A note on a class of problems for a higher-order fully nonlinear equation under one-sided Nagumo-type condition

M.R. Grossinho\textsuperscript{a,*}, F. Minhós\textsuperscript{b}, A.I. Santos\textsuperscript{b}

\textsuperscript{a} Departamento de Matemática e Centro de Matemática e Aplicações à Previsão e Decisão Económica. Instituto Superior de Economia e Gestão. Technical University of Lisbon. Rua do Quelhas, n° 6, 1200-781 Lisboa, Portugal
\textsuperscript{b} Departamento de Matemática e Centro de Investigação em Matemática e Aplicações da U.E. (CIMA-UE). Universidade de Évora. R. Romão Ramalho, n° 59, 7000-671 Évora, Portugal

\textbf{ABSTRACT}

The purpose of this work is to establish existence and location results for the higher-order fully nonlinear differential equation

\[ u^{(n)}(t) = f(t, u(t), u'(t), \ldots, u^{(n-1)}(t)), \quad n \geq 2, \]

with the boundary conditions

\[ u^{(i)}(a) = A_i, \quad \text{for } i = 0, \ldots, n - 3, \]
\[ u^{(n-1)}(a) = B, \quad u^{(n-1)}(b) = C, \]

or

\[ c_1 u^{(n-2)}(a) - c_2 u^{(n-1)}(a) = B, \quad c_3 u^{(n-2)}(b) + c_4 u^{(n-1)}(b) = C, \]

with \( A_i, B, C \in \mathbb{R} \), for \( i = 0, \ldots, n - 3 \), and \( c_1, c_2, c_3, c_4 \) real positive constants.

It is assumed that \( f : [a, b] \times \mathbb{R}^{n-1} \to \mathbb{R} \) is a continuous function satisfying one-sided Nagumo-type conditions which allows an asymmetric unbounded behaviour on the nonlinearity. The arguments are based on the Leray–Schauder topological degree and lower and upper solutions method.

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1. Introduction

Let us consider the nth-order differential equation

\[ u^{(n)}(t) = f(t, u(t), u'(t), \ldots, u^{(n-1)}(t)), \]

for \( n \geq 2 \), with the following boundary conditions

\[ u^{(i)}(a) = A_i, \]
\[ u^{(n-1)}(a) = B, \quad u^{(n-1)}(b) = C, \]

\* Corresponding author.
E-mail addresses: mrg@iseg.utl.pt (M.R. Grossinho), fminhos@dmat.uevora.pt (F. Minhós), aims@dmat.uevora.pt (A.I. Santos).