Chapter 116
Masses and Structure of Heavy Quarkonia and Heavy-Light Mesons in a Relativistic Quark Model

Alfred Stadler, Sofia Leitão, M. T. Peña and Elmar P. Biernat

Abstract Using the framework of the Covariant Spectator Theory, we calculated the masses and vertex functions of heavy and heavy-light mesons, described as quark-antiquark bound states. Our interaction kernel consists of an adjustable mixture of Lorentz scalar, pseudoscalar, and vector linear confining interactions, together with a one-gluon-exchange kernel. We performed a series of fits to the heavy and heavy-light meson spectrum, and we discuss what conclusions can be drawn from it, especially about the Lorentz structure of the kernel.

116.1 Theoretical Framework

We present results of calculations of the mass spectrum and structure of heavy and heavy-light mesons in the framework of the Covariant Spectator Theory (CST) [1, 2]. Among the features that distinguish our CST treatment of $q\bar{q}$ bound states from other covariant approaches, such as Lattice QCD (e.g., [3–6]) and Dyson-Schwinger-Bethe-Salpeter equations [7–12], the more prominent are that we implement confinement through an effective linear confining interaction, and that we work in the physical Minkowski space. An advantage is that we can calculate highly excited meson states rather easily, which is not the case for the Euclidean approaches mentioned above.

The so-called one-channel covariant spectator equation (1CSE) for $q\bar{q}$ bound states is represented graphically in Fig. 116.1. It is obtained from the Bethe-Salpeter equation (BSE) by carrying out the integration over the energy component of the internal loop momentum in the complex plane and using Cauchy's integral formula, but keeping only the residue of the positive-energy propagator pole of the quark (in

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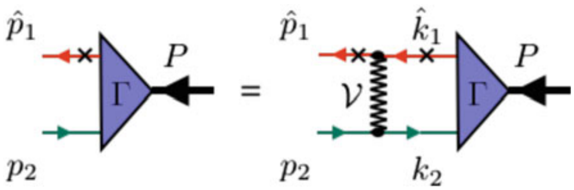

Fig. 116.1 The one-channel spectator equation (1CSE) for the bound-state vertex function Γ of a quark (particle 1) and an antiquark (particle 2), interacting through a kernel $\mathcal{V}$. On-shell four-momenta are characterized through a " ˆ ", and the corresponding lines in the diagram are marked with a "×"

our convention the mass of the antiquark is lower or equal to the mass of the quark). Neglecting the residues of the kernel's poles can lead to faster convergence to the full BS result—with all ladder and crossed ladder diagrams—than the BSE in ladder approximation, because these residues have a tendency to cancel [1].

The 1CSE has the correct one-body limit: when one quark becomes infinitely heavy, the 1CSE becomes an effective one-body (Dirac) equation for the lighter quark. It also has a smooth nonrelativistic limit. The more complicated full four-channel CST equation (4CSE) includes all quark propagator poles, but the 1CSE is a very good approximation for heavy and heavy-light systems. However, in contrast to the 4CSE it is not charge-conjugation symmetric, which means that we cannot assign a C-parity to its solutions for heavy quarkonia. On the other hand, the mass difference between axialvector $C = +$ and $C = -$ pairs is only 5–6 MeV in bottomonium and 14 MeV in charmonium, so this disadvantage is of no practical importance for our purposes.

The dynamic of the $q\bar{q}$ pair is determined by the interaction kernel $\mathcal{V}$. We adopt a kernel consisting of a covariant generalization of the linear (L) confining potential used in [13], a one-gluon exchange (OGE), and a covariantized constant (C) interaction,

$$\mathcal{V} = \left[(1-y)\left(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5\right) - y\,\gamma_1^\mu \otimes \gamma_{2\mu}\right] V_\mathrm{L}(p,k) + \gamma_1^\mu \otimes \gamma_{2\mu}\left[V_\mathrm{OGE}(p,k) + V_\mathrm{C}(p,k)\right] , \tag{116.1}$$

where $p = \hat{p}_1 - P/2$ and $k = \hat{k}_1 - P/2$. The explicit expressions for the functions $V_\mathrm{L}(p,k)$, $V_\mathrm{OGE}(p,k)$, and $V_\mathrm{C}(p,k)$ can be found in [14].

Because the Lorentz structure of the confining interaction is not precisely known, we allow for a mixture of scalar-plus-pseudoscalar and vector structure, controlled by a mixing parameter y that we determine by fitting to the data. Although in principle scalar and pseudoscalar interactions break chiral symmetry, we have shown in [15] that our equal-weight scalar and pseudoscalar linear confining interaction satisfies the axial-vector Ward-Takahashi identity.

Whatever value of y is used, the signs in (116.1) make sure that the nonrelativistic limit of the kernel always yields the same Fourier transform of the Cornell potential, $V(r) = \sigma r - \alpha_s/r - C$. The three coupling strengths, σ, α_s, and C, and in some cases the quark masses, are free parameters of our models. We use Pauli-Villars regularization for the linear and the OGE kernels. The corresponding cut-off parameter Λ is not fitted, but scaled with the heavy quark mass, in the form $\Lambda = 2m_1$.

The 1CSE for the CST vertex function with a kernel of the general form $\mathcal{V} \equiv \sum_K V_K(p,k)\Theta_1^K \otimes \Theta_2^K$, where in our case $\Theta_i^K = \mathbf{1}_i, \gamma_i^5$, or γ_i^μ, is

$$\Gamma(\hat{p}_1, p_2) = -\int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} \sum_K V_K(p,k)\Theta_1^K \frac{m_1 + \hat{\not{k}}_1}{2m_1} \Gamma(\hat{k}_1, k_2) \frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} \Theta_2^K . \tag{116.2}$$

We then reformulate it as an equation for relativistic CST wave functions, which are expanded in a set of basis functions with definite orbital angular momentum and total quark-antiquark spin. We solve the equation for the wave functions numerically by applying techniques developed in [13].

116.2 Numerical Results and Conclusions

Our *global* model parameters (Table 116.1), i.e., taken equal for *all* described mesons, were determined through least square fits to different sets of experimental masses for $J^P = 0^\pm$ and $1^\pm$ mesons. The smallest set consisted of only 9 pseudoscalar (PS) states, whereas the largest set contained a total of 39 states of all kinds (a detailed list of these data sets can be found in [14]).

The mass spectra for the models of Table 116.1 are shown in Fig. 116.2. Model $\mathrm{M0_{S1}}$ was fitted with a fixed $y = 0$ (no Lorentz vector coupling in the confining kernel) and fixed quark masses. It is remarkable that—although it was fitted to a set of 9 PS states alone—it also predicts the full spectrum of $J^P = 0^\pm$ and $1^\pm$ mesons almost as well as the more extensive fits. The results of $\mathrm{M0_{S1}}$ remain almost the same once the PS component in the confining kernel is turned off (the largest difference is about 40 MeV in PS $c\bar{q}$), confirming the expectation that PS coupling does not play an important role in mesons with heavy quarks.

In the fit of $\mathrm{M1_{S3}}$, the quark masses and y were allowed to vary, yielding $y = 0.20$. This seems to indicate that a 20% contribution of vector coupling in the confining kernel is preferred. However, a systematic variation of y showed that the minimum of the least-square-difference at $y = 0.20$ is very shallow, and fixing y anywhere between 0 and 0.3 produces fits of essentially the same quality. As an example, model $\mathrm{M0_{S3}}$ was fitted under the same conditions as $\mathrm{M1_{S3}}$, except that $y = 0$ was imposed. The resulting rms difference to the data is almost as good as the one of $\mathrm{M1_{S3}}$.

We conclude that our models provide a very good description of the heavy and heavy-light meson masses. However, the mass spectrum alone does not constrain the

Table 116.1 Kernel parameters of the models discussed in the text (we use $m_u = m_d \equiv m_q$). N_{st} is the number of states in the data set used in fitting the model, Δ_{rms} is the rms difference to the data for the largest set. The values in boldface were held fixed. The units for the quark masses, Δ_{rms}, and C are GeV, and σ is in GeV^2

Model	σ	α_s	C	y	m_b	m_c	m_s	m_q	N_{st}	Δ_{rms}
$M0_{S1}$	0.2493	0.3643	0.3491	**0.0000**	**4.892**	**1.600**	**0.4478**	**0.3455**	9	0.037
$M1_{S3}$	0.2022	0.4129	0.2145	0.2002	4.875	1.553	0.3679	0.2493	39	0.030
$M0_{S3}$	0.2058	0.4172	0.2821	**0.0000**	4.917	1.624	0.4616	0.3514	39	0.031

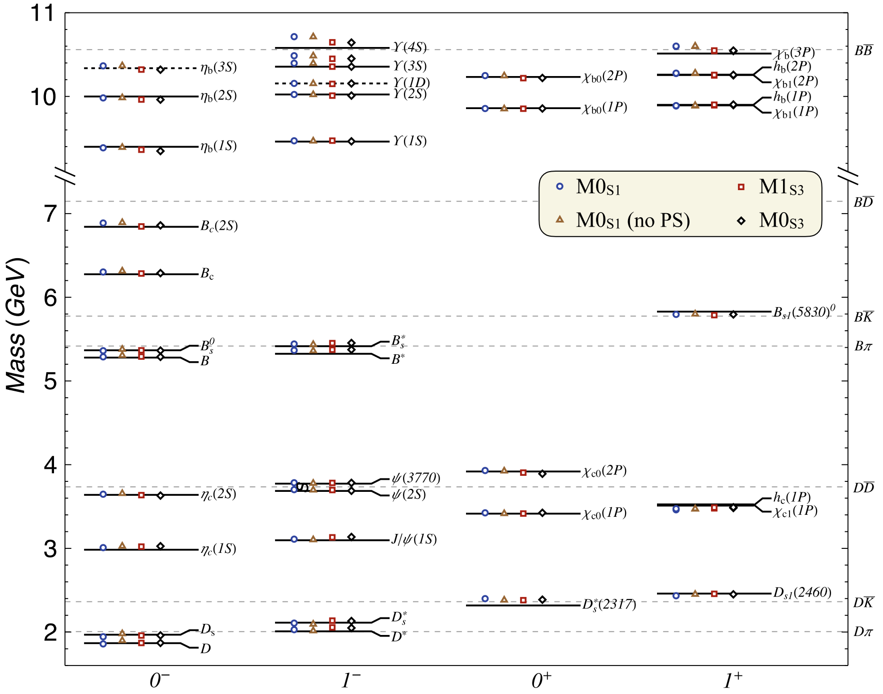

Fig. 116.2 Spectrum of heavy and heavy-light mesons with $J^P = 0^\pm$ and $1^\pm$. The symbols represent results of calculations with models $M0_{S1}$ (circles), $M0_{S1}$ with PS coupling turned off (triangles), $M1_{S3}$ (squares), and $M0_{S3}$ (diamonds). The solid lines are experimental data from the PDG [16]

Lorentz mixing parameter y very much, and we have to look for other observables in order to obtain more detailed information on the Lorentz structure of the confining interaction. In first exploratory calculations of quarkonia decay constants we found strong sensitivity to details of the wave functions, but a systematic study of the y dependence has not yet been done.

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