HEAVY–LIGHT MESONS IN MINKOWSKI SPACE*

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Following up on earlier work on the $q\bar{q}$-bound-state problem using a covariant, chiral-symmetric formalism based upon the Covariant Spectator Theory, we study the heavy–light case for both pseudoscalar and vector mesons. Derived directly in Minkowski space, our approach approximates the full Bethe–Salpeter-equation, taking into account, effectively, the contributions of both ladder and crossed ladder diagrams in the kernel. Results for several mass spectra using a relativistic covariant generalization of a Cornell plus a constant potential to model the interquark interaction are given and discussed.

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1. Introduction

Mesons offer a prime target for studies of various approaches to quantum chromodynamics (QCD), which is widely accepted as the quantum field theory of the strong interaction. While in terms of the number of constituents their appearance is simple at first glance, mesons provide a broad range of phenomena and challenges to both experiment and theory. Namely, recent data from the Belle and CLEO collaborations, experimental programs such as GlueX at Jefferson Lab, together with the LHCb news on puzzling hadronic structures that elude an explanation within the traditional quark model picture, incite theoretical studies on hadron spectroscopy and structure.

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In the present work, mesons are studied by means of what we denote as the CST-Bethe–Salpeter (CST-BS) equations. This set of equations is rooted in field theory and close in spirit to the Dyson–Schwinger/Bethe–Salpeter approach. The obtained CST-BS equations can be viewed as a reorganization of the complete Bethe–Salpeter (BS) equation in order to efficiently sum ladder and cross-ladder contributions. Technical details can be found in [1–5].

Crossed-ladder contributions in the BS kernel are necessary to guarantee that the equation has the correct one-body limit. Furthermore, for a scalar $\varphi^2\chi$ theory, it was possible to sum all ladder and crossed-ladder diagrams in a path-integral calculation using the Feynman–Schwinger representation and compare it to several approximations of the BS equation [6]. The conclusion was that — even far from the one-body limit — neglecting the crossed-ladder diagrams (in the so-called BS ladder approximation) leads to larger discrepancies to the exact bound-state energies, whereas quasi-potential models do better. Whether the same holds for the physical $q\bar{q}$-system is a very interesting question that certainly merits further investigation.

A notable feature of CST-BS equations is that they are established in *physical* Minkowski space. Therefore, the obtained results are not restricted to bound state masses and momentum regions which are free of propagator singularities. This is advantageous over Euclidean formulations (although it entails the difficulty of handling those singularities numerically) because form factors can be computed directly in the timelike region with no need for analytical continuations. Because the CST equations have the correct one-body limit, *i.e.* they reproduce the Dirac equation in the limit of one of the particles being infinitely heavy, it is possible to establish a connection with nonrelativistic approaches, in particular for systems of heavy quarks. In the CST formalism, the requirements of chiral symmetry can also be satisfied, which is imperative for a realistic description of the pion. In particular, reference [7] shows how confining forces with a Lorentz scalar component, which have been suggested to dominate [8–10], are made consistent with spontaneous chiral symmetry breaking (SχSB).

2. CST-BS bound-state equation

To derive the CST-BS bound-state equation, we start with the full BS equation for the vertex function $\mathit{\Gamma}_{\rm BS}$,

$$\mathit{\Gamma}_{\rm BS}(p,P) = i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathcal{V}(p,k;P)S_1(k_1)\mathit{\Gamma}_{\rm BS}(k,P)S_2(k_2)\,, \tag{1}$$

with total momentum P and relative external and internal momentum p and k, respectively. $S_i(k_i) = (m_{0i} - \not{k}_i + \mathit{\Sigma}_i(\not{k}_i) - i\epsilon)^{-1}$ $(i = 1,2)$ is the dressed propagator and $\Sigma_i(\not{k}_i)$ is the self-energy of quark i. The CST equation is then obtained by keeping only the pole contributions from the propagators at $k_{i0} = \pm E_{k_i} = \pm(m_i^2 + \boldsymbol{k}^2)^{1/2}$, when the integration over k_0 is performed. If we symmetrize the contributions from both complex half-planes, we obtain a charge-conjugation symmetric equation that is a three-dimensional reduction of Eq. (1) and has four contributing diagrams, depicted in Fig. 1, each arising from placing one particle on its positive/negative energy mass-shell. When external legs are systematically placed on-shell in the diagrams of Fig. 1, a closed set of coupled equations emerges, the four-channel spectator equation (4CSE). However, to study heavy–light quark systems, with a large bound-state mass, it is sufficient to consider only the positive-energy pole contribution from the heavier particle 1, for $m_1 > m_2$. The resulting equation, the 1CSE, is represented in Fig. 2 and can be written as

$$\Gamma(p,P) = -\int \frac{\mathrm{d}^3}{(2\pi)^3}\frac{m_1}{E_{1k}} V(p,k;P)\mathcal{O}_1^i \Lambda_1\left(\hat{k}_1\right)\Gamma(k,P)S_2(k_2)\mathcal{O}_2^i\,, \tag{2}$$

where $\mathcal{O}_1^i$ and $\mathcal{O}_2^i$ are Dirac matrices of type i (scalar, vector, pseudoscalar), $V(p,k;P)$ is the momentum-dependent part of the interaction, Λ_1 is the positive-energy projector and $\hat{k}_1 = (E_{k_1}, \boldsymbol{k})$ is the on-shell momentum of particle 1.

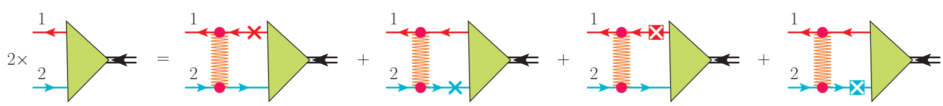

Fig. 1. Contributing diagrams for the 4CSE. A cross on a quark line indicates that only the positive-energy pole contribution of the corresponding propagator is kept in the loop integration, a cross inside a square refers to the respective negative-energy pole.

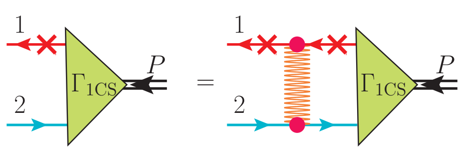

Fig. 2. Diagrammatic representation of the 1CSE.

3. Relativistic kernel

The kernel employed in our calculations with the 1CSE consists of a covariant generalization of the linear (L) confining potential used in [11], a color Coulomb (Coul), and a constant (C) interaction,

$$\mathcal{V}_{\rm L}(p,k) = -8\sigma\pi\left[\left(\frac{1}{q^4}-\frac{1}{\Lambda^4+q^4}\right)-\frac{E_{p_1}}{m_1}(2\pi)^3\delta^3(\boldsymbol{q})\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\frac{m_1}{E_{k'_1}}\right.$$
$$\left.\times\left(\frac{1}{q'^4}-\frac{1}{\Lambda^4+q'^4}\right)\right]\left[(1-y)\left(\mathbf{1}\otimes\mathbf{1}+\gamma^5\otimes\gamma^5\right)-y\left(\gamma^\mu\otimes\gamma_\mu\right)\right]\,, \quad (3)$$

$$\mathcal{V}_{\rm Coul}(p,k) = -4\pi\alpha\left(\frac{1}{q^2}-\frac{1}{q^2-\Lambda^2}\right)\left(\gamma^\mu\otimes\gamma_\mu\right), \quad (4)$$

$$\mathcal{V}_{\rm C}(p,k) = (2\pi)^3\frac{E_{k_1}}{m_1}C\delta^3(\boldsymbol{q})(\gamma^\mu\otimes\gamma_\mu)\,, \quad (5)$$

where $q = p - k$ is the transferred four-momentum. The mixing parameter y allows to dial continuously between a scalar-plus-pseudoscalar structure, suggested recently in [7] due to chiral-symmetry constraints, and a vector structure, while preserving the same nonrelativistic limit. The precise Lorentz structure of the confining interaction is not known, and by fitting the y parameter from the mesonic spectra, some further information can be gained.

The three coupling strengths σ, α and C, are free parameters of the model. Furthermore, a careful analysis of the asymptotic behaviour for large momenta k shows that we need to regularize the kernel under the integral of Eq. (2) in order to have convergence. We used a Pauli–Villars regularization scheme which yields one additional parameter Λ, the cut-off parameter, for both linear and the Coulomb cases.

4. Numerical results and discussion

This first series of results aims to explore the potential of CST framework to do *actual* spectroscopy. For what follows, we used a set of fixed parameters listed in Tables I and II. The linear strength $\sigma = 0.2$ GeV^2 is a typical value reported in lattice calculations and the parameter α has been chosen to reproduce well the heavy sector, in particular the bottomonium and charmonium systems (Figs. 1 and 2). A new set of results determined through a complete fit to data is under preparation.

In the following figures (Figs. 3–5), the 1CSE mass predictions are compared with several experimental states, depicted in grey/blue and dark grey/red, respectively. For both $J^{PC} = 0^{-+}$ and $J^{PC} = 1^{--}$, the agreement is very reasonable. For states composed of lighter constituent quarks, the predictions are not as good as for the heavier systems. This effect can be qualitatively understood by the fact that the one-channel approximation to the CST-BS equations reaches its limit of validity. However, the overall global behaviour of the heavy–light mesons is well-described even without any fit.

TABLE I

List of the constituent quark masses.

Quark q_i	Constituent m_i [GeV]
b	4.66
c	1.25
s	0.60
u/d	0.30

TABLE II

Parameters used in this work to predict *all* spectra with the 1CSE.

σ [GeV^2]	α	C [GeV]	y	Λ [GeV]
0.20	0.30	0.2	0.0 (pure scalar)	$3m_1$

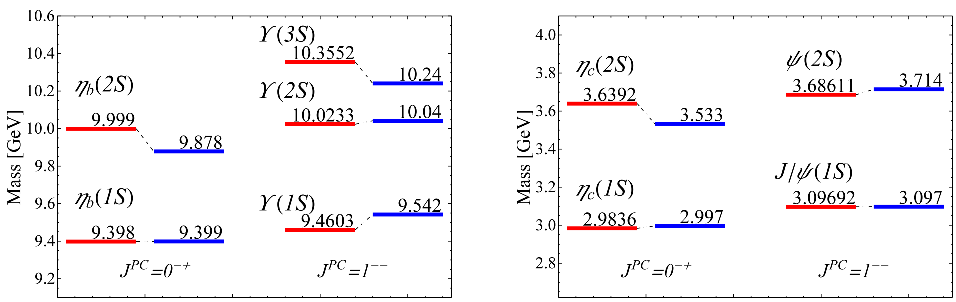

Fig. 3. Left: Bottomonium ($b\bar{b}$). Right: Charmonium ($c\bar{c}$).

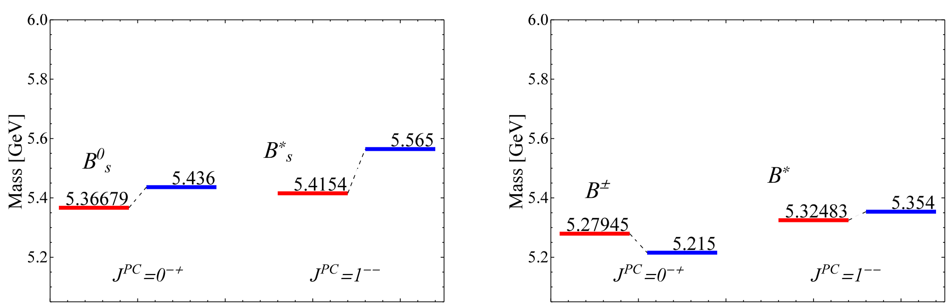

Fig. 4. Left: Bottom+strange ($b\bar{s}$). Right: Bottom+up/down ($b\bar{u}/b\bar{d}$).

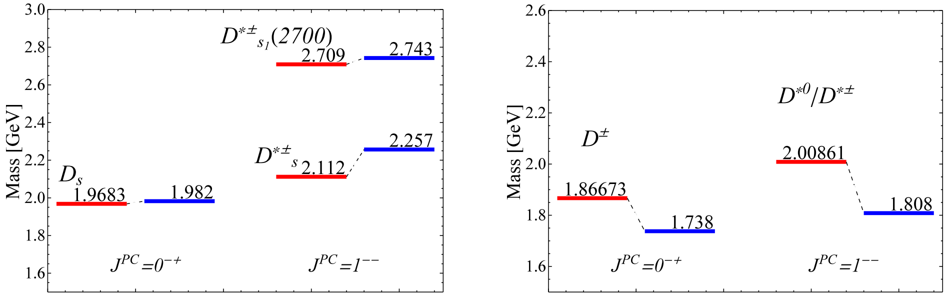

Fig. 5. Left: Charm+strange ($c\bar{s}$). Right: Charm+up/down ($c\bar{u}/c\bar{d}$).

In conclusion, this set of results can be regarded as an important feasibility study of this approach. Furthermore, these preliminary results will be very useful to guide us in global fits and thus to optimize our results for the meson spectrum.

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