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Lecture Notes in Statistics – Proceedings

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Editors

**Workshop on Branching Processes
and Their Applications**

 Springer

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Environmental versus demographic stochasticity in population growth

Carlos A. Braumann

Abstract Demographic stochasticity (sampling variation in births and deaths) and environmental stochasticity (effect of random environmental fluctuations on growth rate) in population growth are usually modeled using different approaches. Branching processes or birth and death processes (BDP) are used to model the effect of demographic stochasticity but do usually assume a constant environment. Stochastic differential equations (SDE) are used to model environmental stochasticity but usually ignore demographic stochasticity. We shall examine the similarities and the differences between these approaches in what concerns extinction and local behavior, using as a benchmark the particular case of the Malthusian (density-independent) models, namely the Galton–Watson process, the simple BDP and the Malthusian SDE model. For SDE density-dependent growth models, we then present a review of the results on extinction and existence of stationary densities. Such results are robust with respect to the form of density-dependence since we use general models (rather than specific models like the logistic). It would be worth studying the results for corresponding general density-dependent demographic stochasticity models.

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