{b} \cdot \widetilde{\boldsymbol{u}} \, \mathrm{d}\Omega_{0t} \\
\int_{\Omega_{0t}} \left\{ \frac{1}{3} \boldsymbol{\tau}_c : \boldsymbol{I} - p \right\} \widetilde{\theta} \, \mathrm{d}\Omega_{0t} &= 0
\end{aligned}
$$

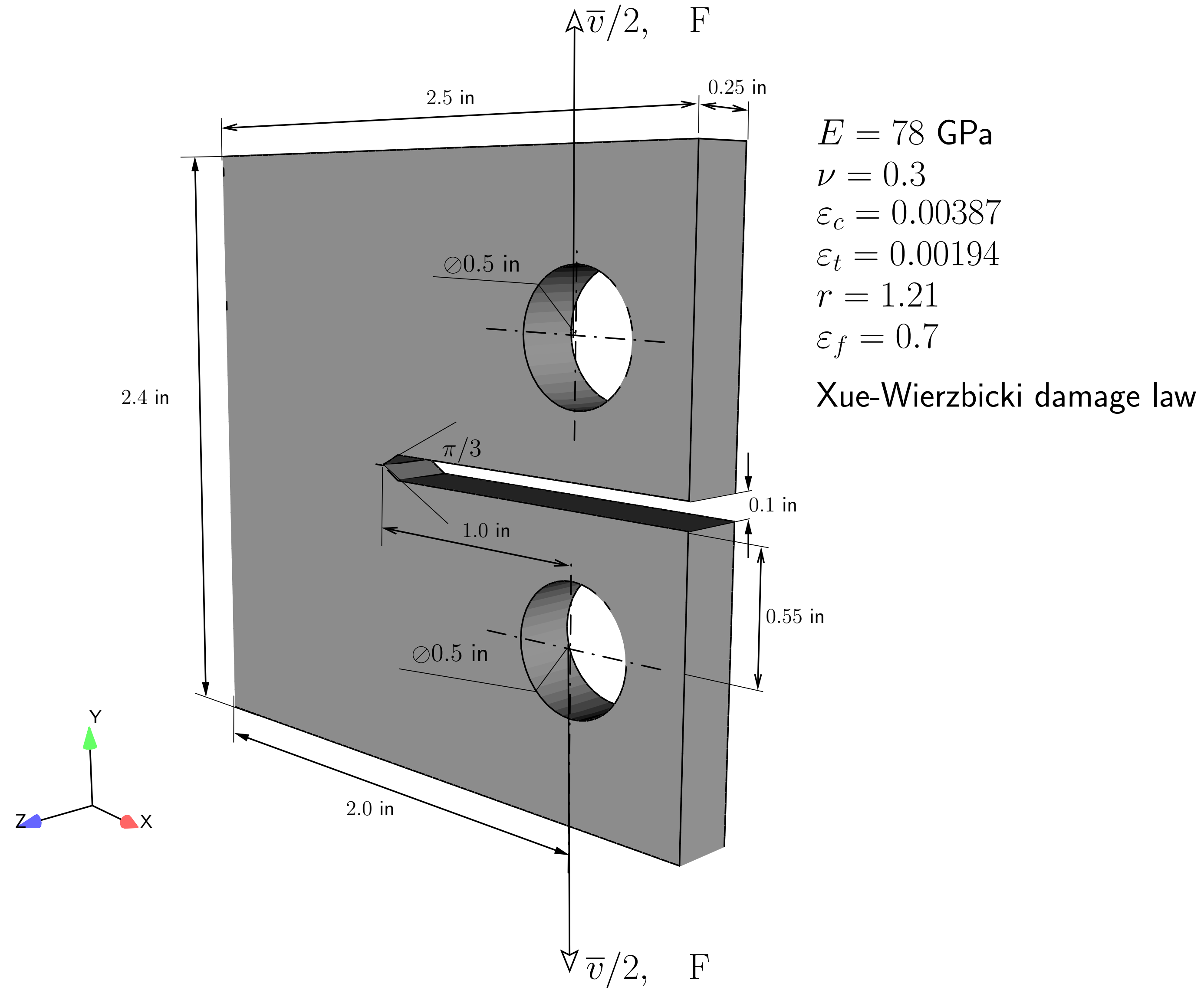
$$
\int_{\Omega_{0t}} \{(\mathscr{P} : \dot{\boldsymbol{\tau}}_c) : \nabla \widetilde{\boldsymbol{u}} - (\mathscr{P} : \boldsymbol{\tau}_c + p\boldsymbol{I}) : (\nabla \widetilde{\boldsymbol{u}} \nabla \dot{\boldsymbol{u}})\} \, d\Omega_{0t} + \int_{\Omega_{0t}} \dot{p}\boldsymbol{I} : \nabla \widetilde{\boldsymbol{u}} \, \mathrm{d}\Omega_{0t} = \delta \dot{W}_{ut}
$$

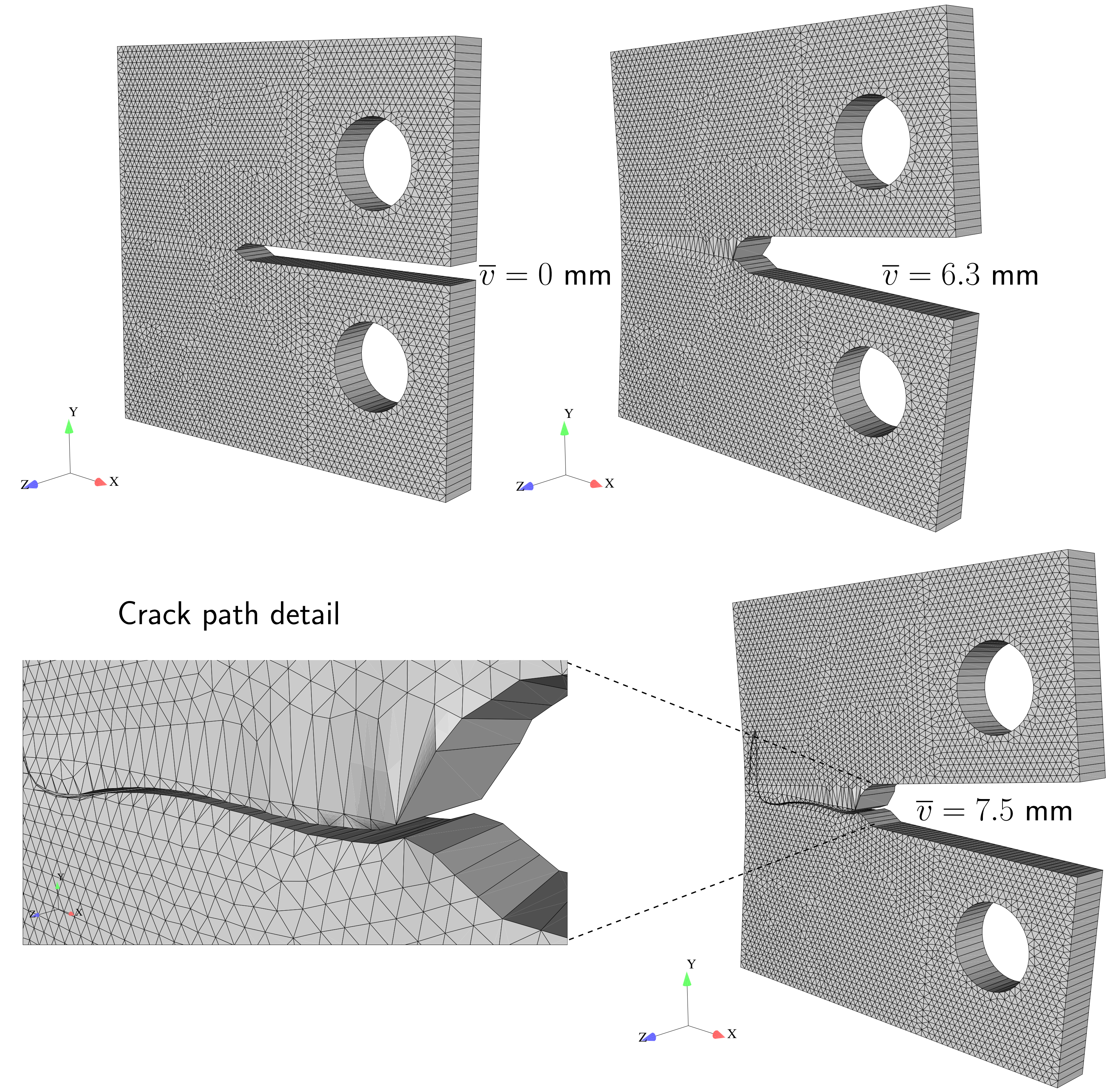
$$
\int_{\Omega_{0t}} \left[ \frac{1}{3} \boldsymbol{I} : \left( \mathscr{C} : \nabla \dot{\boldsymbol{u}} + \boldsymbol{\tau}_c \nabla \dot{\boldsymbol{u}}^T + \nabla \dot{\boldsymbol{u}} \boldsymbol{\tau}_c \right) - \dot{p} \right] \widetilde{\theta} \mathrm{d}\Omega_{0t} = \delta \dot{W}_{pt}
$$

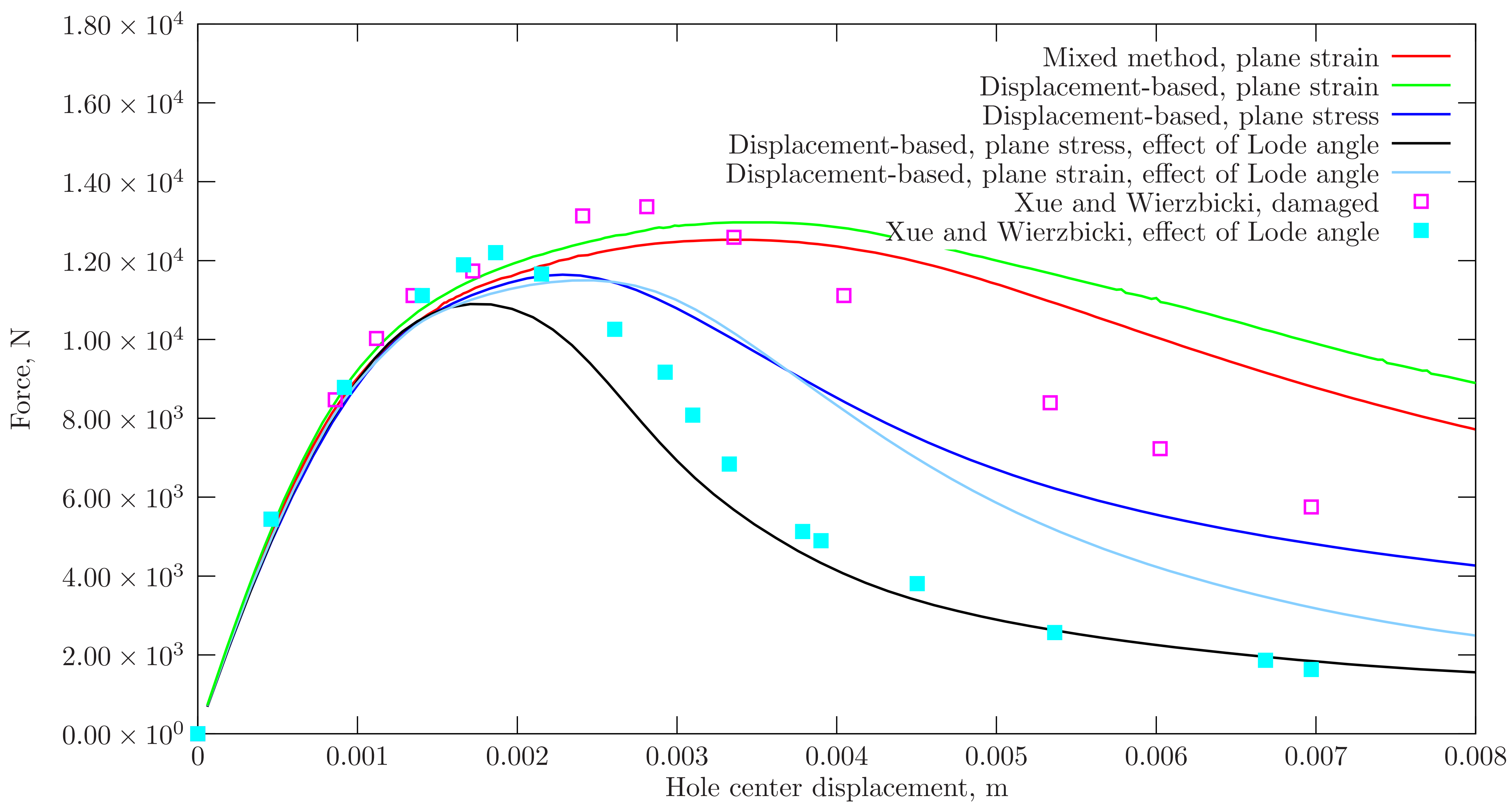
## Technology: bubble displacement
## linear pressure

# Complementarity smoothed and the compact tension test



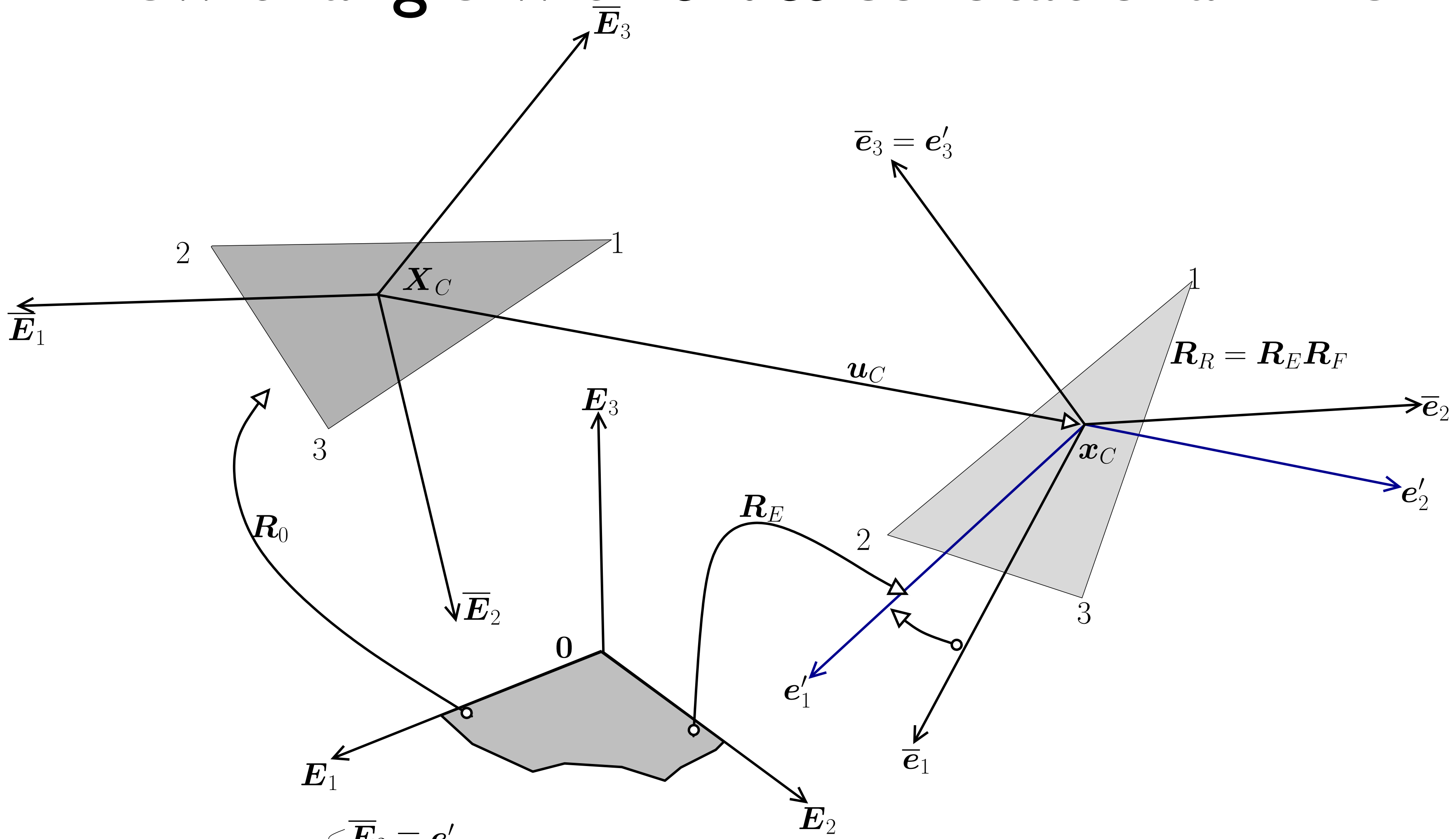



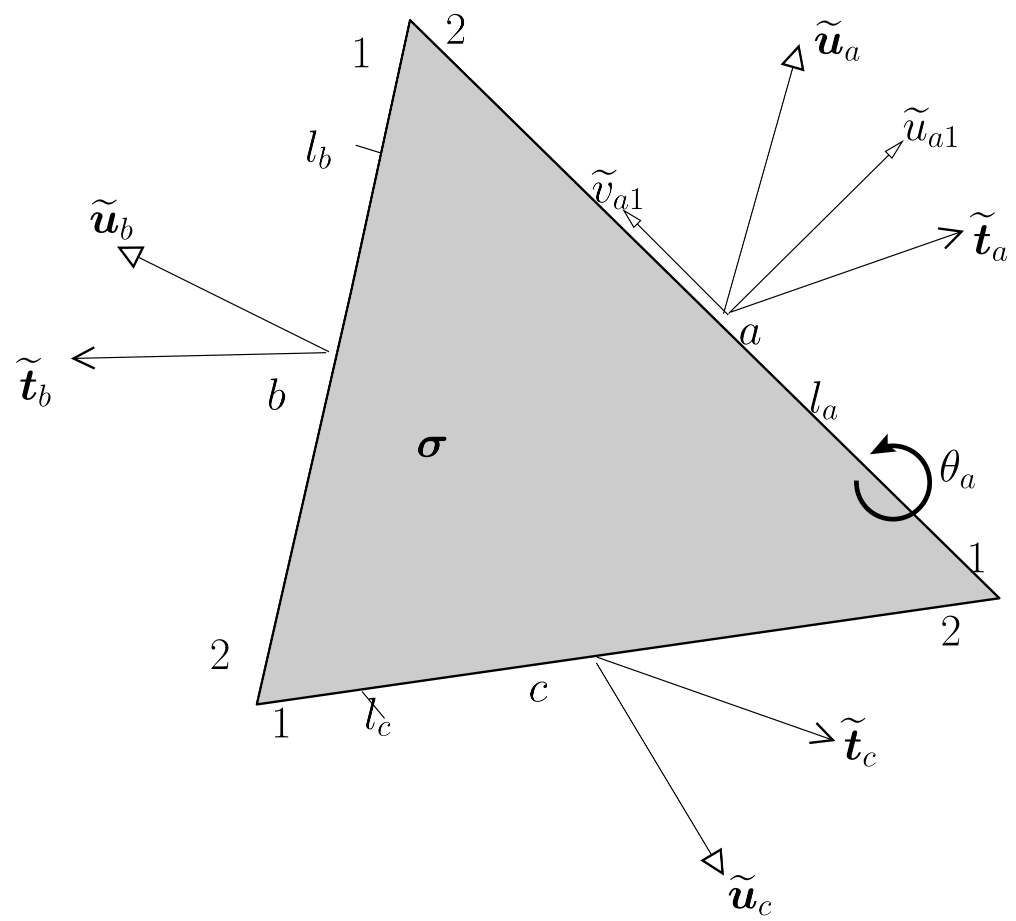




*Exceptionally <u>accurate</u> results with thickness variation*

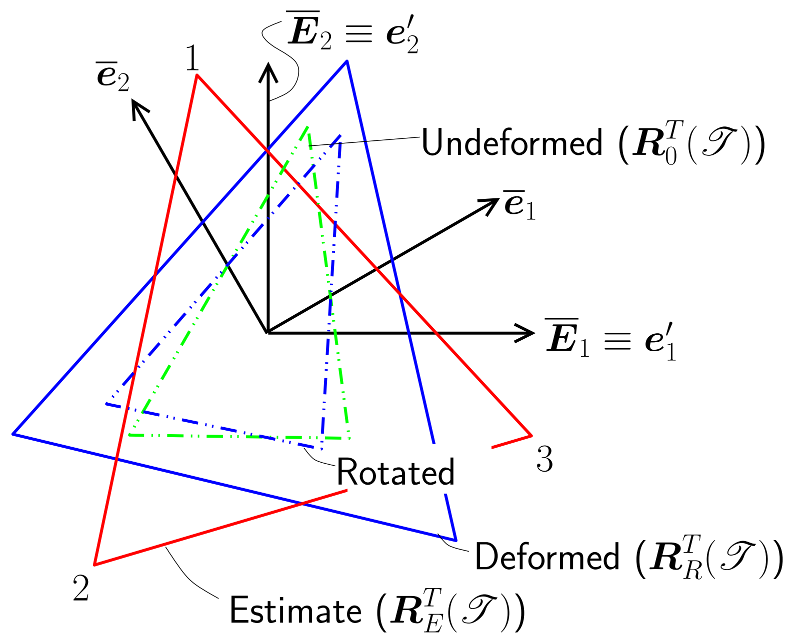
# Shells

## A new triangle with *exact* corotational kinematics for FeFp plasticity



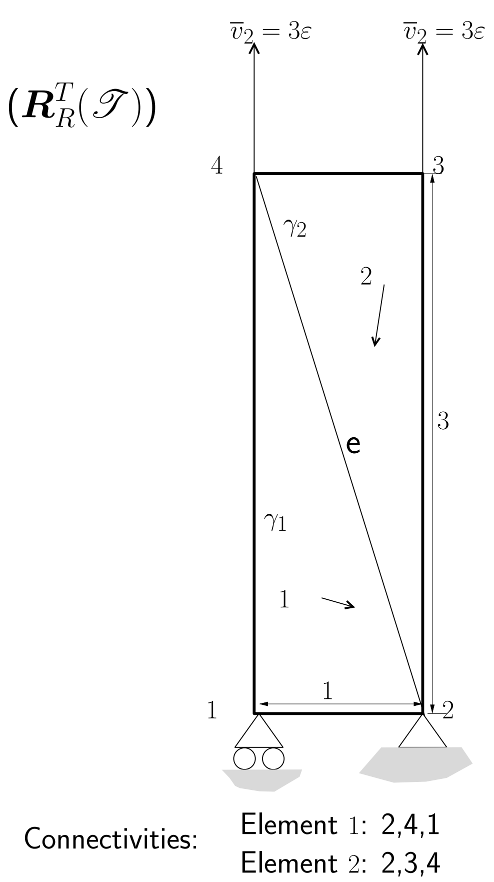



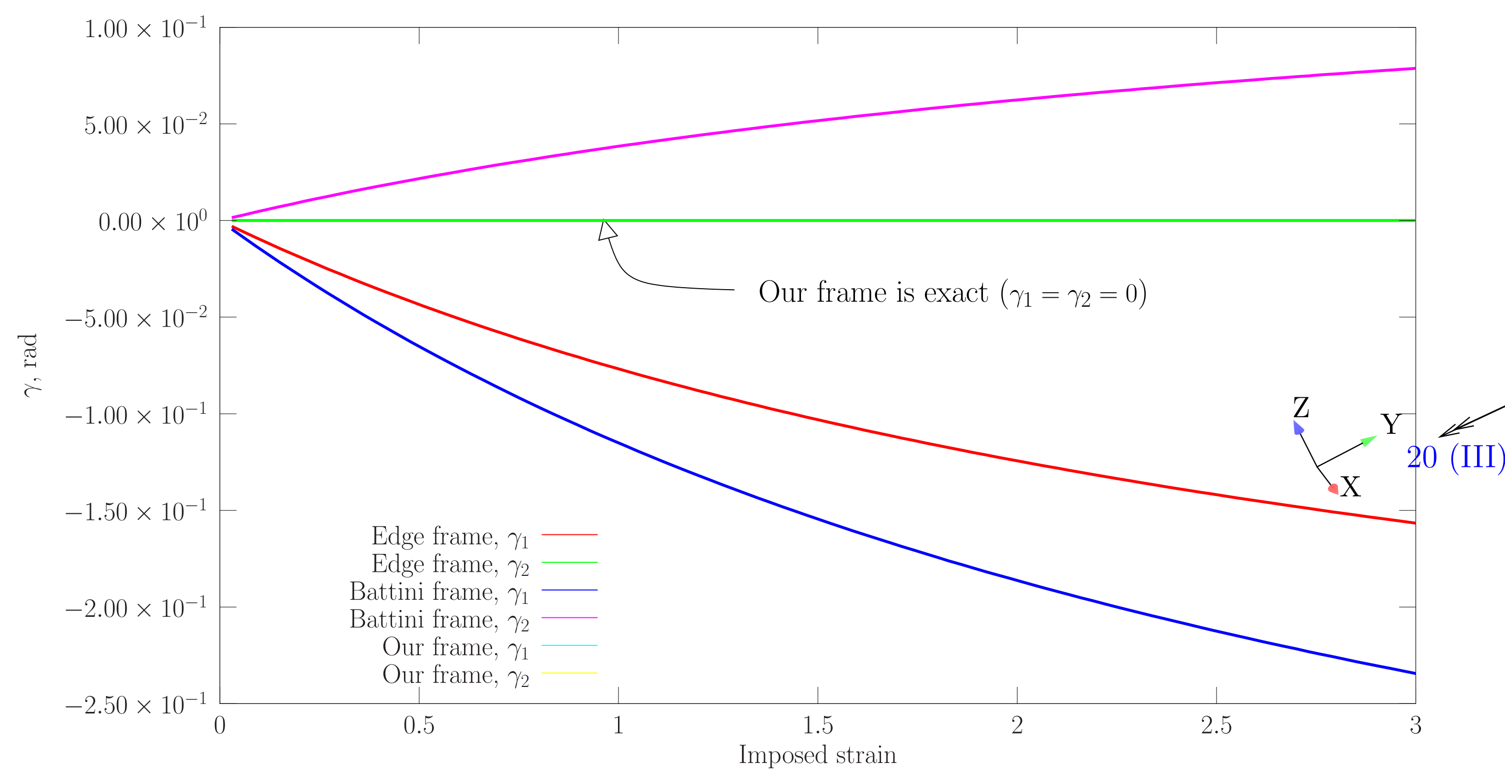

Drilling freedoms



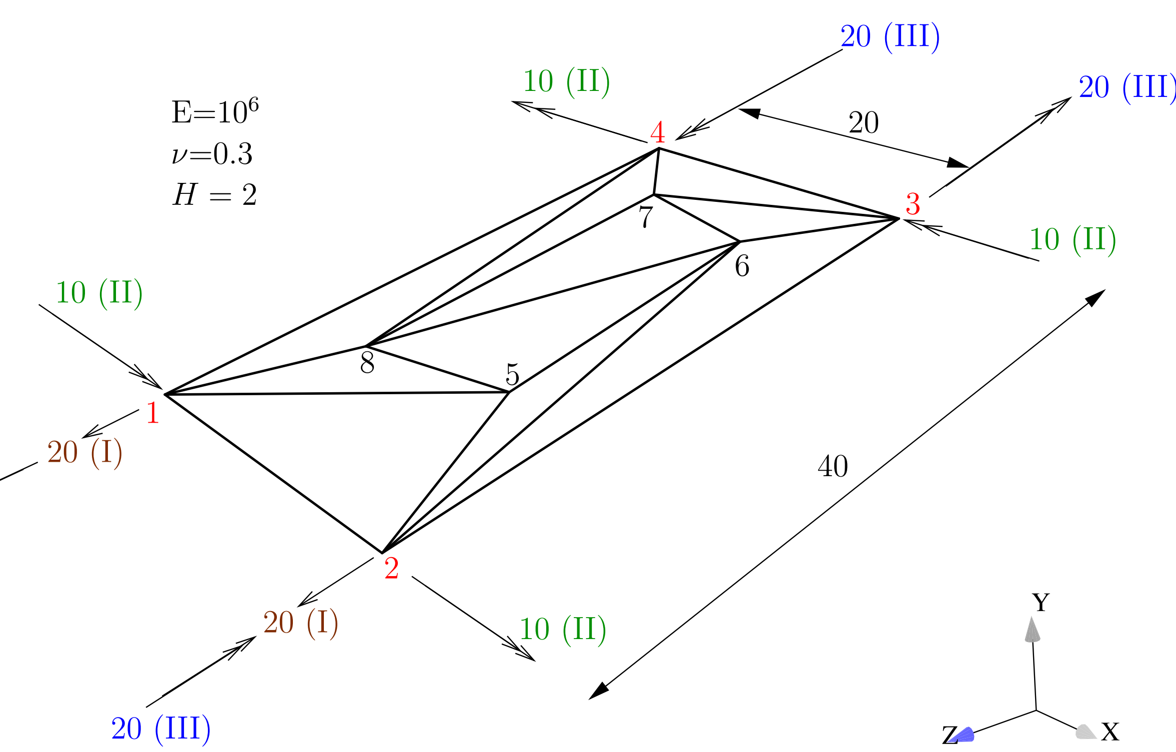

Patch and invariance tests

Frames

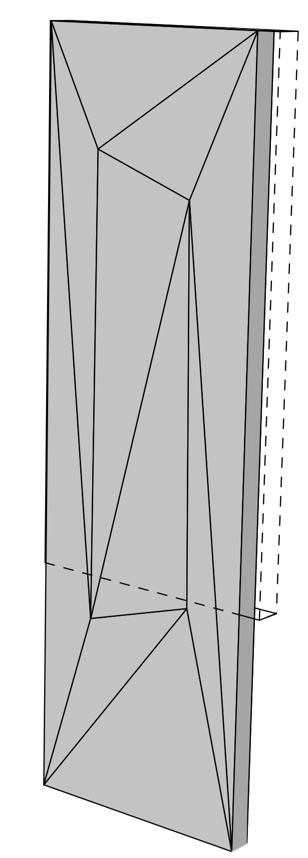


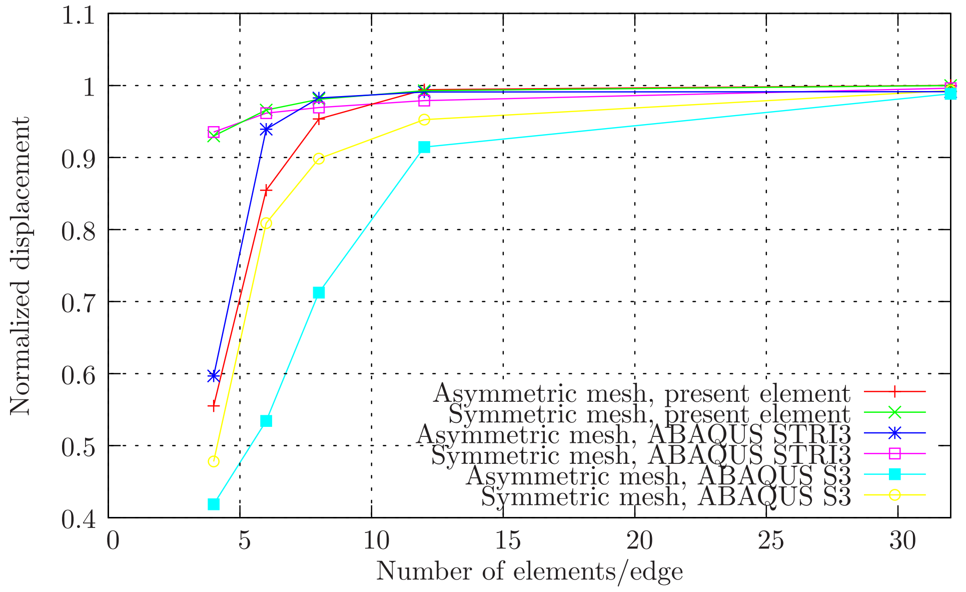

Small strain patch test

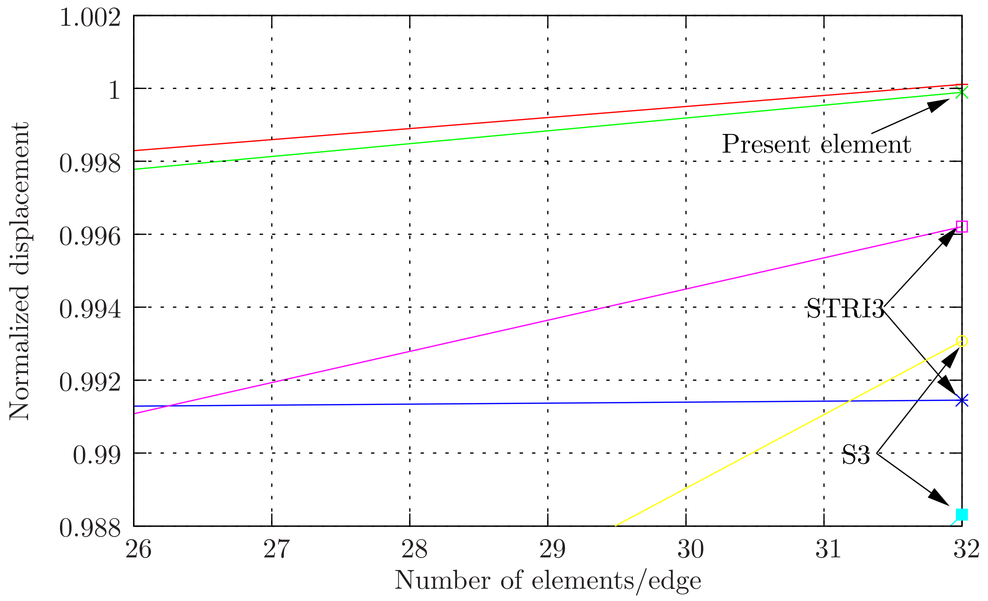



Finite strain patch test

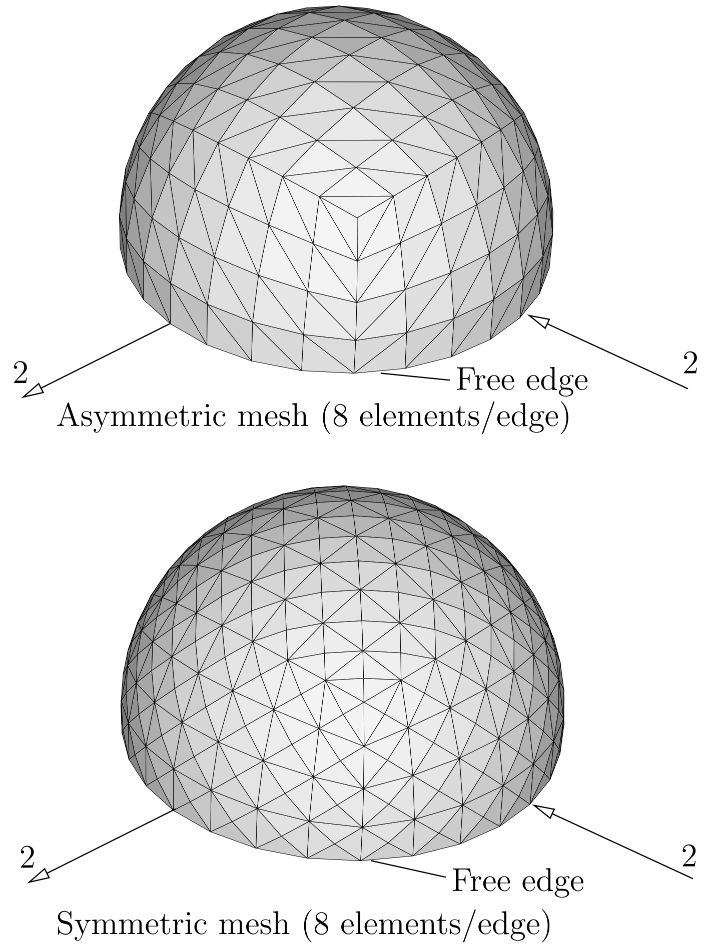


# Benchmark results



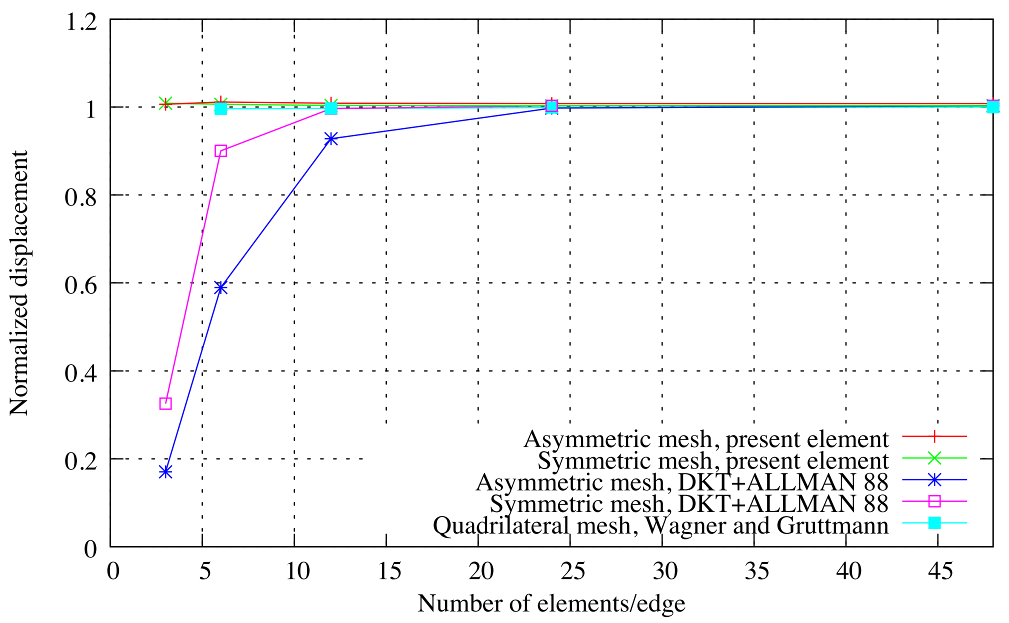




R=10 E=$6.825 \times 10^7$
h=0.04 $\nu = 0.3$

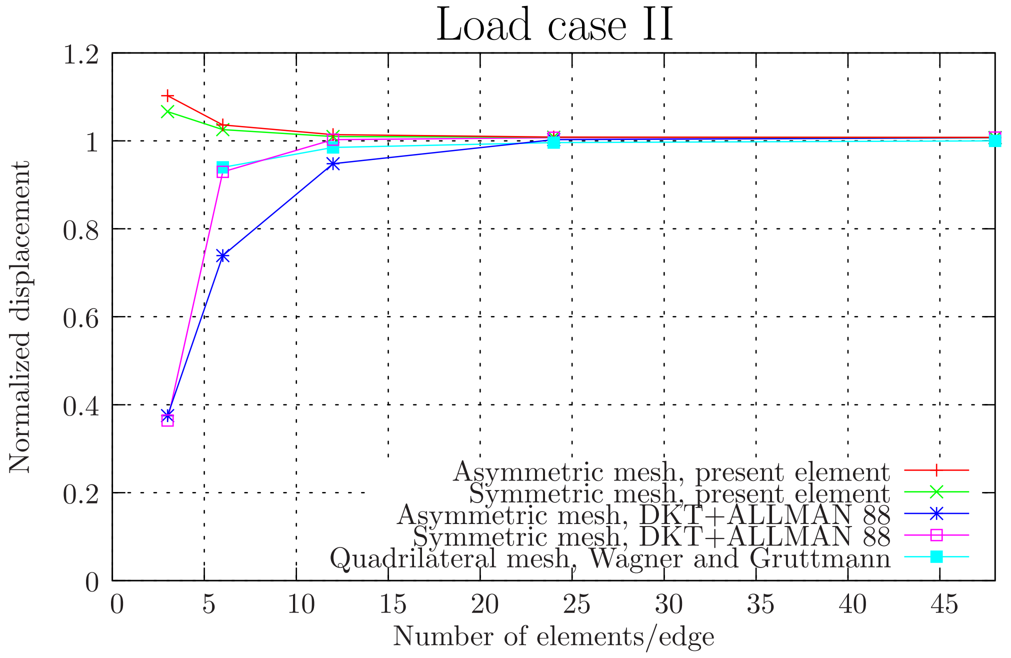



Target displacement=$9.24 \times 10^{-2}$
(Consistent units are used)

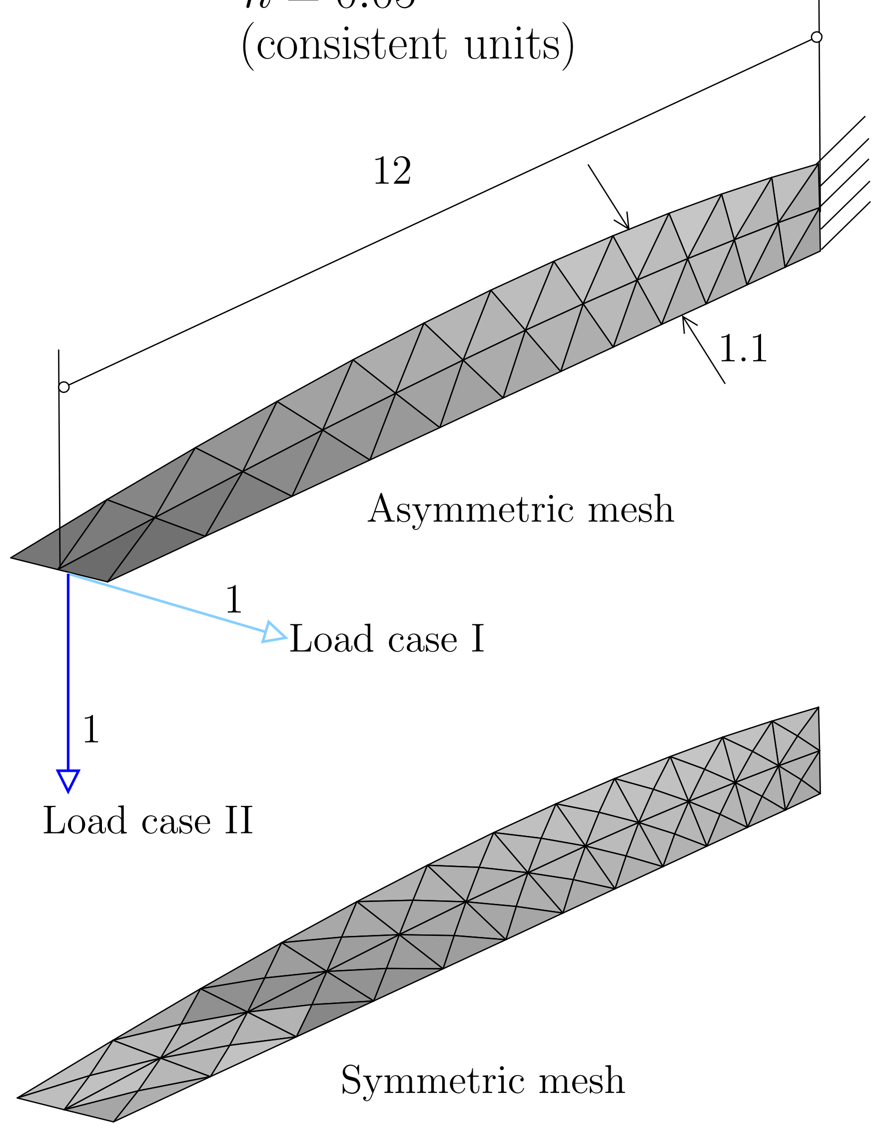
Load case I

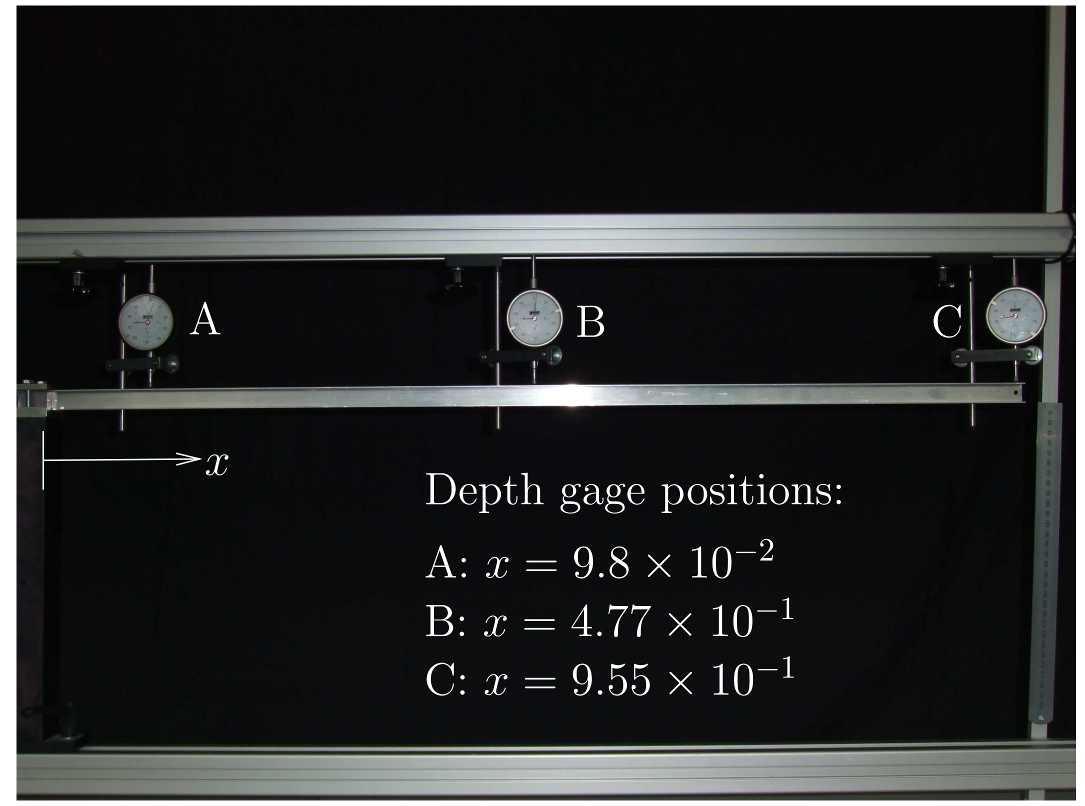


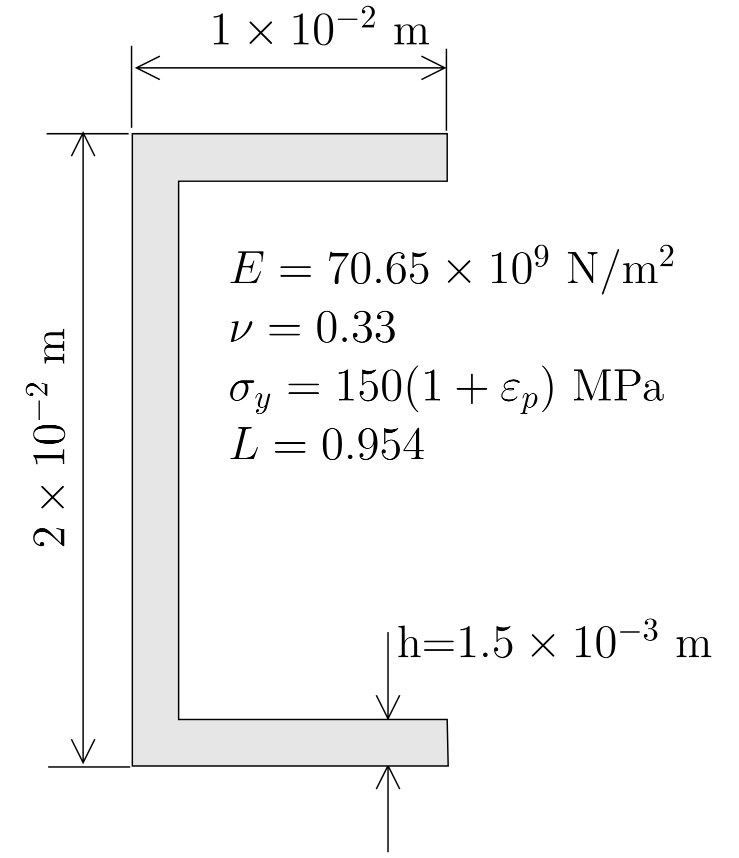

Load case II



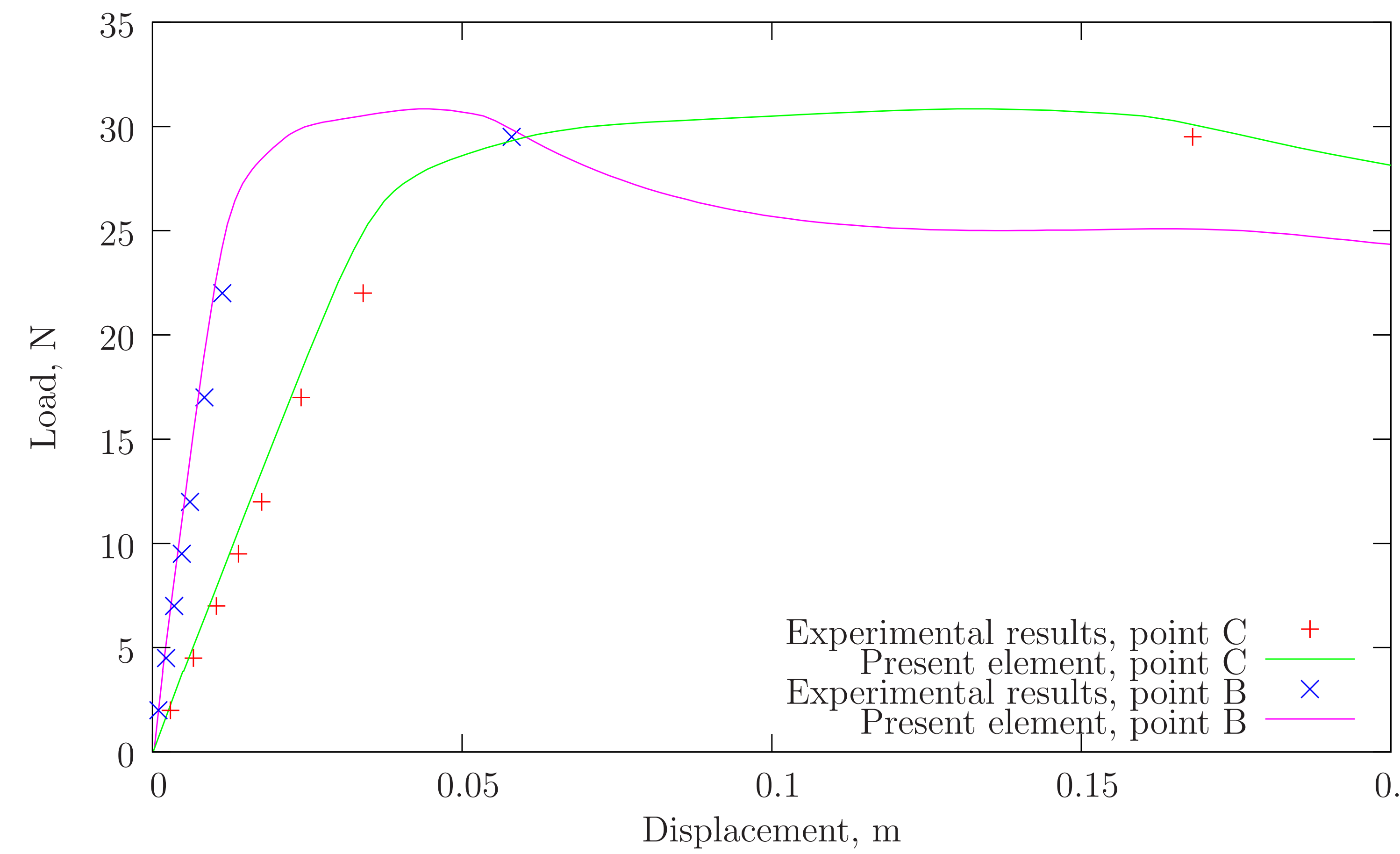

E=$29 \times 10^6$
$\nu = 0.22$
$h = 0.05$
(consistent units)



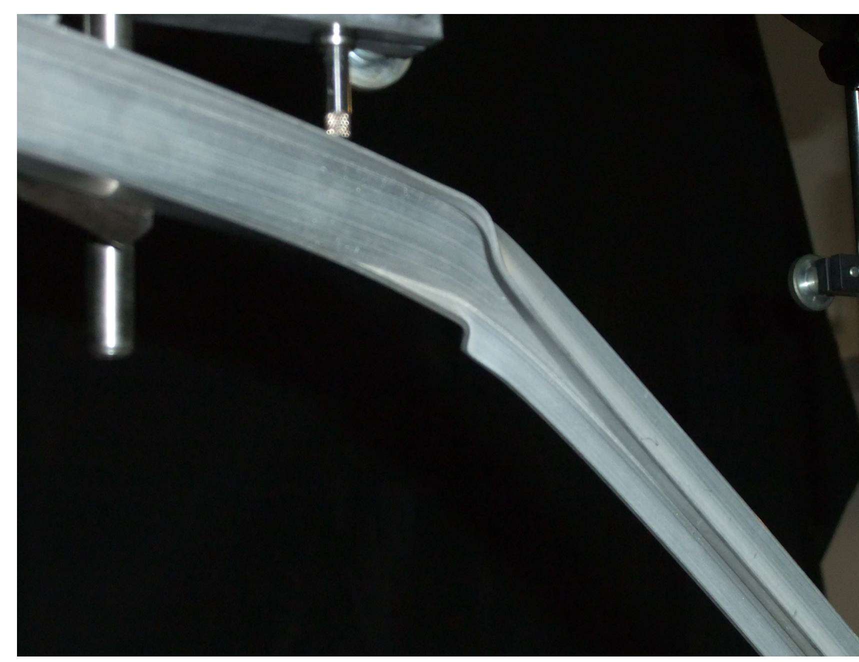



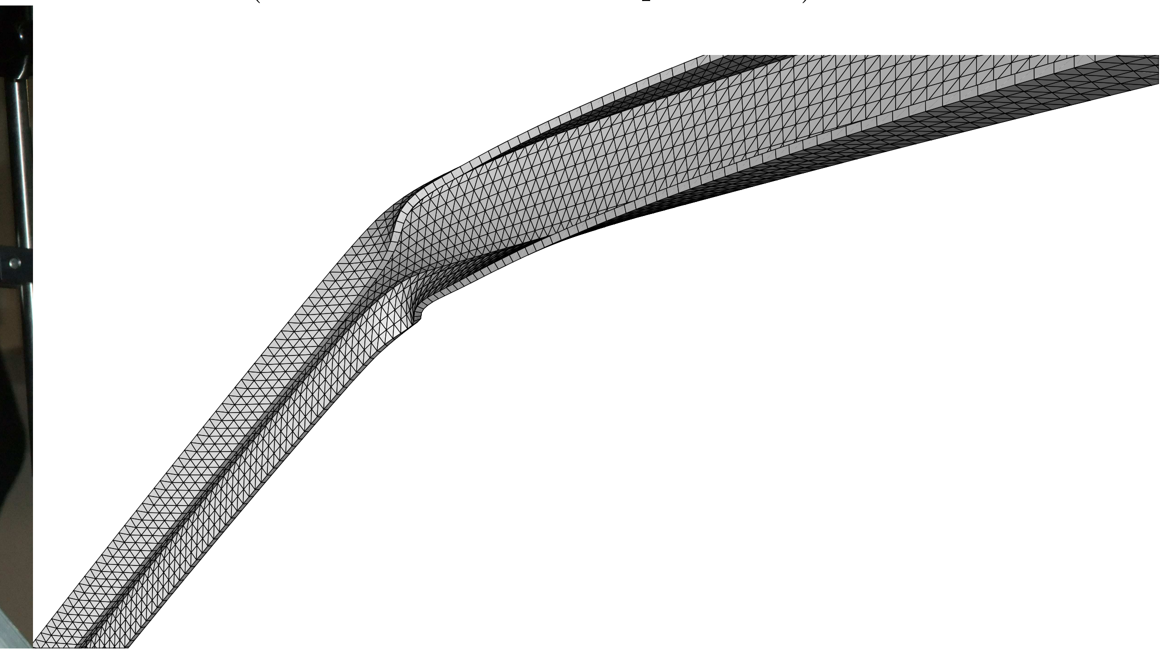

Mesh: $4 \times 8 \times 400$ elements
Asymmetric arrangement is adopted



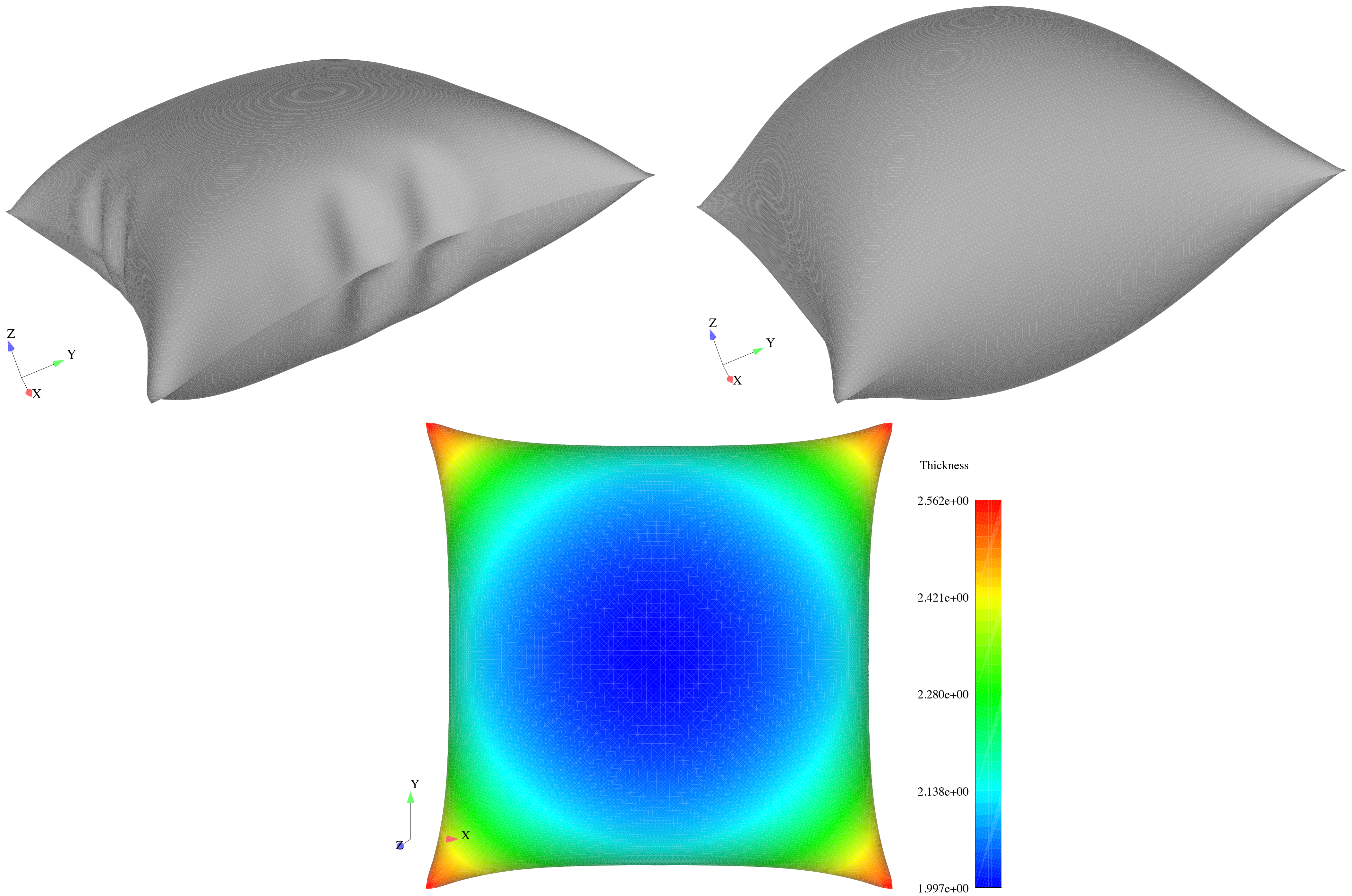

Experimentally observed elasto-plastic flange buckling

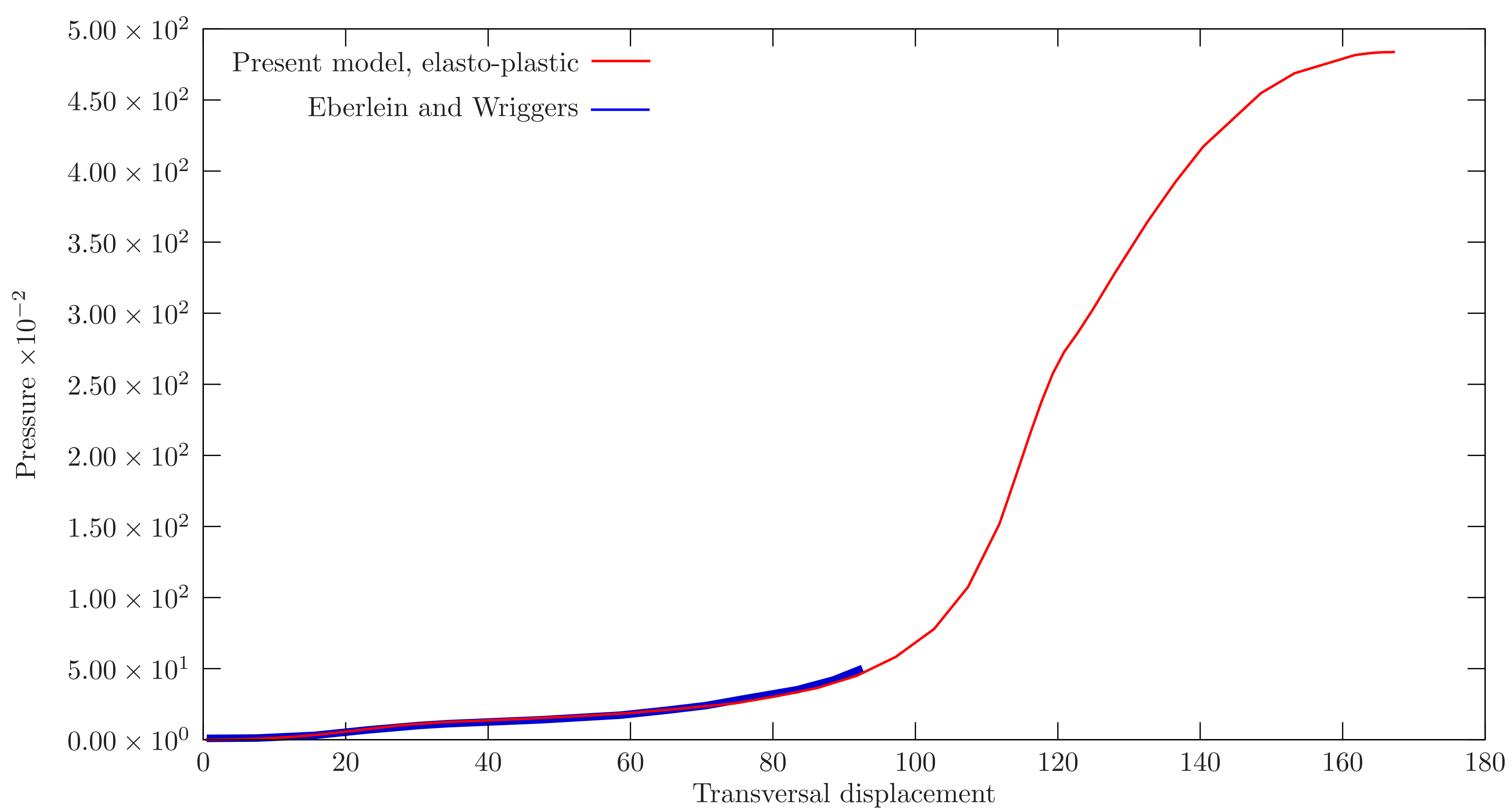


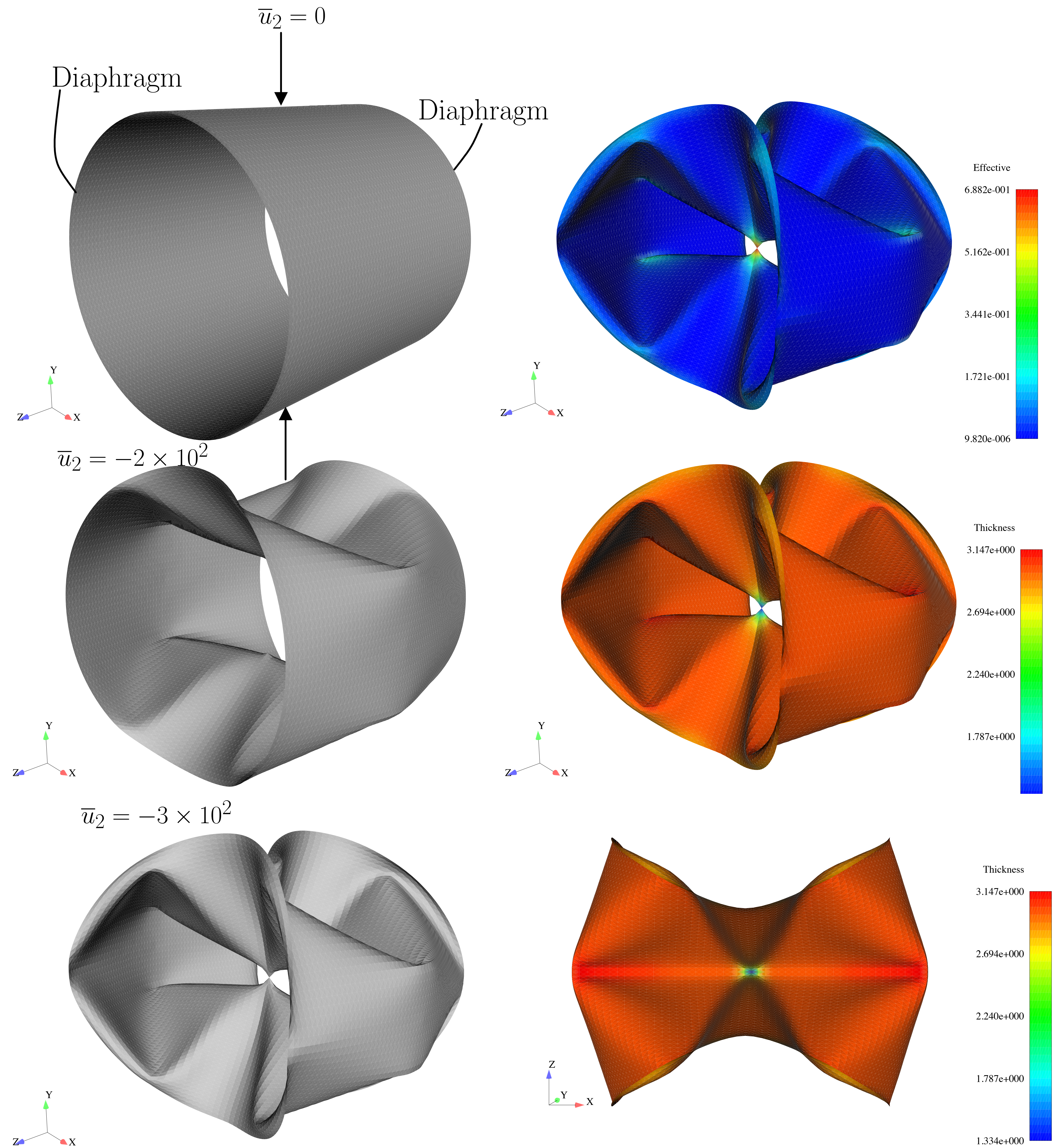
Numerically obtained elasto-plastic flange buckling
(thickness extrusion was performed)



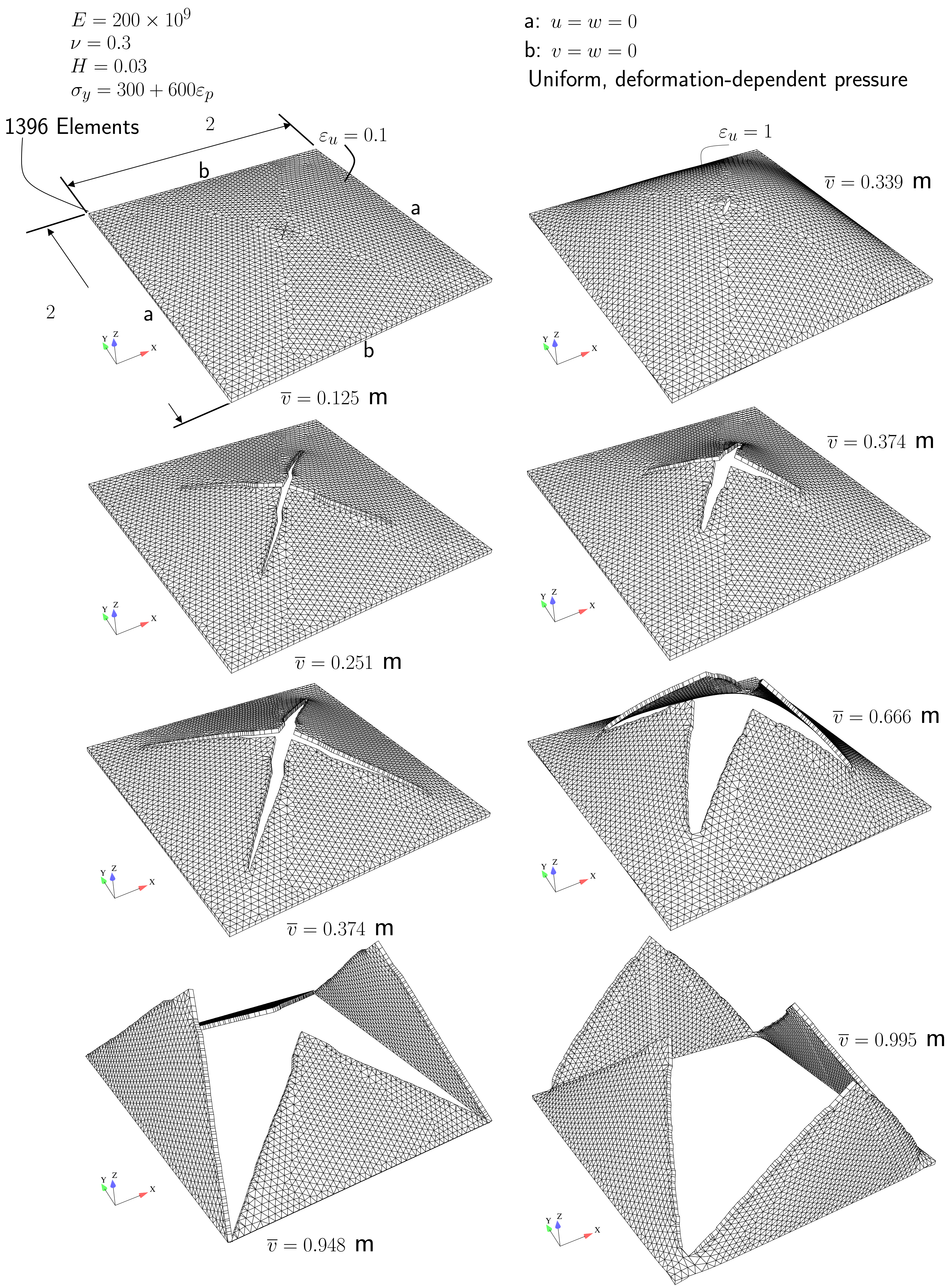
# Much larger deformations than what was reported previously in the literature



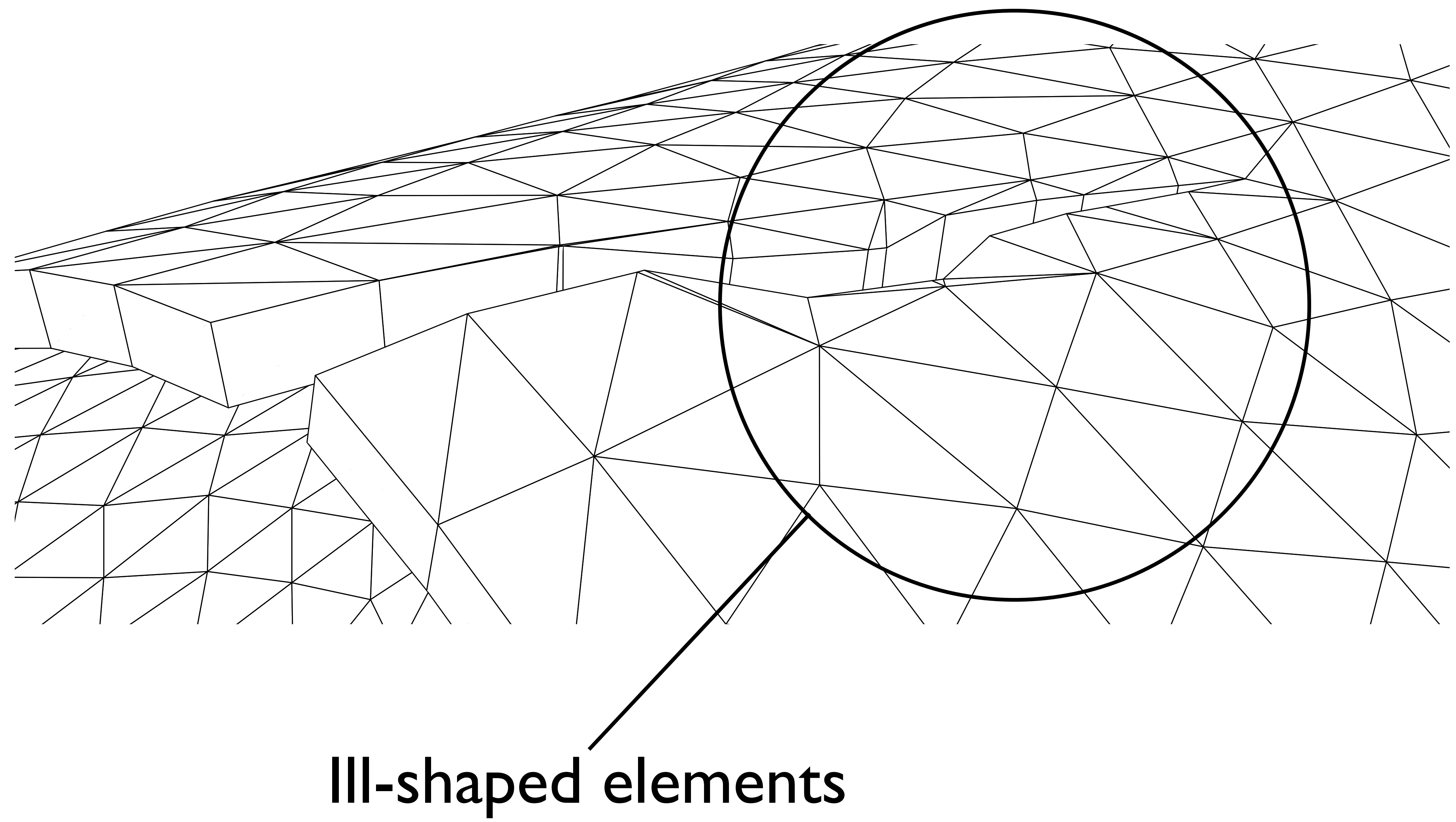
# And *plate* fracture



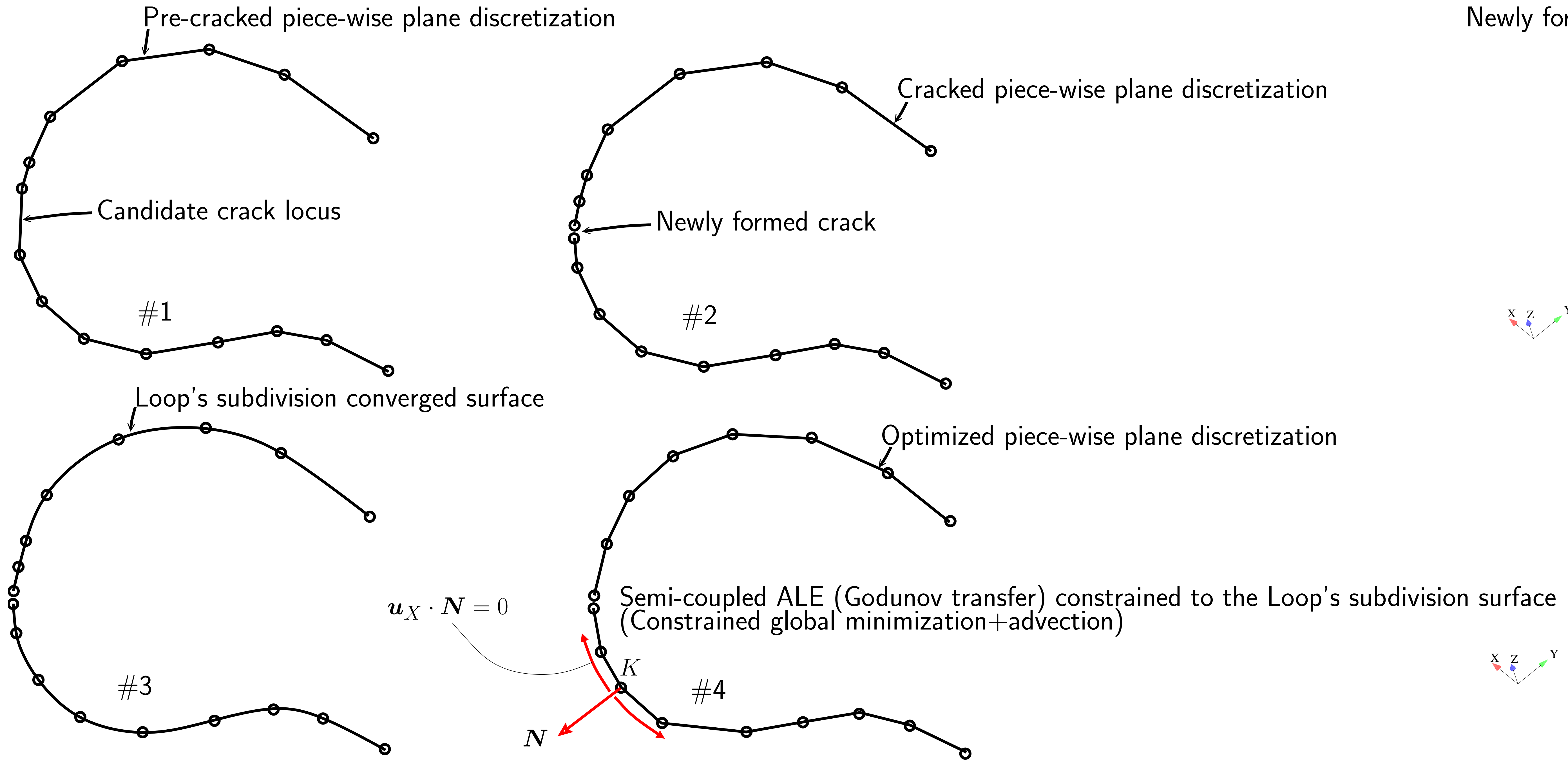

## However, for shells the strategy must be updated due to:

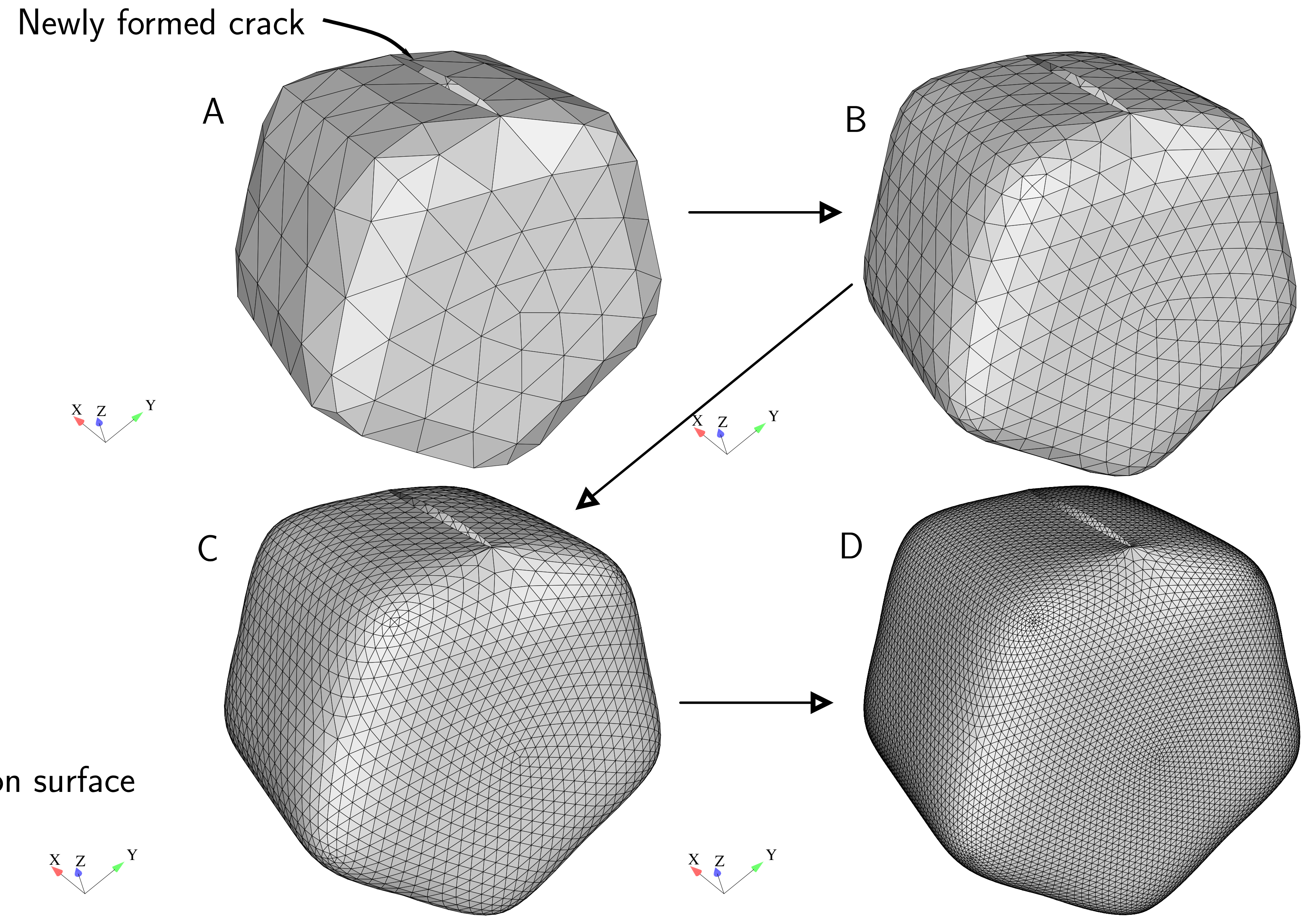
- Non-coplanarity of nodes
- Unknown shape of may surfaces

## Ill-shaped elements naturally occur when the crack advances:



# Our solution to shell fracture



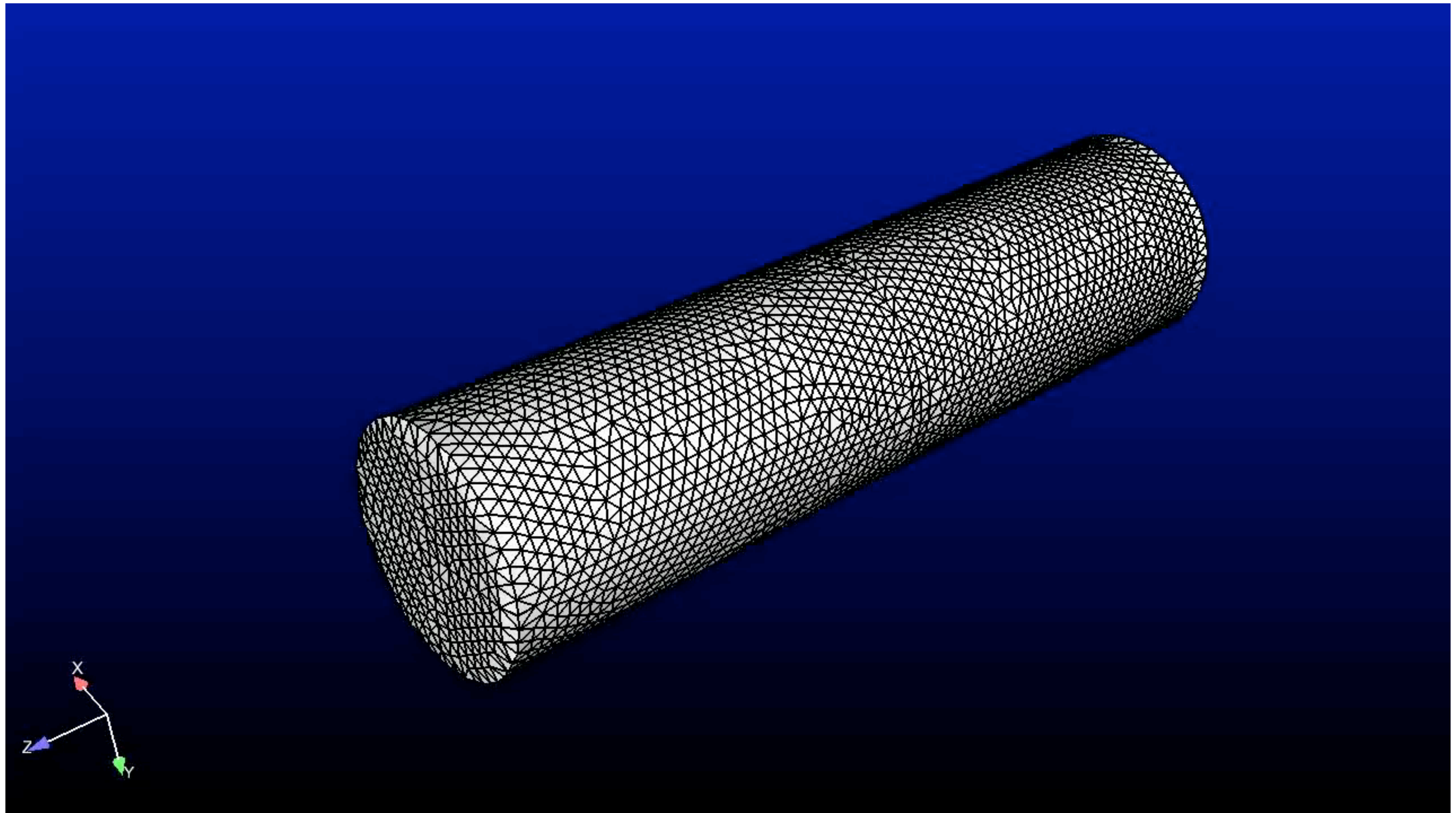




Geometrical operations are all performed in level D

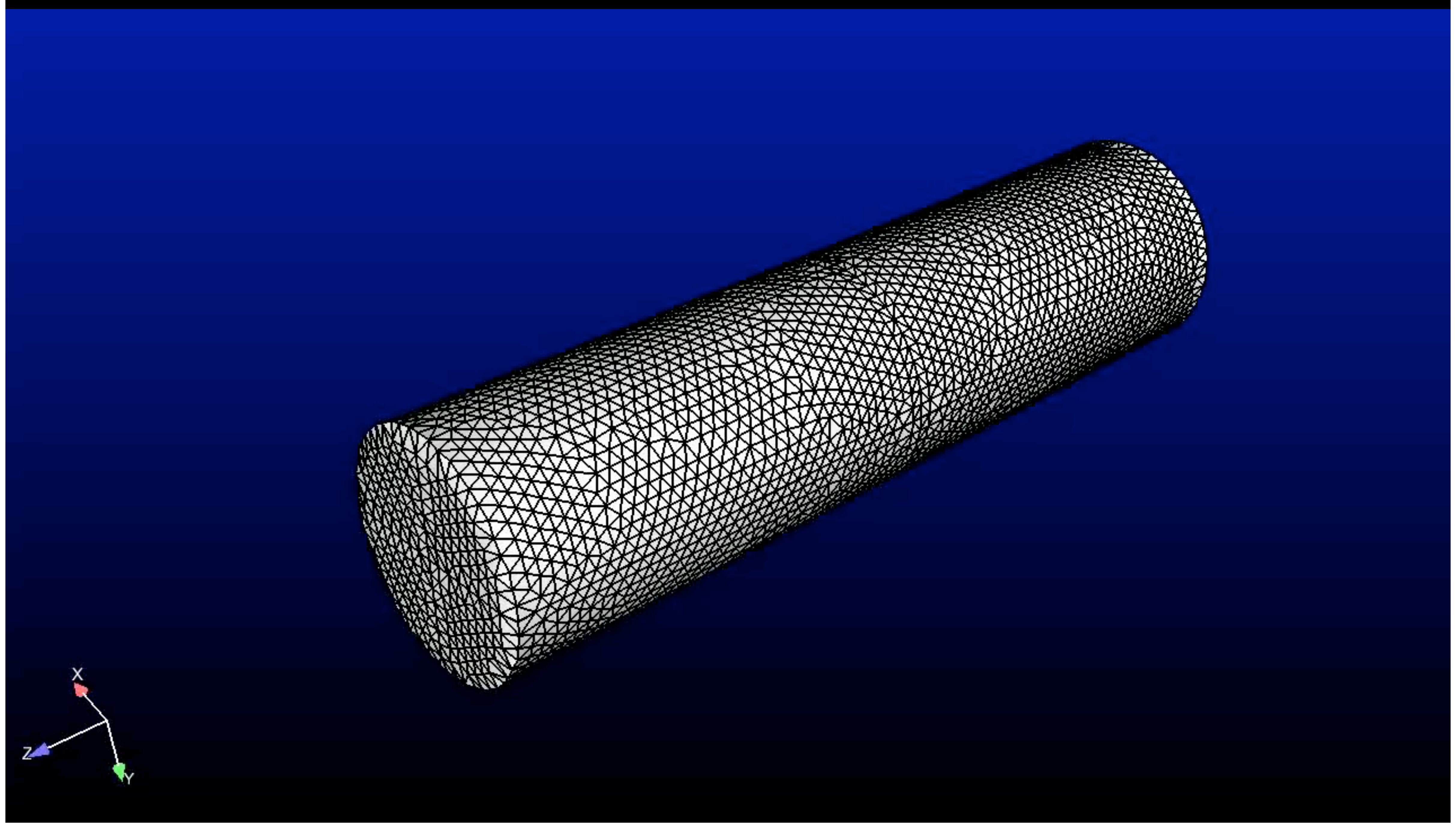
*Hard to code but also very effective*

# The cylinder movies

With our new ALE approach

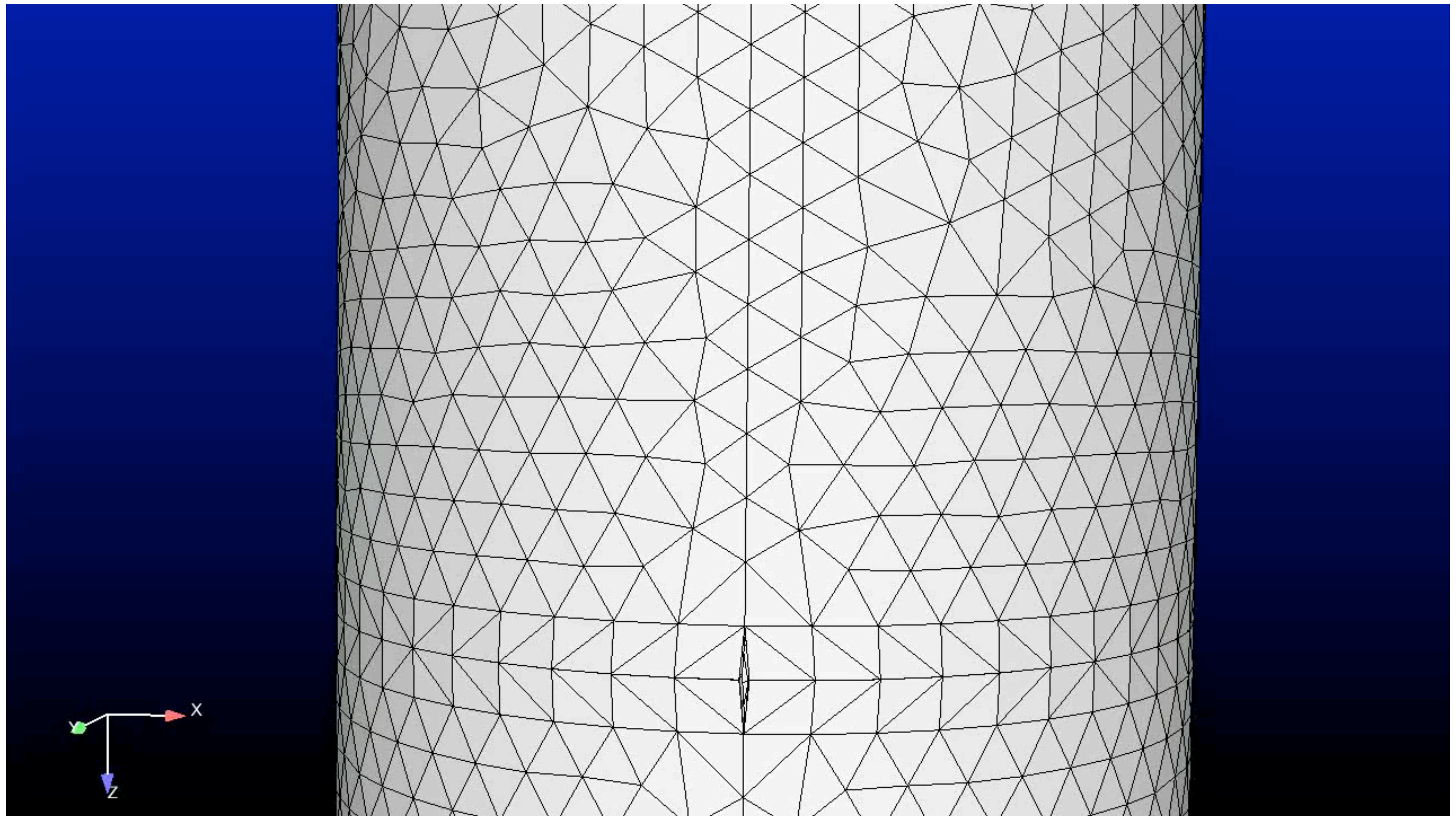



Pristine

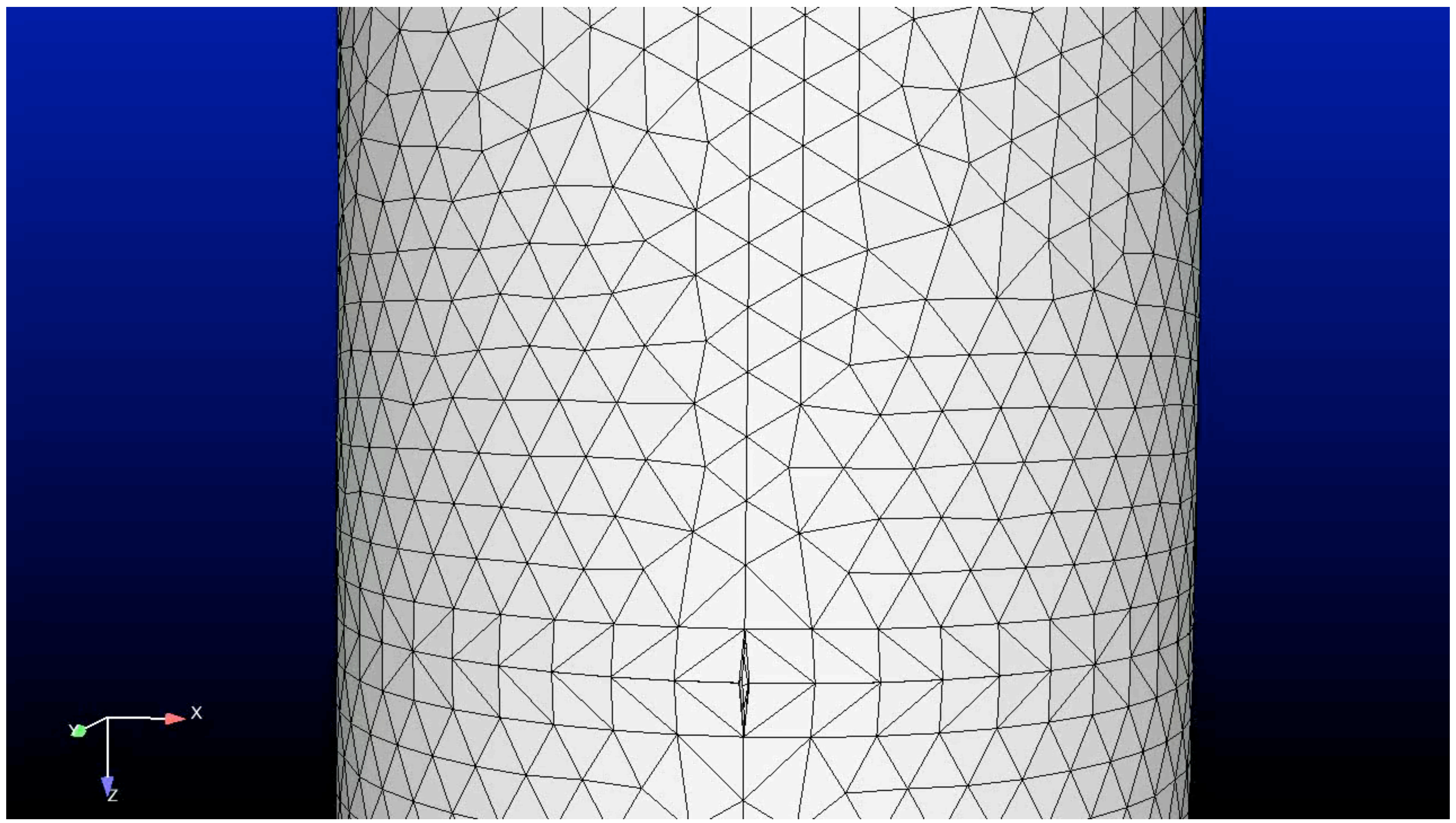



With our new ALE approach

In detail, the effect of *geometrical* elements combined with *structural* elements




Pristine




With our new ALE approach

# Conclusions

- We have alternative approaches to model fracture in a large variety of situations which is based on simple ideas carefully implemented and tested. No enrichment or enhancement approaches are adopted.
- Return mapping techniques are avoided for elasto-plasticity integration.
- Our shell element has been the *best we tested in 14 years of research.*
- A simple Godunov-based ALE approach results very effective in all tests we performed so far.
- The geometrical elements ensure the mesh has a good quality, regardless of the number of cracks.
- For fully 3D problems with *multiple cracks* our tests indicate that a FULL remeshing may be less error prone than tip remeshing.
- With software like ACEGEN, the developer can concentrate on ideas instead of lengthy calculations

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