SOLVABILITY FOR A THIRD ORDER DISCONTINUOUS FULLY EQUATION WITH NONLINEAR FUNCTIONAL BOUNDARY CONDITIONS

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Abstract. We prove an existence and location result for the third order functional nonlinear boundary value problem

$$\begin{aligned} u'''(t) &= f(t,u,u'(t),u''(t)), \text{ for } t \in [a,b], \\ 0 &= L_0(u,u',u(t_0)), \\ 0 &= L_1(u,u',u'(a),u''(a)), \\ 0 &= L_2(u,u',u'(b),u''(b)), \end{aligned}$$

with $t_0 \in [a,b]$ given, $f : I \times C(I) \times \mathbb{R}^2 \to \mathbb{R}$ is a L^1 – Carathéodory function allowing some discontinuities on t and L_0, L_1, L_2 are continuous functions depending functionally on u and u'.

The arguments make use of an *a priori* estimate on u'', lower and upper solutions method and degree theory.

Applications to a multipoint problem and to a beam equation will be presented.

1. Introduction

In this paper it is studied the third order nonlinear functional equation

$$u'''(t) = f(t,u,u'(t),u''(t)), \text{ for } t \in I, \tag{1.1}$$

where $I = [a,b]$, $f : I \times C(I) \times \mathbb{R}^2 \to \mathbb{R}$ is a L^1 – Carathéodory function together with the nonlinear functional boundary conditions

$$\begin{aligned} L_0(u,u',u(t_0)) &= 0, \\ L_1(u,u',u'(a),u''(a)) &= 0, \\ L_2(u,u',u'(b),u''(b)) &= 0, \end{aligned} \tag{1.2}$$

where $t_0 \in I$ is given and L_0, L_1, L_2 are continuous functions satisfying some monotonicity assumptions to be defined in Section 2.

We remark that functional dependence on the solution is allowed in f, moreover functions L_0, L_1 and L_2 depend functionally on the solution of the equation and on the first derivative. Such dependence allows us to consider, amongst others, integro - differential equations, delay equations or equations with maxima coupled with Sturm - Liouville or multipoint boundary value conditions under the same formulation.

This type of fully third order differential equation has been studied by several authors, considering nonlinear boundary conditions (see [5, 7, 14]) or two functional

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