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Allometric, Growth, and Biomass Estimation Models for *Acacia dealbata* Link.: A Case Study in Serra da Estrela Natural Park (Portugal)

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Abstract: Allometric, growth, and biomass estimation models for the quantification of missing parameters in trees are widely used to estimate the productivity of a stand. However, regarding species with lower economic interest, or exotic invasive species, the creation of these tools did not occur in the same way as for other species, lacking the development of relationships to allow a deeper study of these species. Thus, data were collected in a settlement of *Acacia dealbata* Link. in Portugal, in an area known for the current infestation of this species. After a bibliographic review, some of the identified models were tested to select those who best fit the characteristics of this stand, which were used to develop relationships that, based on the data collected, would be able to estimate parameters, such as height, volume, or mass. It was found that the models established, at least for the initial 20 years of the life of the trees, followed close to a linear model. However, the model still presents some weaknesses. For being considered an invasive species in Portugal, *A. dealbata* is frequently controlled, and for this reason it is very difficult to find trees older than 20 years.

Keywords: *Acacia dealbata* Link.; allometric models; growth models; biomass estimation models; invasive species

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1. Introduction

Acacia dealbata Link. is one of the most aggressive invasive tree species worldwide [1]. This species takes advantage of disturbances occurring in ecosystems, competing very effectively with native and other exotic species [2]. For this reason, knowing the parameters that conditioned the dispersion of this invasive species is essential to define the best strategies for its control and eradication [3]. However, despite the existence of studies carried out on growth models, allometric models, or biomass estimation models for many species, very few references addressed the topic for *A. dealbata* [4].

Ecosystems, as described by Levin, are prototypical examples of complex adaptive systems, where patterns at higher levels emerge from localized interactions and selection processes acting at lower levels, which can be verified through the nonlinearity of these systems [5]. This nonlinearity leads to the analysis of historical data and the multiple possible outcomes of its dynamics. Modeling biological parameters presented difficulties precisely caused by the nonlinearity of the processes involved, and by the variables influencing these processes, which also present high correlation factors among them [6]. The processes associated with forest ecosystems, similar to other terrestrial ecosystems, are complex and multivariate, and aroused the interest of researchers regarding the development of representative models of the reality of the system [7]; firstly, because they enable an understanding of the development of relationships and processes occurring within and outside of the forest ecosystem [8]. Secondly, the interest in developing models is to predict, for example, the productivity of a forest supplying raw materials or services [9]. For this reason, Landsberg pointed out the need for the models to combine the predictive capacity and flexibility of process-based models, with the empirical information and descriptive accuracy of conventional measurement-based models [10]. However, as mentioned by Bataglia and Sands, the incorporation of process-based forest productivity models into forest management systems is not easy. There is an awareness that the use of more detailed process-based models can play an important role in validating simpler models, in the development of generalizations applicable over long time scales, and for testing hypotheses about how trees react to stresses and external interactions [11].

Models relating parameters, such as diameter and height, were widely used to describe the growth of trees and the development of stands of diverse species. However, regarding *A. dealbata*, the available bibliography majorly addresses the perspective of territorial dispersion and its direct competition with native species, focusing on territories where *A. dealbata* is considered an exotic and invasive species, as is the case of Portugal. One example was presented by Raposo et al., where the authors presented a mathematical model to quantify the invasiveness rate of the species [12]. However, perhaps the first reference concerning regression equations that can be used as non-destructive methods to calculate total tree dry weight are the equations presented by Senalwa and Sims [13]. As the authors pointed out, these equations were developed for five eucalyptus species, but they may be suitable to other tree species, such as *Pinus radiata* D. Don or *A. dealbata*. Additionally, Medhurst et al. made a similar assumption, associating *A. dealbata* with *E. nitens* [14]. More recently, Ríos-Saucedo et al. presented allometric models for estimating aboveground biomasses in tree sprouts of dendroenergetic crops, including *A. dealbata* [15]. The authors obtained a very accurate prediction of aboveground biomass sprouts. These results were obtained in planted stands for bioenergy, with all trees having similar ages, and with the allometric parameters showing little variability. Thus, the development of a specific model for *A. dealbata*, capable of evaluating stands composed of diverse and heterogeneous specimens, can be very useful.

Modeling the growth rate of a given species allows for testing hypotheses and carrying out virtual experiments that could otherwise take years in natural field conditions [16]. The results obtained by Fourcaud et al. allow the visualization of growth simulations to directly see the outcome of a given model in the current scenario of climate change [17]. The dispersion of an invasive species benefits from intense and frequent disturbances that increase dispersal capacity since they are more resilient than native species, which are less agile reacting to new conditions [18]. Knowing growth evolution on a tree or stand level allows well-founded decisions, regarding which, forest management models should be implemented, concerning strategies to control and eradicate invasive species such as *A. dealbata* [19]. Although there are several research articles on growth, allometric, and biomass quantification estimation models in the literature on the most diverse species, specifically for *A. dealbata*, there are no works on these topics. Thus, this article should be one of the first approaches to the subject, specifically for this species. In the specific case presented in Portugal, this species is perhaps the most important and impacting invasive forest species. The knowledge of its potential for dispersion and growth is important, as it serves as a starting point for studying and planning the best and most effective control actions for this species. In this way, the main contribution that this article brings is the fact that it is the first approach to the subject, using as a case study a region included in a protected area that was devastated by the occurrence of several rural fires, which contributed to the dispersion of this species, which, as it is a pyrophyte, directly benefits from fire. From the analysis of works carried out for other species, what can be seen is that the various authors always refer to the application of models created for a specific area. Subsequently, other authors precisely analyze the differences between the initial studies and the results obtained in the new area under analysis, validating, or not, its applicability outside the area where it was developed. Thus, the objective of this work is to present growth, allometric, and biomass estimation models for the species *A. dealbata* based on sampling carried out in a stand located in Portugal.

2. Materials and Methods

2.1. Location of the Sampling Area

To carry out this study, the União de Freguesias de Cabeça e Vide (Seia, Portugal) territory was selected, since there is available abundant biogeophysical information at this location, and all *A. dealbata* stands were identified and delimited [12,20–23]. Figure 1 shows the location of the sampling area, and Figure 2 shows the sampling and measurement operations on site.

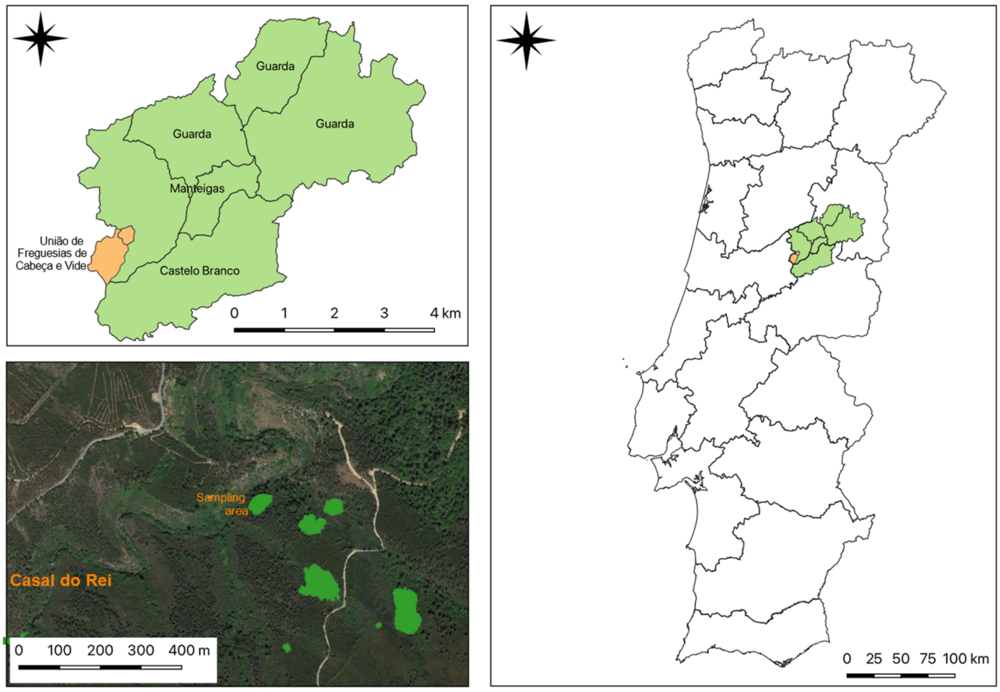


Figure 1. Location of the study and sampling area.

In the stand were marked two circles with a radius of 15 m (approximately 710 m^2 each), resulting in 82 trees of three species (one specimen of *Pinus pinaster*, two specimens of *Arbutus unedo*, and 79 specimens of *A. dealbata*). The specimens of *A. unedo* and *P. pinaster* were excluded from this analysis. From the 79 specimens of *A. dealbata*, 20 were selected and completely measured. In the remaining 59 trees, only DBHs were measured. For all trees, its position in the circles was registered, as presented in Figure 3 and in Table S1 of the Supplementary Materials. A similar procedure was presented by Peng et al., who applied the generated model in specimens belonging to the same population [24].

Figure 2. Collection of samples on site. (**a**) Cutting of trees; (**b**) separation of the different parts (trunk and branches/foliage); (**c**) weighing the different parts.

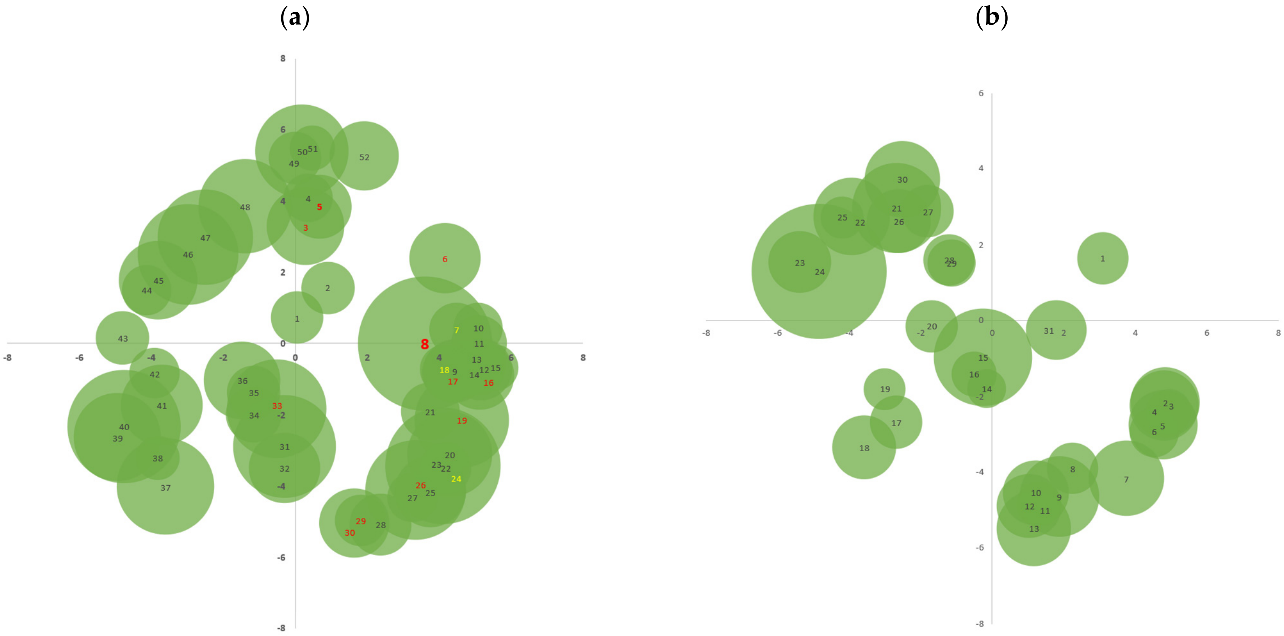


Figure 3. Distribution of the trees in the areas defined to select and collect the samples. (**a**) Parcel 1; and (**b**) Parcel 2.

2.2. Model Selection

2.2.1. Allometric Models

The allometric function is the relationship between the height of the tree and the diameter at breast height (DBH) [25]. The importance of this relationship is greater when there is a need to find expeditious methods for determining the height of trees in situations where obtaining this parameter becomes difficult [26]. The determination of hypsometric curves, which are usually established for a particular species and for a particular location, results from the combination of pairs of values, (d; h), which are obtained from a sample population, and can later be used in the determination of the volume of trees of a given species, and for which only the DBH is known [27,28]. Depending on the species, the height–diameter relationship may follow a sigmoidal type of evolution, with an inflection point occurring at the end of the data projection.

Despite the extensive research carried out on the establishment of allometric models for many species, there are no known models applied to *A. dealbata*, which is a species with potential for applications as an energy crop, or, depending on the region where it is found, can be considered an invasive species needing frequent control actions. The estimation of *A. dealbata* growth allows, in the case of its use and exploitation as a short rotation coppice for bioenergy, the optimization of the associated logistics and a better scheduling of the management operations. Regarding the case when a species presents an invasive behavior, predicting its growth is very important, as it allows for the estimation of the evolution of populations, and their impacts on ecosystems. Moreover, by estimating the quantities of biomass, it may potentiate the creation of value chains to enhance the sustainability of the control and eradication actions.

Pretzsch stated that the biological variability observed in ecosystems makes it difficult to establish strict functional laws, as happens with physical and mathematical processes, since in the case of biological systems, stochastic relationships are more appliable [29]. The establishment of growth relationships, at both the tree and stand levels, together with the biological variability, are of great interest for the analysis of forest growth. Levin et al. stated that the prediction and understanding of the mechanisms beyond the evolution of ecosystems can be based on observed patterns, since those present a unique behavior in any dimension or scale, and will always have their origin in unique causes, leading to unique biological consequences [30]. The authors concluded that the observation of biological phenomena lead to the study of how patterns change, showing a certain variability depending on the scale of description, which influences the development of laws for simplification, aggregation, and scaling. The models were chosen among those described by Schmidt, Lappi, Peng et al., and Pretzsch (Table 1) [24,29,31,32].

Table 1. Selected height-diameter relationships, where h—height; d—diameter; a, b and c—nonlinear coefficients, as described by Schmidt, Lappi, Peng et al. and Pretzsch (adapted from [24,29,31,32]).

Model	Equation	
Assmann (1943)	$h = a + b \times d + c \times d^2$	(1)
Prodan (1951)	$h = 1.3 + \frac{d^2}{a + b \times d + c \times d^2}$	(2)
Petterson (1955)	$h = 1.3 + \left(\frac{d}{a + b \times d}\right)^3$	(3)
Korsun (1935)	$h = e^{\left[a + b \times \ln(d) + c \times (\ln(d))^2\right]}$	(4)
Logaritmic	$h = a + b \times \ln d$	(5)
Freeze (1964)	$h = e^{[a + b \times \ln d + c \times d]}$	(6)
Loetsch et al. (1973)	$h = 1.3 + \left(\frac{d}{a + b \times d}\right)^2$	(7)

2.2.2. Growth Models

As stated by Dale et al., tree growth models project the growth and development of forest ecosystems by increasing the size of each tree used in the simulation for an annual period, or other periods, depending on the objective [33]. These models express changes in size, both of individual trees and the forest itself, and represent the evolution of the diameter (d) and height (h), the dependent variables, as a function of time (t), and can be described by a sigmoidal or concave curve. Depending on the species, this curve can be more or less accentuated, since species such as *A. dealbata* show a very rapid growth at the beginning of its life in order to reach direct access to sunlight as quickly as possible. Thus, in the initial phase, growth tends to be markedly close to a straight line [34]. Zeide, Burkhart and Tomé present a list of growth equations (Table 2) [35,36]. As can be seen, most of the equations present a set of parameters equal to or greater than three. As stated by Zeide, the flexibility of an equation depends on the number of parameters [35]. However, Richard Woollons, in a personal communication referred to by Zeide, states that there are not significant differences between having two or three parameters for fitting the models [35]. Thus were selected equations that use fewer parameters.

Table 2. Growth equations, where h—height, e—Euler's number (approximately 2.71828); t—time; k, b, n, m and c—parameters such as number of samples, constants, and nonlinear coefficients, as described by Zeide, Burkhart and Tomé [35,36].

Model	Equation	
Schumacher	$f(h) = h \times e^{-\frac{k}{t}}$	(8)
Johnson–Schumacher	$f(h) = h \times e^{-\frac{k}{t+b}}$	(9)
Lundqvist–Korf	$f(h) = h \times e^{-k \times \frac{1}{t^n}}$	(10)
Monomolecular	$f(h) = h \times \left(1 - c \times e^{-k \times t}\right)$	(11)
Logistic	$f(h) = \frac{h}{1 + c \times e^{-k \times t}}$	(12)
Gompertz	$f(h) = h \times e^{-c \times e^{-k \times t}}$	(13)
Sloboda	$f(h) = h \times e^{-c \times e^{-k \times t^b}}$	(14)
Hossfeld IV	$f(h) = \frac{t^k}{c + \frac{t^k}{h}}$	(15)
Richards	$f(h) = h \times \left(1 - c \times e^{-k \times t}\right)^{\frac{1}{1-m}}$	(16)
Chapman–Richards	$f(h) = h \times \left(1 - e^{-k \times t}\right)^{c}$	(17)
Bertalanffy	$f(h) = h \times \left(1 - e^{-k \times t}\right)^{3}$	(18)
Weibull	$f(h) = h \times \left(1 - e^{-k \times t^b}\right)$	(19)
Levakovic I	$f(h) = h \times \left(\frac{t^k}{c_1 + t^k}\right)^{c_2}$	(20)
Levakovic II	$f(h) = h \times \left(\frac{t^2}{c_1 + t^2}\right)^{c_2}$	(21)
Korf (or Bailey and Clutter)	$f(h) = h \times e^{-k \times t^{-c}}$	(22)
Yoshida I	$f(h) = h \times \frac{t^k}{c_1 + t^k} + c_2$	(23)

2.2.3. Biomass Estimation

Allometric models characterize the causality of deterministic size relations in and between organisms [37,38]. The principle of similarity, extrapolated by Galileo Galilei from the realm of physics to the study of the form and function of plants, was used to establish the allometric models for the length, volume and quantity of biomass by several authors, as presented in Equation (24) [37,39], and can be rewritten as presented in Equation (25).

$$y = a \times x^b \tag{24}$$

$$w_{biomass} = h \times d^a \tag{25}$$

where $w_{biomass}$ is the biomass weight produced by a tree with total height h and diameter at breast height d, and a corresponds to the allometric exponent.

3. Results and Discussion

3.1. Tree Height–Diameter Relationships

The data collected (see Table S2 of Supplementary Materials) were analyzed using Microsoft® Excel version 16.54 (Redmond, WA, USA), and IBM® SPSS Statistics version 27.0.1.0, 64-bit edition (Armonk, NY, USA). The results are presented in Table 3.

Table 3. Results of the numerical interactions.

Equations	Parameter	Estimate	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	R^2
Assmann (1943)	*a*	4.037	3.276	−2.874	10.948	0.641
	b	1.741	0.490	0.707	2.776	
	c	−0.036	0.015	−0.069	−0.003	
Prodan (1951)	*a*	1.200	1.396	−1.744	4.145	0.648
	b	0.059	0.285	−0.544	0.661	
	c	0.039	0.012	0.014	0.065	
Petterson (1955)	*a*	0.674	0.146	0.367	0.981	0.643
	b	0.321	0.012	0.295	0.348	
Korsun 1935)	*a*	0.427	1.004	−1.691	2.545	0.647
	b	1.647	0.822	−0.087	3.381	
	c	−0.245	0.164	−0.591	0.101	
Logaritmic	*a*	−1.499	3.685	−9.240	6.242	0.630
	b	8.411	1.520	5.218	11.605	
Freeze (1964)	*a*	1.186	0.532	0.064	2.308	0.645
	b	0.901	0.336	0.191	1.610	
	c	−0.035	0.025	−0.087	0.016	
Loetsch et al. (1973)	*a*	0.595	0.048	0.493	0.696	0.640
	b	0.018	0.003	0.011	0.024	

For the model that used the expression proposed by Assmann (1943) were obtained three parameters with values of, respectively, 4.037 for coefficient *a*, 1.741 for coefficient *b*, and −0.036 for coefficient *c*. The standard errors for the coefficients *a*, *b*, and *c* were, respectively, 3.276, 0.490, and 0.015. The lower and upper bounds for a 95% confidence interval, for the coefficients *a*, *b*, and *c*, were, respectively, −2.874 and 10.948, 0.707 and 2.776, and −0.069 and −0.003. Since the coefficient *a* has a negative lower bound and a positive upper bound, it includes the value Ø, which is why it is statistically non-significant. The lower and upper bounds of coefficients *b* and *c* are, respectively, both positive and both negative, not including the Ø value, so they are statistically significant. For the model that used the expression proposed by Prodan (1951) were obtained three parameters with values of, respectively, 1.200 for coefficient *a*, 0.059 for coefficient *b*, and 0.039 for coefficient *c*. The standard errors for the coefficients *a*, *b*, and *c* were, respectively, 1.396, 0.285, and 0.012. The lower and upper bounds for a 95% confidence interval, for the coefficients *a*, *b*, and *c*, were, respectively, −1.744 and 4.145, −0.544 and 0.661, and 0.014 and 0.065. Since the coefficients *a* and *b* have a negative lower bound and a positive upper bound, it means that the value Ø is included, which is why they are statistically non-significant. The lower and upper bounds of coefficient *c* are both positive, not including the Ø value, so it is statistically significant. For the model that used the expression proposed by Petterson (1955) were obtained two parameters with values of, respectively, 0.674 for coefficient *a* and 0.321 for coefficient *b*. The standard errors for the coefficients *a* and *b* were, respectively, 0.146 and 0.012. The lower and upper bounds for a 95% confidence interval, for the coefficients *a* and *b*, were, respectively, 0.367 and 0.981, and 0.295 and 0.348. The lower and upper bounds of coefficients *a* and *b* are both positives, not including the Ø value, so they are statistically significant. For the model that used the expression proposed by Korsun (1935) were obtained three parameters with values of, respectively, 0.427 for coefficient *a*, 1.647 for coefficient *b*, and −0.245 for coefficient *c*. The standard errors for the coefficients *a*, *b*, and *c* were, respectively, 1.004, 0.822, and −0.245. The lower and upper bounds for a 95% confidence interval, for the coefficients *a*, *b*, and *c*, were, respectively, −1.691 and 2.545, −0.087 and 3.381, and −0.591 and 0.101. Since coefficient *a*, *b*, and *c* have a negative lower bound and a positive upper bound, it means that it includes the value Ø, which is why they are statistically non-significant. For the model that used the Logaritmic equation were obtained two parameters with values of, respectively, −1.499 for coefficient *a* and 8.411 for coefficient *b*. The standard errors for the coefficients *a* and *b* were, respectively, 3.685 and 1.520. The lower and upper bounds, for a 95% confidence interval, for the coefficients *a* and *b*, were, respectively, −9.240 and 6.242, and 5.218 and 11.605. Since the coefficient *a* has a negative lower bound and a positive upper bound, it means that it includes the value zero, which is why it is statistically non-significant. The lower and upper bounds of coefficient *b* are both positive, not including the value zero, so is statistically significant. For the model that used the expression proposed by Freeze (1964) were obtained three parameters with values of, respectively, 1.186 for coefficient *a*, 0.901 for coefficient *b*, and −0.035 for coefficient *c*. The standard errors for the coefficients *a*, *b*, and *c* were, respectively, 0.532, 0.336, and 0.025. The lower and upper bounds, for a 95% confidence interval, for the coefficients *a*, *b*, and *c*, were, respectively, 0.064 and 2.308, 0.191 and 1.610, and −0.087 and 0.016. The lower and upper bounds of coefficients *a* and *b* are both positive, not including the Ø value, so they are statistically significant. Since the coefficient *c* has a negative lower bound and a positive upper bound, it means that it includes the value Ø, which is why it is statistically non-significant. For the model that used the expression proposed by Loetsch et al. (1973) were obtained two parameters with values of, respectively, 0.595 for coefficient *a* and 0.180 for coefficient *b*. The standard errors for the coefficients *a* and *b* were, respectively, 0.048 and 0.003. The lower and upper bounds, for a 95% confidence interval, for the coefficients *a* and *b*, were, respectively, 0.493 and 0.696, and 0.011 and 0.024. The lower and upper bounds of coefficients *a* and *b* are both positive, not including the Ø value, so they are statistically significant. The value of R^2 varied between the minimum value presented by the Logaritmic equation, with 0.630, and the maximum value presented by the Prodan equation (1951), with 0.648.

The projection of the estimated values around the expected normal, the projection of the deviation of the estimated values from the normal, and the error of the estimated values are presented in Figure S1 of the Supplementary Materials. Figure S2 of the Supplementary Materials presents the projections of the predicted values and corresponding residuals to check homoscedasticity. The coefficients generated in the nonlinear regression are presented in Table 4.

Table 4. Equations generated based on the coefficients shown in Table 3.

Model	Equation	
Assmann (1943)	$h = 4.037 + 1.741 \times d - 0.036 \times d^2$	(26)
Prodan (1951)	$h = 1.3 + \frac{d^2}{1.2 + 0.059 \times d + 0.039 \times d^2}$	(27)
Petterson (1955)	$h = 1.3 + \left(\frac{d}{0.674 + 0.321 \times d}\right)^3$	(28)
Korsun (1935)	$h = e^{[0.427 + 1.647 \times \ln(d) - 0.245 \times (\ln(d))^2]}$	(29)
Logarithmic	$h = 1.0 - 0.971 \times \ln d$	(30)
Freeze (1964)	$h = e^{[1.186 + 0.901 \times \ln d - 0.035 \times d]}$	(31)
Loetsch et al. (1973)	$h = 1.3 + \left(\frac{d}{0.595 + 0.180 \times d}\right)^2$	(32)

Note: *d* is the diameter at breast height (DBH); *e* is the Euler's number; *h* is the height of the tree.

It was found that five of the seven equations have coefficients with lower and upper bounds presenting negative and positive results simultaneously. This allows us to conclude that, in these situations, the value zero can be chosen. So, these coefficients are considered as being statistically non-significant. Thus, the equations from Assmann (1943), Prodan (1951), Korsan (1935), and Logarithmic and Freeze (1973) are excluded, since they present coefficients that are statistically non-significant. On the other hand, in the equations presented by Petterson (1955) and Loetsch et al. (1973), the coefficients have lower and upper bounds, both negative or both positive, for a confidence interval of 95%, indicating they do not include the zero in any situation. For this reason, the coefficients obtained from the models by Petterson (1955) and Loetsch et al. (1973) can be considered statistically significant. Regarding standard errors, these two models also present significant differences compared to the others, as they present standard errors that are lower than those for the other equations. In fact, the equation by Petterson (1955) has a value of 0.146 for the standard error of coefficient a, and 0.012 for the standard error of coefficient b, whereas the equation presented by Loetsch et al. (1973) presents a value of 0.048 for the standard error of coefficient a, and 0.003 for the standard error of coefficient b. The R^2 values for the seven models presented values between 0.630 for the Logarithmic model, and 0.648 for the Prodan model (1951). These R^2 values mean that the models explain 63.0% and 64.8% of the total variation in tree height, respectively. The Assmann (1943), Prodan (1951), Korsun (1935), logarithmic, and Freeze (1964) models, despite presenting interesting R^2 values, with even the model of Prodan (1951) presenting the value of a higher R^2 at 0.648, should not be considered for the reasons mentioned above. Thus, the values of R^2 for the model of Petterson (1955), which was 0.643, means that this model explains 64.3% of the total variation in tree height, whereas the model by Loetsch et al. (1973), which presented an R^2 value of 0.640, means that this model explains 64.0% of the total variation in tree height. Regarding homoscedasticity, all models presented a good dispersion of data, constituted by the standardized predicted values and the standardized residual values. However, there is a slight asymmetry, as all models have a distribution of standardized residual values mostly below the origin line. Concretely, of the 20 projected values, 12 are below this line, whereas 8 are above it. This distribution points to a slight asymmetry of the models. The models can be considered as being well-adjusted, as all, without exception, present the residuals randomly dispersed around zero, showing a constant variance, with the data obtained by measurement concentrated between −2 and 2.

3.2. Tree Growth Models

The cut samples were weighed and measured. Afterwards, they were sliced every 50 cm, and growth rings counted and the annual increment measured, as shown in Table S3 of the Supplementary Materials and in Figure 4.

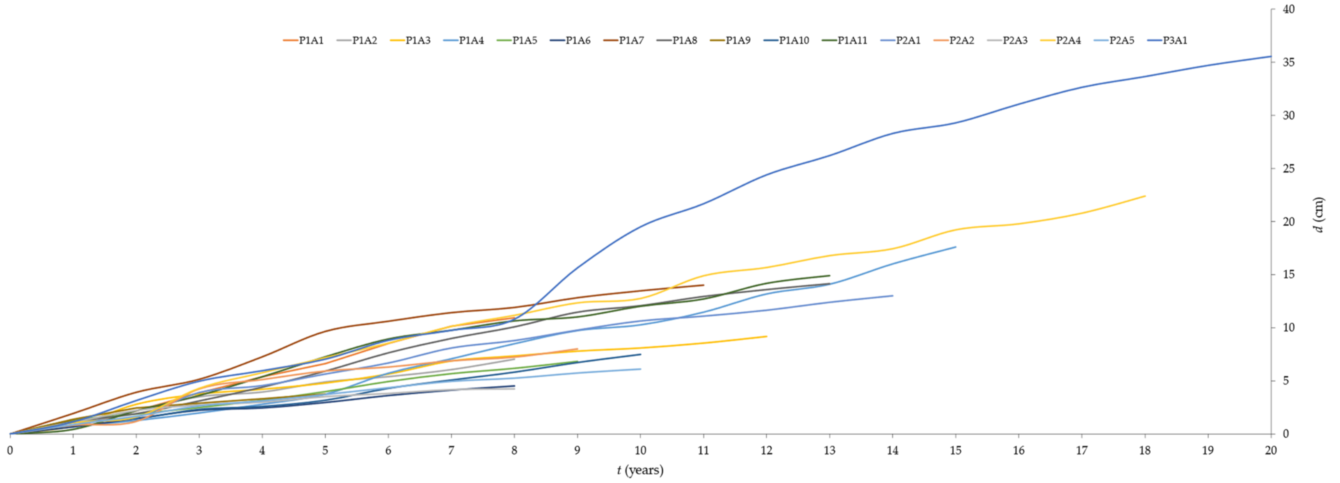


Figure 4. Relationship $d = f(t)$ for the trees used in the study.

As can be seen, the trees present a fast initial growth rate, which is confirmed by the linearity of the increase in diameter. Subsequently, available data were increased by being converted into cumulative data corresponding to each of the growth years, as presented in Table S4 of the Supplementary Materials. These data were used in the numerical iterations to determine the parameters of the Schumacher and Bertalanffy equations. Data configurations were allowed to transform the 17 initial samples into 191, giving greater significance to the results from a statistical point of view. Then, the models were used to estimate the parameters for the Schumacher and Bertalanffy equations (Table 5).

Table 5. Results from the numerical interactions using the software IBM® SPSS Statistics.

Model	Parameter	Estimate	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	R^2
Schumacher	a	13.778	0.276	13.233	14.324	0.967
	k	10.454	0.204	10.051	10.858	
Bertalanffy	a	9.331	0.170	8.995	9.667	0.971
	k	0.162	0.003	0.156	0.167	

The results obtained for the Schumacher equation lead to a value of 13.776 for the parameter a, which corresponds to the growth asymptote of *A. dealbata*, of 13.778, with a standard error of 0.276, framed in a lower bound of 13.233 and an upper bound of 14.324. The parameter k, which corresponds to the allometric exponent, presented a value of 10.454, with a standard error of 0.204, framed in a power bound of 10.051 and an upper bound of 10.858. For the Bertalanffy equation, a value for the parameter a of 9331 was obtained, with a standard error of 0.170, framed in a lower bound of 8.995 and an upper bound of 9.,667. The parameter k was obtained with a value of 0.162, with a standard error of 0.003, framed in a lower bound of 0.156 and an upper bound of 0.167. The values obtained for R^2 were, respectively, for the Schumacher and Bertalanffy equations, 0.967 and 0.971.

With the data presented in Table S4 of the Supplementary Materials and the parameters presented in Table 6, it was possible to formulate the equations shown in Table 6. In a first analysis and looking only at the parameter a referring to the asymptote, the Schumacher equation presents a value of 13.778, which corresponds to the maximum height from which tree growth stops, although tree development continues (for example, increasing trunk diameter). This value, about 3.5 m higher than what is presented for the equation formulated using the model proposed by Bertalanffy, seems to be more in line with the values observed in the measurements performed, as all observed values are above this value, except for two of the measured trees.

Table 6. Equations generated based on the coefficients shown in Table 5, where h—height; e—Euler's number; t—time.

Model	Equation	
Schumacher	$h = 13.778 \times e^{-\frac{10.454}{t}}$	(33)
Bertalanffy	$h = 9.331 \times \left(1 - e^{-0.162 \times t}\right)^3$	(34)

The obtained R^2 values indicate that the calculated parameters influence 96.7% and 97.1% the determination of the height, respectively, for the Scumacher anad Bertalaffy equations.

3.3. Biomass Weight Estimation

Table 7 presents the results obtained using Equation (26).

Table 7. Results obtained in the numerical interactions.

Parameter	Estimate	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	R^2
h	0.823	0.035	0.749	0.897	0.754

The allometric exponent a presented a value of 0.823, with a standard error of 0.035, framed in a lower bound of 0.749 and in an upper bound of 0.897, with an R^2 of 0.754. The predicted results obtained showed a mean value of 157.86 kg, with a standard error of 28.60, with a lower bound of 96.89 and an upper bound of 218.82, for a 95% confidence interval. The calculated standard deviation was 114.40. The minimum value verified was 33.83, and the maximum value was 413.15. Based on the coefficients generated in the nonlinear regression iterations for the model selected, it was possible to build the equation presented in Table 8.

Table 8. Equation generated based on the coefficients shown in Table 7, where $w_{biomass}$ is the biomass quantity calculated; *h*—height; *d*—diameter.

Model	Equation	
Biomass estimation model	$w_{biomass} = h \times d^{0.823}$	(35)

The Q–Q normal plot of predicted values is presented in Figure 5a. Figure 5b shows the projection of the standardized predicted values and the standardized residual values.

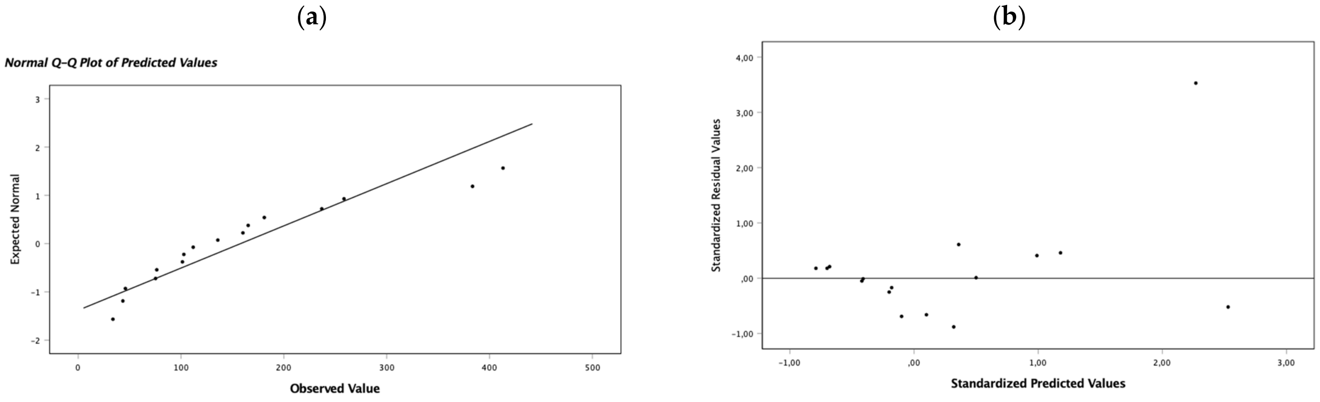


Figure 5. (**a**) Normal Q–Q plot of the predicted values; and (**b**) standardized predicted values and standardized residual values.

As can be seen in Figure 5a, the observed values are arranged in a very regular way along the line that represents the expected normal, indicating a tendency to overlap the line and a reduced variance. This tendency to overlap should be greater with a greater number of data, but with the number of data currently available, it is already possible to verify a good fit of the model. On the other hand, in the graph shown in Figure 5b, from the projection of the standardized predicted values and the standardized residual values, the values are found to be between −2 and 2, with only two points to be placed in the interval between 2 and 3. This distribution indicates a good fit of the model.

4. Performance Analysis of the Models

4.1. Height–Diameter Relationship

For the performance analysis of the models, Petterson (1955) and Loetsch et al. (1973) equations were applied to the DBH data collected by direct measurement from the 59 trees that were part of the two transepts delimited in the study area, and which were not used in the construction of the mathematical models. The results of applying the two equations are presented in Table S5 of the Supplementary Materials. The normality of the results was evaluated as shown in Figure 6.

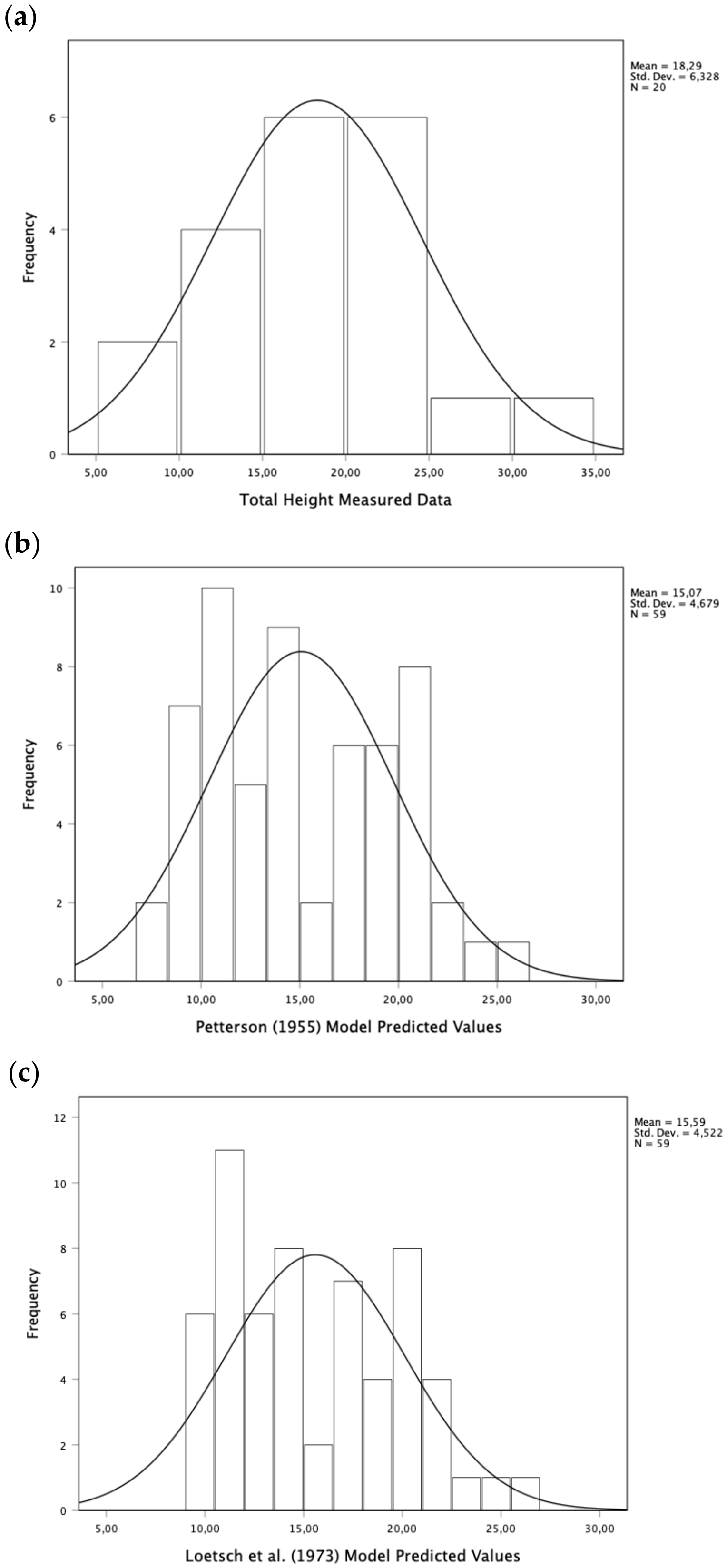


Figure 6. Histograms corresponding to the distribution of data (**a**) obtained by directly measuring the trees used in the construction of the models; (**b**) obtained by applying the model of Petterson (1955); and (**c**) obtained by applying the model by Loetsch et al. (1973).

As can be seen from the analysis of the histogram shown in Figure 6a, the data tend towards the normal distribution, as shown by the application of the Kolmogorov–Smirnov test (since $n < 30$), which presented a $p > 0.05$, and validated the H_0 hypothesis for this reason. That is, the results point to a normal distribution. On the other hand, in the histograms shown in Figure 6b,c, it is possible to observe some deviations from the normal distribution, which are also confirmed with the application of the Shapiro–Wilk test (for $n > 30$) that presented $p < 0.05$ for both situations, leading to the choice of the alternative hypothesis, H_1, which points to the fact that the distribution is not normal. This non-normality of the data obtained by the application of the models may be related to the fact that the sample used for the application of the models, given the scarcity of some of the diameter classes, may affect the type of distribution verified in the data. When analyzing the projection of the observed values and their relationship with the expected normal, which is shown in Figure 7, it is verified that the observed values overlap very well with the expected normal line, except for the values corresponding to the largest diameters. This situation occurs, once again, probably due to the lack of data on these diameter classes, but as mentioned above, it is very difficult to find specimens older than 20 years.

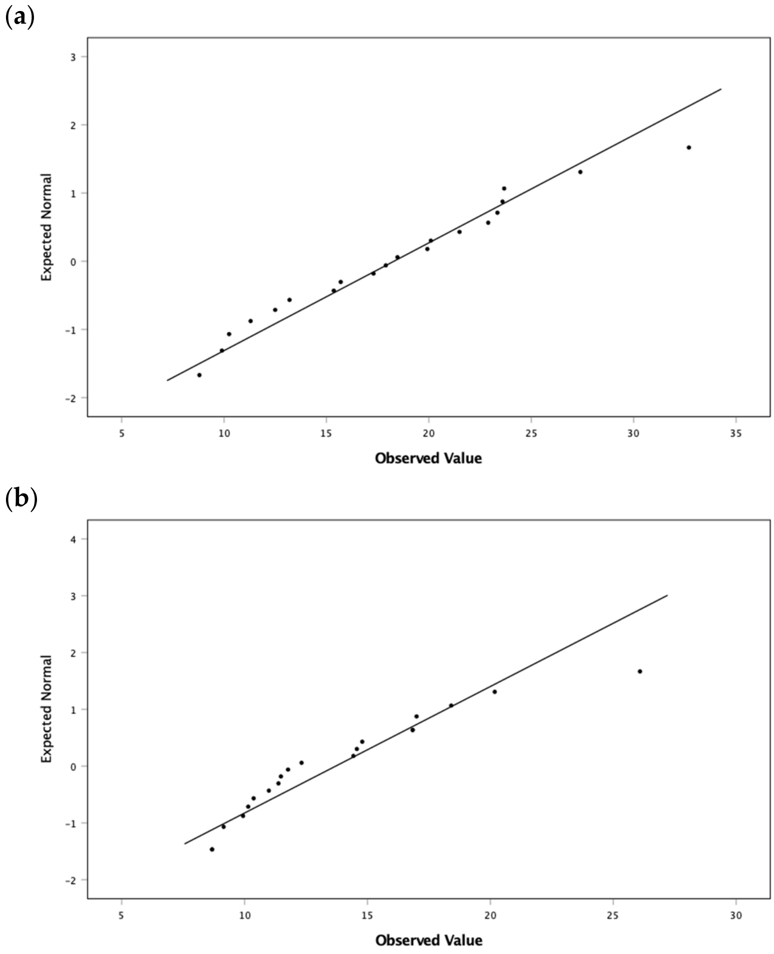


Figure 7. *Cont.*

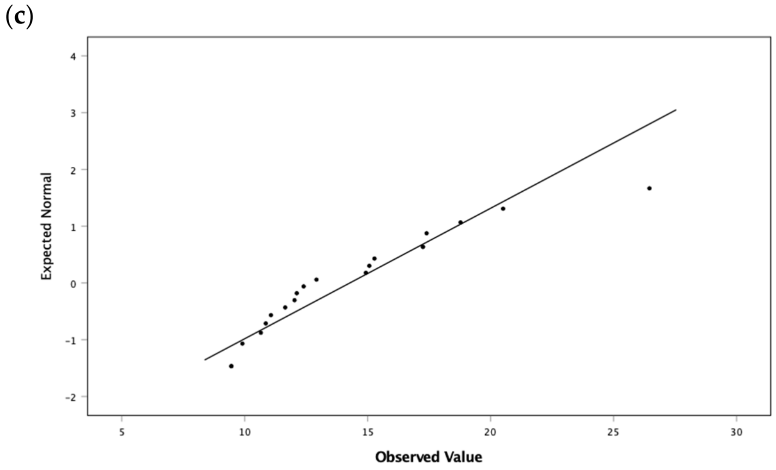


Figure 7. Normal Q–Q plot of (**a**) the total height of the measured data; (**b**) Petterson's (1955) predicted value; e (**c**) Loetsch et al.'s (1973) predicted value.

The comparison of the descriptive parameters is shown in Table 9.

Table 9. Comparison of the descriptive parameters calculated for each of the datasets.

		Total Height Measured Data	Petterson (1955) Predicted Values	Loetsch et al. (1973) Predicted Values
Mean		18.29	13.70	14.27
Standard deviation		6.33	4.49	4.35
n		20	59	59
Confidence interval (95%)	Lower bound	15.33	11.60	12.24
	Upper bound	21.23	15.80	16.31
Variance		40.05	20.16	18.92

From the direct comparison of the data, it appears that those obtained by applying the model generated from Loetsch et al. (1973) are the closest to the parameters obtained by directly measuring the sampled trees used to build the model. As can be seen from the data presented for the total height of the measured data, the variance is much higher than in the other datasets, which is most likely related to the number of samples used, and the type of *A. dealbata* stand used as reference. The variance of the remaining datasets was significantly lower, reaching its minimum value in the dataset referring to the predicted values using the Loetsch et al. (1973) equation. This group also presented a smaller standard deviation, as well as a smaller difference between the lower and upper bounds for a confidence interval of 95%, with a value of 4.07, whereas the dataset referring to the predicted values using the Petterson (1955) equation presented a value of 4.2. Thus, it is understood that the most adjusted model seems to be the equation created from the model presented by Loetsch et al. (1973):

$$h = 1.3 + \left(\frac{d}{0.595 + 0.180 \times d} \right)^2 \tag{36}$$

This model, despite the good adjustment it presents, must be conditioned in its current use to specimens under the age of 20 and their corresponding DBH, since, in the model generation, it was not possible to use specimens in representative quantity to give significance to these classes of diameters. As *A. dealbata* is considered an invasive species in Portugal, stands are frequently controlled, making it difficult to find trees older than 20 years. In fact, among the 79 trees analyzed, only one specimen was 20 years old at the date the samples were cut (in October 2018), and all the other trees were younger.

The ability of *A. dealbata* to grow very quickly, especially in height, as a way of conquering direct access to the sunlight, seems to be in accordance with the growth represented by a curve with a concave format. On the other hand, species that present slower growth, as shown in the studies by Jackson et al., who analyzed the growth of several species of the genus *Pinus* and the genus *Quercus*, found that in species with faster initial growth, the projection curve of the allometric model followed the concave type, whereas in species with slower growth, the allometric model is best represented by a sigmoidal curve [40]. This relationship can also be better verified the greater the number of specimens in the sample is, and, mainly, the longer the available growth period of the trees is. This is because the existence of very old specimens will allow for confirming whether the growth of the trees already reached the asymptote and stabilized, as may be the case that the growth of the trees is in the ascending phase and yet to reach the inflection point towards stabilization. The relationships between the different elements of a stand can also interfere with growth, both positively and negatively, as mentioned by Enquist and Niklas. As such, organizing principles are needed to link organism, community, and ecosystem attributes across spatial and temporal scales [41]. However, the type of very fast growth in the phase immediately after germination seems to be related to, and is more common in, heliophile species, as is the case of *A. dealbata*. However, the development of trees for the first 20 years follows a growth that can be well adjusted to a linear model. In this way, establishing a linear relationship between diameter and height, which is statistically significant, can allow for the quick assessment of one of the most critical characteristics of forest stands, especially when it comes to stands destined to the supply of industrial raw materials. Diameter is one of the properties of trees that determine, for example, their cutting moment or the use of the wood. Equation (39) shows the linear relationship between diameter and height, with $R^2 = 0.807$.

$$d = -1.84 + 2.92 \times h \tag{37}$$

4.2. Growth Model

For the validation of the models, the equations were applied to a time sequence of 50 years to verify the evolution of the height of the trees over this period (Figure 8). The data generated with the descriptive statistical parameters presented in Table 10 are very close to the values verified for the height measured in the collected samples, thus indicating a good fit for the model. The results from the Schumacher equation presented a mean value of 3.27, with a standard deviation of 2.28, a lower bound of 2.95, an upper bound of 3.60, and a variance of 5.18. On the other hand, the results from the Bertalanffy equation presented a mean value of 3.26, with a standard deviation of 2.30, a lower bound of 2.93, an upper bound of 3.59, and a variance of 5.30.

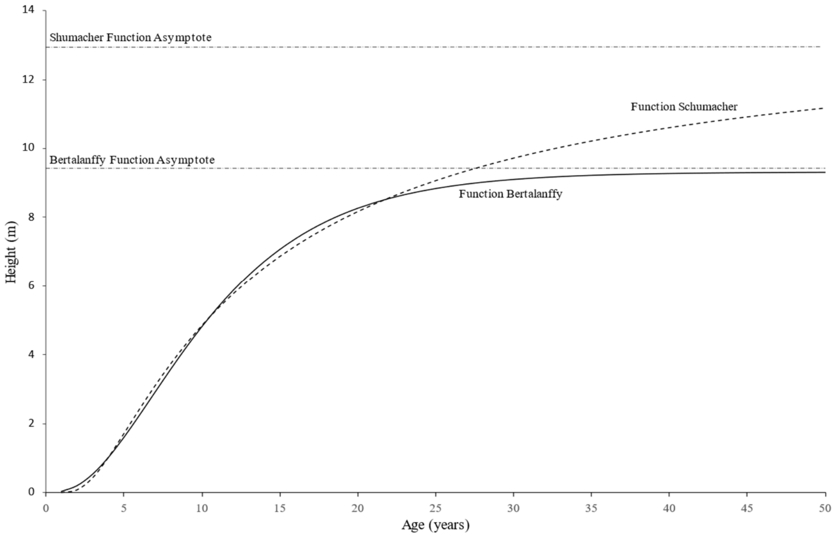


Figure 8. Evolution of height growth models for *A. dealbata* calculated from the Schumacher and the Bertalanffy equations.

Table 10. Comparison of the descriptive parameters calculated for each of the datasets.

		Height Measured Data	Schumacher Predicted Values	Bertalanffy Predicted Values
Mean		3.38	3.27	3.26
Standard deviation		2.15	2.28	2.30
n		191	191	191
Confidence interval (95%)	Lower bound	3.07	2.95	2.93
	Upper bound	3.69	3.60	3.59
Variance		4.62	5.18	5.30

The descriptive statistical parameters presented by the results of the Schumacher equation are closer to the measured height values than the values obtained from the Bertalanffy equation. This observation could, at first sight, suggest a better fit for the Schumacher model compared to the Bertalanffy model. However, from the projection shown in Figure 9, the two models practically overlap in the first 20 years of tree growth, allowing the assumption that both models fit well to the initial growth period. However, it appears that the Schumacher model never reaches the asymptote during the simulation established for the 50-year period, whereas the Bertalanffy model reaches the asymptote and presents a perfect curve. This observation, although based on a relatively small number of data, may indicate a better fit for the Bertallanfy model. However, both models need to be fine-tuned with the inclusion of data obtained from specimens aged over 20 years to confirm the height asymptote.

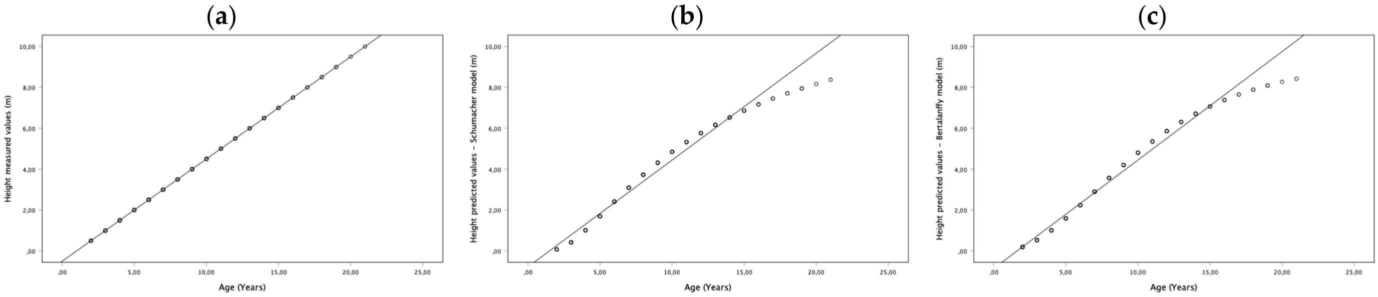


Figure 9. (**a**) Linear model of the projection of height and age determined for the samples used in the study; (**b**) linear model of the predicted height projection determined using the Schumacher model; and (**c**) linear model of the predicted height projection determined using the Bertalanffy model.

As mentioned, the available data do not encourage the extrapolation of the results to analyze growth for periods longer than 20 years. However, the two generated models present an acceptable fit for the initial 20 years of the life of *A. dealbata* trees. As can be seen in Figure 9, the initial growth seems to fit, in both cases, to a linear model.

The following trend adjustment lines resulted from the projection of the values described by the equations presented in Table 11.

Table 11. Linear models for the height versus age projections for the data obtained by measurement and for the data obtained through the application of the Schumacher and Bertalanffy models.

Equation	Model	R^2	
$h = -0.5 + 0.5 \times t$	Measured data	1.00	(38)
$h = -0.779 + 0.522 \times t$	Data from the Schumacher model	0.974	(39)
$h = -0.858 + 0.530 \times t$	Data from the Bertalanffy model	0.981	(40)

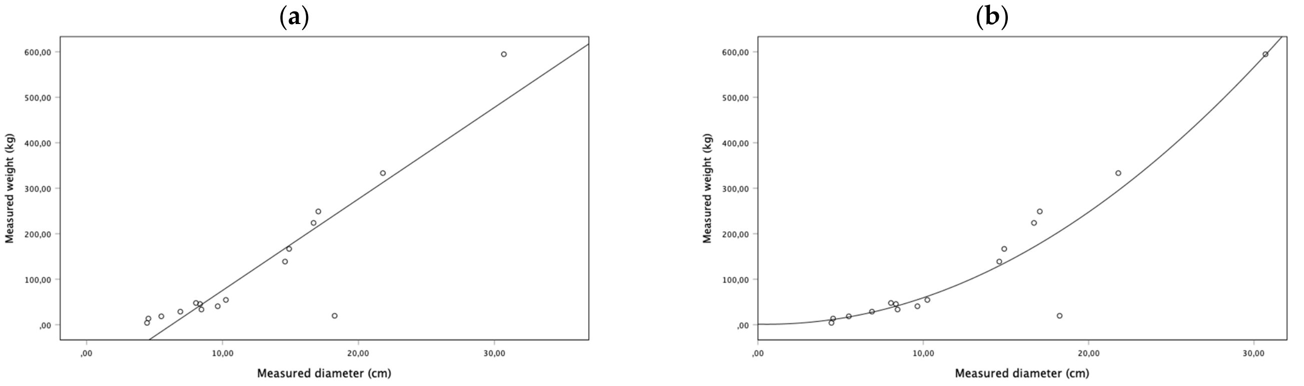
The linear relationships generated from the data obtained using the Schumacher and Bertalanffy models are very close with the linear model generated from the measured values, as can be seen through the values of R^2, which, in the case of the Schumacher model, presenting a value of R^2 = 0.974, and for the case of the Bertalanffy model, which presents a value of R^2 = 0.981. These values are both very close to the R^2 presented by the linear model defined by the measured values. In all situations, the normality tests performed (Kolmogorov–Smirnov test) presented $p > 0.05$, indicating that the data follow a normal distribution.

4.3. Biomass Estimation

For the validation of the formula generated for the biomass quantification, the biomass quantity was estimated based on the data presented in Table S3 of Supplementary Materials, namely, the diameter and the height data. Then, with the results obtained, the estimated dataset was compared with the dataset obtained by weighing the samples collected in the area selected for the study. For this comparison, the non-parametric Wilcoxon test was used, which presented $p > 0.05$. Thus, with this result, hypothesis H_0 is accepted. That is, the two sets of data are identical to each other. As the value of Z generated in the Wilcoxon test was negative, the negative ranks predominate over the positive ranks, so it can also be concluded that the estimated values tend to be lower than the real measured values. This fact must be considered when estimating the amount of biomass of *A. dealbata* using this model. However, a linear relationship was also established between the biomass weight of the collected samples and the diameter of these same samples. It was found that the generated trend line (Equation (41)), despite being close to most of the projected values with an R^2 = 0.819, could be easily improved with the adjustment of a quadratic function (Equation (42)), which presented a result of R^2 = 0.882, as shown in Figure 10.

$$w = -126 + 20.12 \times d \tag{41}$$

$$w = 1.35 - 0.72 \times d + 0.65 \times d^2 \tag{42}$$



(**a**) (**b**)

Figure 10. (**a**) Establishment of a linear relationship between the measured weights of the samples collected and the diameters of these samples; and (**b**) the establishment of a quadratic-type relationship between the measured weights of the collected samples and the diameters of these same samples.

4.4. Model Application

The estimation of biomass is important from the perspective of forest productivity when it comes to species of interest for the supply of raw materials or for bioenergy, but also from a long-term perspective, for example, for estimating the capacity of forest species for carbon capture and sequestration, as a measure to mitigate climate change. The application of the models can be used in the quantification, for example, of income from the sale of wood, whether it is intended to produce sawmill products, or biomass to energy. In places where *A. dealbata* is considered invasive, as is the case of Portugal, the estimation of biomass can be used, firstly, for better organization of control and eradication actions. On the other hand, estimating the amount of biomass allows for better management of productive planted forests. The use of the simplified linear and quadratic models developed seems to indicate the possibility of a use adjusted to stands or trees under 20 years of age. The use of

Equations (37), (40), and (42) sequentially established a relationship for measuring growth rates of the type $h = f(t)$, $d = f(h)$, and $w = f(d)$, allowing for the creation of the abacus shown in Figure 11.

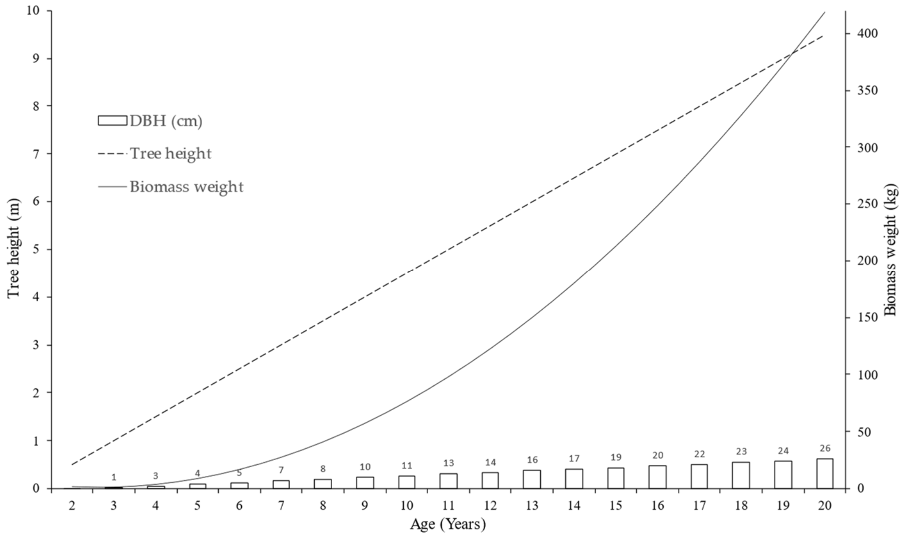


Figure 11. Height, diameter, and biomass weight estimation as a function of time.

The use of a tool, such as the one shown in Figure 11, can allow for the quick determination of missing parameters in field forest surveys and forest inventory, but also in a perspective of optimizing the control operations of this invasive species in Portugal, for example, by estimating the quantities of biomass available in a certain location, allowing the creation of a value chain through energy recovery.

5. Conclusions

Allometric, growth, or biomass estimation models can be used to determine the dimensions of trees in a non-destructive way when it is not possible to obtain the desired information. There are several models available, which are already applied to different species. Regarding species that may acquire an invasive behavior, as is the case of *A. dealbata* in Portugal, models that were developed specifically for this species are not available, so it is presented as an innovative and necessary development, given the importance that the study of this species presents, particularly due to the need for its control and eradication. In this way, the models selected from the available bibliography were applied to data collected in Portugal, and, from the application of non-linear methods, equations derived from the selected models were obtained, which were statistically analyzed to assess their significance. This analysis resulted in the selection of the equations that were shown to be the most significant from a statistical point of view, which were then validated by applying them to a set of trees that were previously measured. The models obtained presented better significance, mainly for the allometric models (Petterson, 1955; Loetsch et al., 1973) and for the growth models (Schumacher; Bertalanffy). It is verified that the results obtained follow a trend close to a linear model. Thus, for trees aged up to 20 years, which was the limit for which the models were validated, due to the lack of specimens aged over 20 years, the models can be simplified using linear-type models. However, the use of the models presented here should, for now, be limited to young trees and stands, and it is necessary to complete the models with new data, mainly from trees over 20 years old and from other locations. This study (which can be considered preliminary) will serve as a basis for a comprehensive analysis, now in different regions of the rest of the national territory (Portugal), so that in the future, a more general model can be created.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/environments9080104/s1, Figure S1. Projection of the predicted values around the expected normal, the projection of the deviation of the predicted values from the normal, and the error of the predicted values; Figure S2. Homoscedas-ticity evaluation; Table S1. Results from the measurements to the trees, P—Parcel; Spec.—Species; T nr.—Tree number; DC—Distance to the center of the circle (Parcel); Dir.—Direction; d1—Diameter; d2—Diameter; dmed—Average diameter; DHB class—Diameter at breast high class; ht—Trunk height; hc—Canopy height; dcanopy—Canopy diameter; Wt—Trunk weight; wb—branches weight; A.d.—*Acacia dealbata*; A.u.—Arbutus unedo; P.p.—*Pinus pinaster*; Table S2. Evolution of growth increments (in mm) of each of the trees selected for this study; Table S3. Data obtained in the sampling operations and used to create the models; Table S4. Cumulative data per year of growth for height and diameter; Table S5. DBH of the 59 trees located in the two reference transepts and the values obtained from the application of the equation generated according to the model of Petterson (1955) and from the application of the equation generated according to the model of Loetsch et al. (1973).

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