



# Bidding strategy of wind-thermal energy producers



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## ABSTRACT

This paper presents a stochastic mixed-integer linear programming approach for solving the self-scheduling problem of a price-taker thermal and wind power producer taking part in a pool-based electricity market. Uncertainty on electricity price and wind power is considered through a set of scenarios. Thermal units are modelled by variable costs, start-up costs and technical operating constraints, such as: forbidden operating zones, ramp up/down limits and minimum up/down time limits. An efficient mixed-integer linear program is presented to develop the offering strategies of the coordinated production of thermal and wind energy generation, having as a goal the maximization of profit. A case study with data from the Iberian Electricity Market is presented and results are discussed to show the effectiveness of the proposed approach.

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## 1. Introduction

Because of the negative environmental impact of fossil fuels and the desire to achieve energy independence [1], a number of countries have attempted to promote electricity generation from renewable resources by implementing various supporting mechanisms and policies to provide incentive or subsidy for renewable energy generation [2–4]. As the wind power technology matures and reaches breakeven costs, subsidies turn out to be less relevant and wind producers become interested in participating in electricity markets to maximize their profits [5]. Also, the incentives for renewable energy exploitation are feasible for modest penetration levels, but the incentives will become flawed as wind power penetration increases [6]. In 2014, 43.7% of all new renewable installations were based on wind power and was the seventh year running that over 55% of all additional power capacity in the EU was renewable [7]. Despite the growing renewable energy integration worldwide in last decade [8], energy supply is still significantly dependent on fossil fuel since more than 60% of the electricity generated worldwide in 2012 was based on fossil fuel thermal plants [9].

In a deregulated electricity market, the generation companies (GENCOs) operate under a high competition degree due to the nodal variations of electricity prices [10] in order to obtain the best profit bidding in the day-ahead market [11]. For the wind power producers (WPP), wind power and the market-clearing electricity price uncertainties are to be addressed in order to know how much to produce in order to formulate realistic bids, because in case of excessive or moderate bids, other producers must reduce or increase production to fill the so-called deviation, implying penalties causing economic losses [12]. For thermal power producers, only market-clearing electricity price uncertainties have to be addressed.

This paper presents a formulation for GENCO owners of thermal plants and wind farms, which allows to submit electrical energy offers at the day-ahead market based on a profitable coordinated exploitation of wind power with thermal power.

## 2. State of the art

For the thermal producers, optimization methods to solve the unit commitment (UC) problem have been used since the old priorities list method [13–15] to the classical mathematical programming methods, like dynamic programming [16] and Lagrangian Relaxation [17,18] until the more recently reported artificial intelligence methods [19,20].

Although, easy to implement and requiring a small computation

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time, the priority list method does not ensure an economic convenient solution near a global optimal one, implying a higher operation cost [21]. Within the classical methods are included dynamic programming and Lagrangian relaxation-based techniques [22]. Dynamic programming methods are flexible but suffer from the “curse of dimensionality”, due to the increase in the problem size related with the number of thermal units to be committed and the number of states considered for modelling the thermal behaviour of each unit during the time horizon, implying an eventually huge use of computation memory and processing time [23]. Although the Lagrangian relaxation [24] can overcome the previous limitation, does not always lead to a conveniently feasible solution, requiring the satisfaction of some violated constraints using heuristics in order to set a feasible solution, undermining the optimality. Artificial intelligence methods based on artificial neural networks [25], genetic algorithms [26], evolutionary algorithms [27] and simulating annealing [28] have also been applied. However, the major limitation of the artificial intelligence methods is the likelihood to obtain a convenient solution near global optimum, especially with a few thermal units.

Mixed integer linear programming (MILP) has been applied with success for solving the UC problem [29]. MILP is one of the most successful explored methods for scheduling activities because of rigorouslyness, flexibility and extensive modelling capability [30]. Although, nonlinear constraints have to be converted into linear ones by piecewise linear approximation, MILP allows an easily inclusion of new constraints that makes the problem formulation more appropriated to support the management decision.

For the WPP in a deregulated market, wind power and the market-clearing electricity price uncertainties are passed on the variables of the problems [31] to be addressed by the WPP in order to know how much to produce in order to formulate realistic bids. In order for upholding a WPP in a competitive environment, three main lines of action have been proposed in the technical literature: One of them is based on the use of wind power with energy storage technologies [32–34]; another one is the use of financial options as a tool for WPP to hedge against wind power uncertainty [35]; and the final one focuses on designing stochastic models intended to produce optimal offer strategies for WPP participating in an electricity market [36–39], without depending on third-parties or governmental regulations. The third line of action is a stochastic formulation explicitly modelling the uncertainties faced by the scheduling activities of a WPP [40], using uncertain measures and multiple scenarios built by wind power forecast [41–43] and market-clearing electricity price forecast [44–46] applications.

### 3. Problem formulation

#### 3.1. Market balancing

Trading energy associated risks which must be taken in account in offering strategies. The risks are due to the uncertainties in available wind, energy prices and imbalance penalties. These uncertainties are addressed in the problem formulation.

System imbalance is the difference between the energy demand and the energy offer. The system imbalance is positive if there is an excess of generation in the power system, the system balance is negative if there is a deficit of generation in the power system.

The system operator seeks to minimize the system imbalance through a mechanism based on energy prices applied to the deviation of the energy produced by a producer and its energy offer in the day-ahead market. These prices,  $\lambda_t^+$  and  $\lambda_t^-$ , are applied in the balancing market to the energy deviations of the GENCO:  $\lambda_t^+$  is the price paid to the GENCO for its excess of generation and  $\lambda_t^-$  the price to be charged for its deficit.

If the system imbalance is negative, the system operator will keep the price of the day-ahead market,  $\lambda_t^D$ , for the energy in excess for those producers producing in excess of their offers, and will pay a premium price for the energy produced above the offer. If the system imbalance is positive, the system operator will keep the price of the day-ahead market for the energy in deficit for those producers producing in deficit of their offers (this is to be paid by the producers in deficit), and will pay for the energy in excess a price equal or lower than price of the day-ahead market. The applied prices to the deviations from the bids are explained in detail in [47].

The uncertainties in the available wind may result in differences between the energy traded by a GENCO with wind power and the actual energy generated. The revenue  $R_t$  of the GENCO for hour  $t$  is stated as:

$$R_t = \lambda_t^D P_t^{offer} + I_t \quad (1)$$

In (1),  $P_t^{offer}$  is the power traded by the GENCO in the day-ahead market and  $I_t$  is the imbalance income resulting from the balancing process.

The total deviation for hour  $t$  is stated as:

$$\Delta_t = P_t^{act} - P_t^{offer} \quad (2)$$

where  $P_t^{act}$  is the actual power for hour  $t$ .

If we define:

$$r_t^+ = \frac{\lambda_t^+}{\lambda_t^D}, \quad r_t^+ \leq 1 \quad (3)$$

$$r_t^- = \frac{\lambda_t^-}{\lambda_t^D}, \quad r_t^- \geq 1 \quad (4)$$

Then:

$$I_t = \lambda_t^D r_t^+ \Delta_t, \quad \Delta_t \geq 0 \quad (5)$$

$$I_t = \lambda_t^D r_t^- \Delta_t, \quad \Delta_t < 0 \quad (6)$$

A GENCO that needs to correct its energy deviations in the balancing market incurs an opportunity cost because in the day-ahead market energy is traded at a more competitive price. The deviation in (2) will cause an imbalance cost given by:

$$C_t = \lambda_t^D (1 - r_t^+) \Delta_t, \quad \Delta_t \geq 0 \quad (7)$$

$$C_t = -\lambda_t^D (r_t^- - 1) \Delta_t, \quad \Delta_t < 0 \quad (8)$$

The term  $\lambda_t^D P_t^{offer}$  in (1) is the maximum revenue that the GENCO can collect from trading its energy in a situation without production uncertainty.

Because of the uncertainties, we will include a set of scenarios  $\Omega$  for wind, energy prices and system imbalances. Each scenario  $\omega$  will be weighted with its probability of occurrence  $\pi$ .

#### 3.2. Thermal production

The operating  $F_{\omega i t}$  cost for a thermal unit can be stated as:

$$F_{\omega i t} = A_i u_{\omega i t} + d_{\omega i t} + b_{\omega i t} + C_i z_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (9)$$

The total operating cost in (9) is composed of four terms,

namely: the fixed production cost,  $A_i$ , associated with the units operating at the minimum power, the added variable cost  $d_{\omega i t}$  part derived from the amount of fossil fuel consumed by the units, and the start-up and shut-down costs,  $b_{\omega i t}$  and  $C_i$ , of the units.

The typical non-differentiable and nonconvex functions used to quantify the variable costs of thermal units is replaced by piecewise linear approximations in order to use and to have the advantage of the stochastic MILP approach [1]. Hence, the variable cost function,  $d_{\omega i t}$  is stated as:

$$d_{\omega i t} = \sum_{l=1}^L F_i^l \delta_{\omega i t}^l \quad \forall \omega, \quad \forall i, \quad \forall t \quad (10)$$

$$p_{\omega i t} = p_i^{\min} u_{\omega i t} + \sum_{l=1}^L \delta_{\omega i t}^l \quad \forall \omega, \quad \forall i, \quad \forall t \quad (11)$$

$$(T_i^1 - p_i^{\min}) t_{\omega i t}^1 \leq \delta_{\omega i t}^1 \quad \forall \omega, \quad \forall i, \quad \forall t \quad (12)$$

$$\delta_{\omega i t}^1 \leq (T_i^1 - p_i^{\min}) u_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (13)$$

$$(T_i^l - T_i^{l-1}) t_{\omega i t}^l \leq \delta_{\omega i t}^l \quad \forall \omega, \quad \forall i, \quad \forall t, \quad \forall l \\ = 2, \dots, L-1 \quad (14)$$

$$\delta_{\omega i t}^l \leq (T_i^l - T_i^{l-1}) t_{\omega i t}^{l-1} \quad \forall \omega, \quad \forall i, \quad \forall t, \quad \forall l \\ = 2, \dots, L-1 \quad (15)$$

$$0 \leq \delta_{\omega i t}^L \leq (p_i^{\max} - T_i^{L-1}) t_{\omega i t}^{L-1} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (16)$$

The variable cost in (10) is the sum of the product of the slope of each segment,  $F_i^l$  by the segment power  $\delta_{\omega i t}^l$ . In (11), the power generation of the unit  $i$  is given by the minimum power generation plus the sum of the segment powers associated with each segment. The binary variable  $u_{\omega i t}$  ensures that the power generation is equal to 0 if the unit  $i$  is offline. In (12), if the binary variable  $t_{\omega i t}^1$  has a null value, then the segment power  $\delta_{\omega i t}^1$  can be less than the segment 1 maximum power; otherwise and in conjunction with (13), if the unit is on, then  $\delta_{\omega i t}^1$  is equal to the segment 1 maximum power. In (14), from the second segment to the second last one, if the binary variable  $t_{\omega i t}^l$  has a null value, then the segment power  $\delta_{\omega i t}^l$  can be less than the segment  $l$  maximum power; otherwise and in conjunction with (15), if the unit is on, then  $\delta_{\omega i t}^l$  is equal to the segment  $l$  maximum power. In (16), the segment power must be between zero and the last segment maximum power.

The exponential nature of the start-up costs function,  $b_{\omega i t}$  is approximated by a linear formulation [2]. This linear formulation is stated as:

$$b_{\omega i t} \geq K_i^\beta \left( u_{\omega i t} - \sum_{r=1}^{\beta} u_{\omega i t-r} \right) \quad \forall \omega, \quad \forall i, \quad \forall t \quad (17)$$

$$b_{\omega i t} \geq 0 \quad \forall \omega, \quad \forall i, \quad \forall t \quad (18)$$

Notice that if the unit  $i$  is online at hour  $t$  and has been offline in  $\beta$  preceding hours, the expression in parentheses in (17) is equal to 1, i.e., a start-up happen and the respective cost is considered.

The constraints to limit the power generated for each unit  $i$  are stated as:

$$p_i^{\min} u_{\omega i t} \leq p_{\omega i t} \leq p_{\omega i t}^{\max} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (19)$$

$$p_{\omega i t}^{\max} \leq p_i^{\max} (u_{\omega i t} - z_{\omega i t+1}) + SD \quad \forall \omega, \quad \forall i, \quad \forall t \quad (20)$$

$$p_{\omega i t}^{\max} \leq p_{\omega i t-1}^{\max} + RU \quad u_{\omega i t-1} + SU \quad y_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (21)$$

$$p_{\omega i t-1} - p_{\omega i t} \leq RD \quad u_{\omega i t} + SD \quad z_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (22)$$

In (18), the generating limits of the units are set. In (19) and (20), the upper bound of  $p_{\omega i t}^{\max}$  is set, which is the maximum available power of the unit. This variable considers: unit's actual capacity, start-up/shut-down ramp rate limits, and ramp-up limit. In (22), the ramp-down and shut-down ramp rate limits are considered. In (20)–(22), the relation between the start-up and shut-down variables of the unit are given, using binary variables and their weights.

The minimum down time constraint is imposed by a linear formulation stated as:

$$\sum_{t=1}^{J_i} u_{\omega i t} = 0 \quad \forall \omega, \quad \forall i \quad (23)$$

$$\sum_{t=k}^{k+DT_i-1} (1 - u_{\omega i t}) \geq DT_i \quad z_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall k \\ = J_i + 1 \dots T - DT_i + 1 \quad (24)$$

$$\sum_{t=k}^T (1 - u_{\omega i t} - z_{\omega i t}) \geq 0 \quad \forall \omega, \quad \forall i, \quad \forall k \\ = T - DT_i + 2 \dots T \quad (25)$$

$$J_i = \min\{T, (DT_i - s_{\omega i 0})(1 - u_{\omega i 0})\}$$

If the minimum down time has not been achieved (33) ensures that the unit remains offline at hour 0. In (24), the minimum down time will be satisfied for all the possible sets of consecutive hours of size  $DT_i$ . In (25), the minimum down time will be satisfied for the last  $DT_i-1$  hours.

The minimum up time constraint is also imposed by linear formulation stated as:

$$\sum_{t=1}^{N_i} (1 - u_{\omega i t}) = 0 \quad \forall \omega, \quad \forall i \quad (26)$$

$$\sum_{t=k}^{k+UT_i-1} u_{\omega i t} \geq UT_i \quad y_{\omega i t} \quad \forall \omega, \quad \forall i, \quad \forall k \\ = N_i + 1 \dots T - UT_i + 1 \quad (27)$$

$$\sum_{t=k}^T (u_{\omega i t} - z_{\omega i t}) \geq 0 \quad \forall \omega, \quad \forall i, \quad \forall k = T - UT_i + 2 \dots T \quad (28)$$

$$N_i = \min\{T, (UT_i - U_{\omega i 0})u_{\omega i 0}\}$$

If the minimum up time has not been achieved, (36) ensures that the unit remains online at hour 0. In (27), the minimum up

time will be satisfied for all the possible sets of consecutive hours of size  $UT_i$ . In (28), the minimum up time will be satisfied for the last  $UT_i - 1$  hours.

The relation between the binary variables to identify start-up, shutdown and forbidden operating zones is stated as:

$$y_{\omega i t} - z_{\omega i t} = u_{\omega i t} - u_{\omega i t-1} \quad \forall \omega, \quad \forall i, \quad \forall t \quad (29)$$

$$y_{\omega i t} + z_{\omega i t} \leq 1 \quad \forall \omega, \quad \forall i, \quad \forall t \quad (30)$$

The total power generated the thermal units is stated as:

$$p_{\omega t}^g = \sum_{i=1}^I p_{\omega i t} \quad \forall \omega, \quad \forall t \quad (31)$$

The total operating costs,  $F_{\omega t}^T$  of the thermal units is stated as:

$$F_{\omega t}^T = \sum_{i=1}^I F_{\omega i t} \quad \forall \omega, \quad \forall t \quad (32)$$

### 3.3. Objective function

The offer submitted by the GENCO is the sum of the power offered from the thermal units and the power offered from the wind farm. The offer is stated as:

$$p_{\omega t}^{offer} = p_{\omega t}^{th} + p_{\omega t}^D \quad \forall \omega, \quad \forall t \quad (33)$$

The actual power produced by the GENCO is the sum of the power generated by the thermal units and the power generated by the wind farm. The actual power is stated as:

$$p_{\omega t}^{act} = p_{\omega t}^g + p_{\omega t}^d \quad \forall \omega, \quad \forall t \quad (34)$$

In (34),  $p_{\omega t}^g$  is the actual power generated by the thermal units and  $p_{\omega t}^d$  is the actual power generated by the wind farm for scenario  $\omega$ .

For a 24 h period the expected revenue of the GENCO is stated as:

$$\sum_{\omega=1}^{N_\omega} \sum_{t=1}^{N_T} \pi_\omega \left[ (\lambda_{\omega t}^D p_{\omega t}^{offer} + \lambda_{\omega t}^D r_{\omega t}^+ \Delta_{\omega t}^+ - \lambda_{\omega t}^D r_{\omega t}^- \Delta_{\omega t}^-) - F_{\omega t}^T \right] \quad \forall \omega, \quad \forall t \quad (35)$$

Subject to:

$$0 \leq p_{\omega t}^{offer} \leq p_{\omega t}^M \quad \forall \omega, \quad \forall t \quad (36)$$

$$\Delta_{\omega t} = (p_{\omega t}^{act} - p_{\omega t}^{offer}) \quad \forall \omega, \quad \forall t \quad (37)$$

$$\Delta_{\omega t} = \Delta_{\omega t}^+ - \Delta_{\omega t}^- \quad \forall \omega, \quad \forall t \quad (38)$$

$$0 \leq \Delta_{\omega t}^+ \leq P_{t \omega} d_t \quad \forall \omega, \quad \forall t \quad (39)$$

In (36),  $p_{\omega t}^M$  is maximum available power, limited by the sum of the installed capacity in the wind farm,  $p^{Emax}$ , with the maximum thermal production. The maximum available power is stated as:

$$p_{\omega t}^M = \sum_{i=1}^I p_{\omega i t}^{\max} + p^{Emax} \quad \forall \omega, \quad \forall t \quad (40)$$

Some system operators require non-decreasing offers to be

submitted by GENCO. This new constraint for equation (35) is stated as:

$$(p_{\omega t}^{offer} - p_{\omega' t}^{offer}) (\lambda_{\omega t}^D - \lambda_{\omega' t}^D) \geq 0 \quad \forall \omega, \quad \omega', \quad \forall t \quad (41)$$

## 4. Case study

The effectiveness of the proposed stochastic MILP approach is illustrated by a case study using one set of data from the Iberian electricity market, comprising 10 days of June 2014 [28].

The market data is to be used by a GENCO that owns both thermal and wind production. For the thermal generation are considered 8 units with a total installed capacity of 1440 MW and wind farm installed capacity is 360 MW. In a first step, we obtained the results for the described installed capacities and then we change the capacities first for the wind farm and then for the thermal units and investigate the gain variation of the coordinated approach versus the uncoordinated approach.

The eight thermal unit data, minimum/maximum power, ramp up/down values, start-up and shut-down ramp rate values, minimum up/down time, fixed and shut-down costs are shown in Table 1.

The variable costs of the thermal units have been modelled through piecewise linear approximations with three segments. Also, the start-up costs are modelled through stepwise approximations with ten intervals, as shown in Table 2.

The energy prices and the energy produced from wind are shown in Fig. 1.

The energy produced is obtained using the total energy produced from wind scaled to the maximum power of the installed capacity in the wind farm, 360 MW.

The system operator seeks to match the total energy production to the system needs. This is achieved by defining the price multipliers given by  $r_t^+$  and  $r_t^-$ . These price multipliers are shown in Table 3 and in Table 4.

The expected results for two different approaches, with and without coordination, are shown in Table 5.

The uncoordinated approach implies that two bids have to be submitted independently, one for the wind farm plant and other for the thermal plant. While the coordinated approach has only one bid to be submitted, having an expected profit higher. Also, the coordinated approach does not represent a burden in computational resources when compared with the uncoordinated one. The execution time measured by Gams is about the same for both approaches, since the wind execution time is irrelevant when compared with the thermal execution time in the uncoordinated approach. In the coordinated approach, the wind part is again irrelevant since the execution time for thermal uncoordinated is almost the same as the coordinated execution time.

The energies traded for each scenario, with and without coordination, are shown in Fig. 2 and in Fig. 3.

The weight of the wind in the total energy traded, with and without coordination, is shown in Fig. 4.

For the thermal part of the energy, the weight of the energy traded on the actual energy considered for the cost is shown in Fig. 5.

The actual energy is the energy considered for the thermal cost and is subjected to the technical constraints of the thermal plants. The offered energy, is only limited by the sum of the maximum possible energy generated by each thermal unit,  $p_{\omega t}^{th}$ .

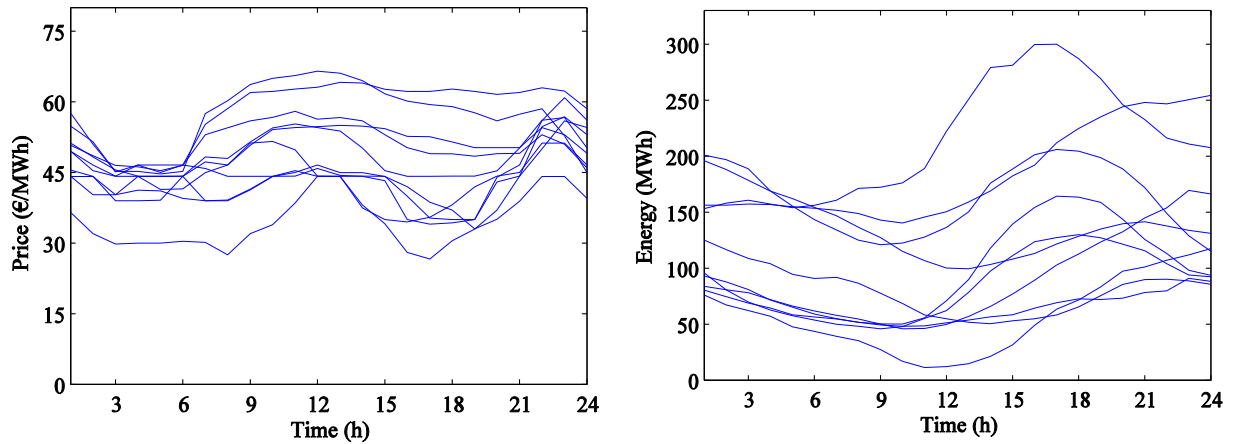
Most energy markets require non decreasing energy offers. The non-decreasing energy offers for hour 5 and hour 15 are shown in Fig. 6.

**Table 1**  
Thermal units' characteristics.

Unit	$p_i^{\min}$ (MW)	$p_i^{\max}$ (MW)	$RU_i$ (MW)	$RD_i$ (MW)	$SU_i$ (MW)	$SD_i$ (MW)	$UT_i$ (h)	$DT_i$ (h)	$A_i$ (Eur/h)	$C_i$ (Eur/h)
U1	70	125	45	40	100	95	5	4	2900	170
U2	110	160	60	50	125	140	8	4	3150	215
U3	60	125	55	55	90	80	5	3	3060	120
U4	90	170	40	60	100	100	6	3	2995	155
U6	90	170	40	60	100	100	6	3	2995	155
U6	80	145	35	40	90	105	9	6	3225	120
U7	145	215	45	70	160	170	6	4	3810	110
U8	220	330	70	60	230	245	10	6	4490	135

**Table 2**  
Stepwise approximations of the start-up costs (Eur/h).

Unit capacity (MW)	$K_i^1$	$K_i^2$	$K_i^3$	$K_i^4$	$K_i^5$	$K_i^6$	$K_i^7$	$K_i^8$	$K_i^9$	$K_i^{10}$
<125	654	1347	1896	2254	2533	2684	2733	2767	2813	2853
$\geq 125$ and $\leq 215$	1046	2155	3034	3606	4053	4294	4373	4427	4501	4565
>215	2224	4580	6446	7664	8612	9126	9292	9408	9564	9700

**Fig. 1.** Iberian market June 2014 (ten days); left: energy price, right: energy from wind.**Table 3**  
Price multipliers, system long.

Time (h)	Scenario $r_t^+$									
	1	2	3	4	5	6	7	8	9	10
1	0.75	1.00	1.00	0.70	1.00	1.00	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	0.80	1.00	1.00	1.00	1.00	1.00	1.00
3	0.00	1.00	1.00	1.00	1.00	0.76	1.00	1.00	1.00	1.00
4	0.00	1.00	1.00	1.00	1.00	0.47	1.00	1.00	1.00	1.00
5	0.00	1.00	1.00	1.00	1.00	0.63	1.00	1.00	1.00	1.00
6	0.00	1.00	1.00	1.00	1.00	0.70	1.00	1.00	1.00	1.00
7	0.00	0.59	1.00	1.00	1.00	0.74	0.79	1.00	1.00	1.00
8	0.00	1.00	1.00	0.53	1.00	1.00	0.72	1.00	1.00	1.00
9	1.00	0.79	1.00	0.52	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	0.66	0.63	0.68	1.00	1.00	1.00	1.00	1.00	0.23
11	1.00	0.74	0.39	0.00	0.01	1.00	0.66	1.00	1.00	1.00
12	1.00	0.76	0.72	0.15	0.00	1.00	0.70	1.00	1.00	1.00
13	1.00	1.00	0.80	0.23	0.29	1.00	0.08	1.00	1.00	1.00
14	1.00	1.00	0.78	0.21	1.00	1.00	0.01	1.00	1.00	1.00
15	1.00	1.00	0.63	0.20	1.00	1.00	0.01	1.00	1.00	1.00
16	1.00	1.00	0.78	1.00	1.00	1.00	0.65	1.00	1.00	1.00
17	1.00	1.00	0.75	1.00	1.00	0.00	0.70	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	0.00	0.53	1.00	1.00	1.00
19	1.00	1.00	0.74	1.00	1.00	0.01	0.79	1.00	1.00	1.00
20	1.00	1.00	0.65	1.00	1.00	0.11	0.55	1.00	1.00	1.00
21	1.00	1.00	0.75	0.67	1.00	1.00	1.00	1.00	1.00	1.00
22	1.00	1.00	0.56	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	1.00	1.00	0.59	1.00	1.00	1.00	0.64	1.00	1.00	1.00
24	1.00	1.00	0.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00

**Table 4**  
Price multipliers, system short.

Time (h)	Scenario $r_t^-$									
	1	2	3	4	5	6	7	8	9	10
1	1.00	1.32	1.25	1.00	1.14	1.00	1.08	1.17	1.24	1.31
2	1.25	1.19	1.32	1.00	1.07	1.00	1.15	1.17	1.43	1.12
3	1.00	1.28	1.28	1.00	1.11	1.00	1.17	1.34	1.55	1.17
4	1.00	1.17	1.19	1.01	1.02	1.00	1.07	1.38	1.50	1.04
5	1.00	1.10	1.21	1.00	1.10	1.00	1.00	1.47	1.60	1.00
6	1.00	1.08	1.24	1.00	1.05	1.00	1.00	1.53	1.65	1.10
7	1.00	1.00	1.19	1.00	1.12	1.00	1.00	1.26	1.24	1.04
8	1.00	1.00	1.21	1.00	1.03	1.00	1.00	1.28	1.17	1.09
9	1.32	1.00	1.00	1.00	1.02	1.02	1.11	1.23	1.11	1.00
10	1.23	1.00	1.00	1.00	1.00	1.09	1.14	1.23	1.13	1.00
11	1.00	1.00	1.00	1.00	1.00	1.26	1.00	1.22	1.13	1.27
12	1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.20	1.14	1.22
13	1.04	1.17	1.00	1.00	1.00	1.00	1.00	1.18	1.16	1.53
14	1.13	1.00	1.00	1.00	1.27	1.00	1.00	1.16	1.19	1.43
15	1.09	1.01	1.00	1.00	1.39	1.03	1.00	1.20	1.22	1.26
16	1.16	1.04	1.00	1.32	1.39	1.17	1.00	1.20	1.26	1.26
17	1.22	1.09	1.00	1.23	1.46	1.00	1.00	1.20	1.32	1.33
18	1.29	1.22	1.14	1.38	1.19	1.00	1.00	1.20	1.33	1.22
19	1.14	1.29	1.00	1.23	1.21	1.00	1.00	1.19	1.39	1.34
20	1.05	1.23	1.00	1.09	1.10	1.00	1.00	1.16	1.43	1.26
21	1.00	1.23	1.00	1.00	1.00	1.00	1.29	1.14	1.36	1.29
22	1.05	1.13	1.00	1.26	1.07	1.14	1.20	1.16	1.34	1.25
23	1.00	1.22	1.00	1.26	1.00	1.02	1.00	1.21	1.52	1.61
24	1.00	1.23	1.00	1.30	1.00	1.00	1.19	1.25	1.69	1.42

**Table 5**  
Results with and without coordination.

Case	Expected profit (€)	Imbalance cost (€)	Execution time (s)
Wind uncoordinated	119200	–17826	0.02
Thermal uncoordinated	516848	229398	0.13
Sum uncoordinated wind and thermal	636048		0.15
Coordinated Wind and thermal	642326	3643	0.13
Gain (%)	0.99		

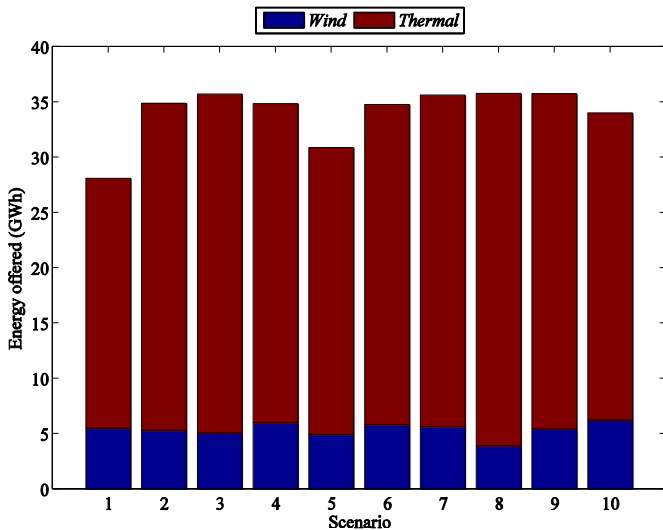


Fig. 2. Energy traded with coordination.

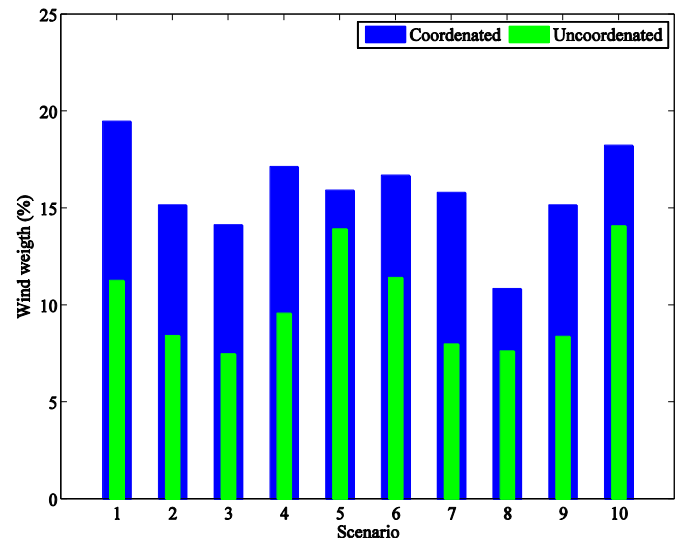


Fig. 4. Wind weight with and without coordination.

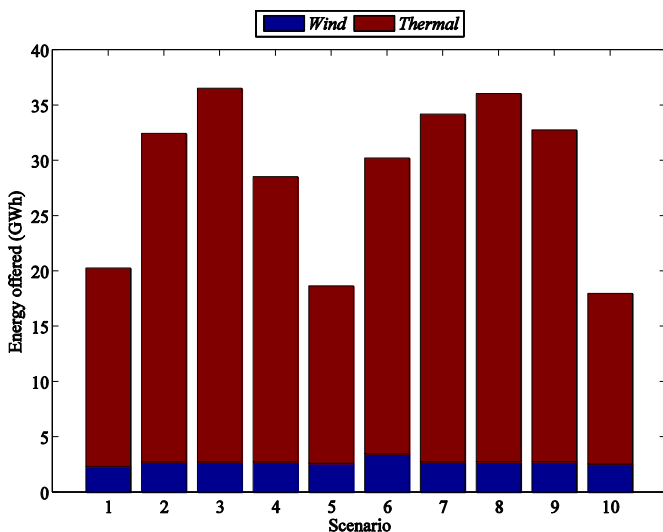


Fig. 3. Energy traded without coordination.

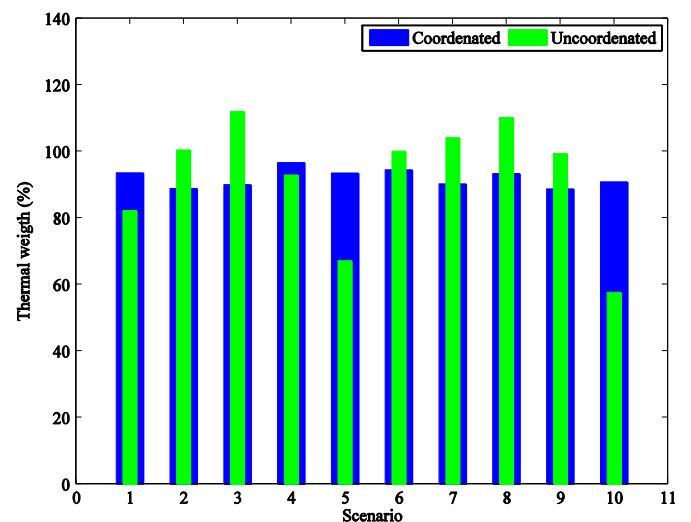


Fig. 5. Weight of the energy traded on the actual energy, thermal part.

For the uncoordinated approach both wind and thermal bids are submitted for each hour, which means that each individual bid has to be non-decreasing. The total curve for hour 5 and hour 15 are the sum of the wind and thermal bids.

For the coordinated approach, only a bid is submitted for each hour. In Fig. 6 is represented the contribution of the wind and thermal generation for the energy to bid. Since only the energy to bid has to be non-decreasing, the wind and thermal contributions

don't have to be non-decreasing. This is the case for hour 15.

The energy offer and committed for scenario 3 and scenario 10 are shown in Fig. 7.

In Fig. 7 we can see that wind parcel of the energy offer is higher for the coordinated approach for both scenarios, what is in accordance with Fig. 4. For the coordinated approach the thermal bid behaviour tends to be the opposite of the wind behaviour: when the wind parcel of the energy offer increases, the thermal parcel of



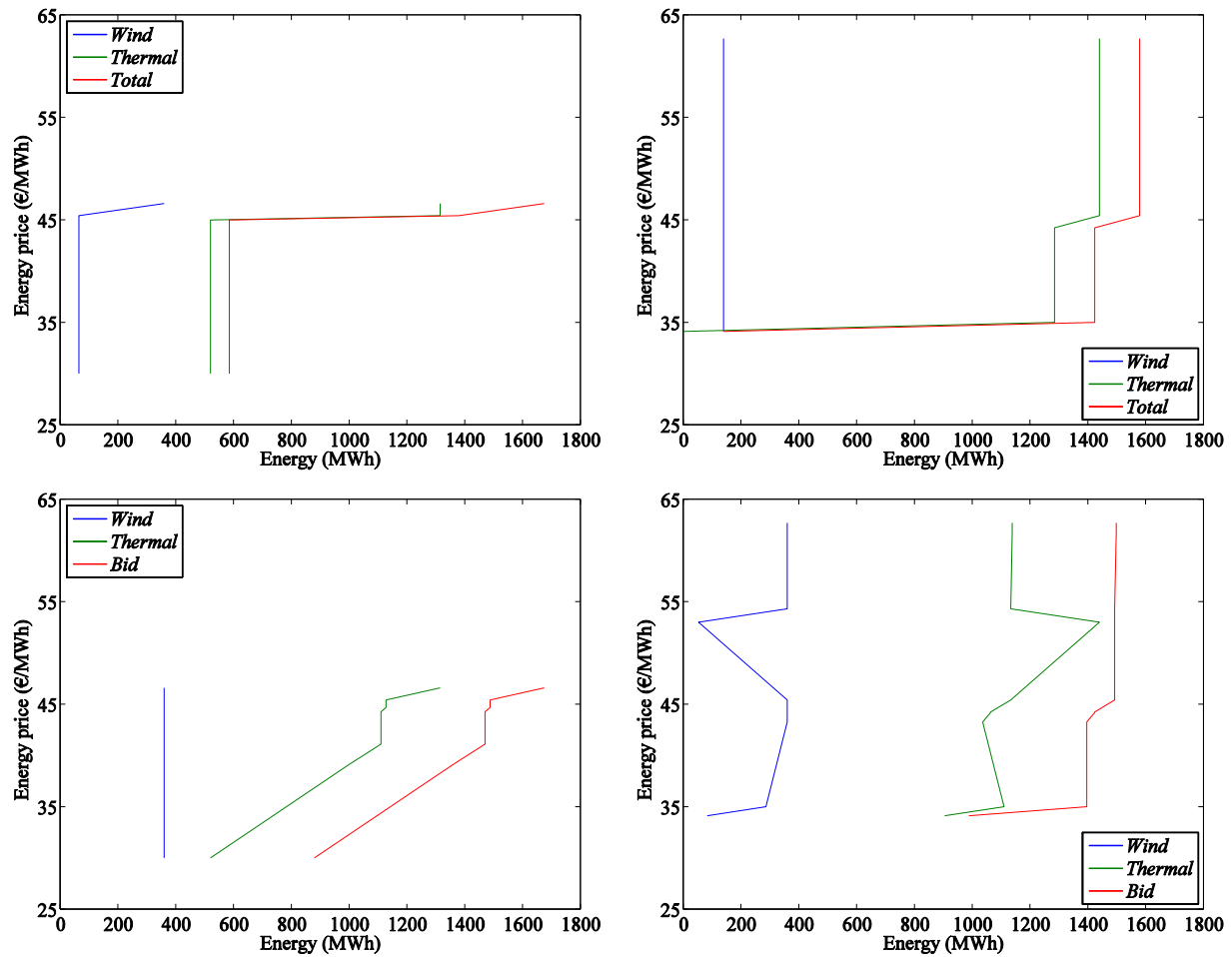


Fig. 6. Bidding energy offers. Top: uncoordinated; bottom: coordinated. Left: hour 5, right: hour 15.

the energy offer decreases, when the wind parcel of the energy offer decreases, the thermal parcel of the energy offer increases. The higher values of the wind parcel of the energy offer for the coordinated approach when compared with the uncoordinated approach is compensated in the coordinated approach by the decreasing of the thermal parcel of the energy offer, so the imbalance is minimized.

The expected gain results for the two different approaches, with and without coordination, are shown in Table 6 as a function of the wind farm capacity, keeping constant the thermal units installed power at 1440 MW.

The results expressed in Table 6 are for the same thermal units and the same scenarios of Table 5. The gain is dependent of the ratio between the wind farm and the thermal units. The maximum, 2.03%, is achieved when the wind farm capacity is about 1.5 times the thermal units installed capacity.

The expected gain results for the two different approaches, with and without coordination, are shown in Table 7 as a function of the thermal units installed power, keeping constant the wind farm installed power at 360 MW.

The results expressed in Table 7 are for a wind farm capacity of 360 MW. For the thermal units installed power, the engineering units expressed in [MW] were scaled down by the same ratio of the thermal units capacity in Table 7 and 1440 MW. The same operation was performed on the ramp start-up costs. The gain is dependent of the ratio between the wind farm and the thermal

units. The maximum, 3.20%, is achieved when the thermal units installed capacity is about 2.3 times the wind farm installed capacity.

## 5. Conclusions

A stochastic MILP approach for solving the offering strategy and the self-scheduling problem of a price-taker wind power and thermal power producer is developed in this paper. The main results are the short-term bidding strategies and the optimal schedule of the thermal units. A mixed-integer linear formulation is used to model the main technical and operating characteristics of thermal units.

The coordinated offer of wind power with thermal power allows to provide better results than the sum of the isolated offers. But the results concerning the indirect objective function, the developed function, parametrized in function of the power capacities of the wind farm and of the thermal units are dependent of the ratio between these capacities. So, in order to have a profitable exploitation of wind power with thermal power the power capacities have to be conveniently seized. For instance, if there is an excess of thermal power in the coordination in comparison with the seized one, eventually this power should be submitted as an isolated offer if better exploitation is achieved. The approach for solving the offering strategy and the self-scheduling problem allows to take a decision about this eventually isolated offer.

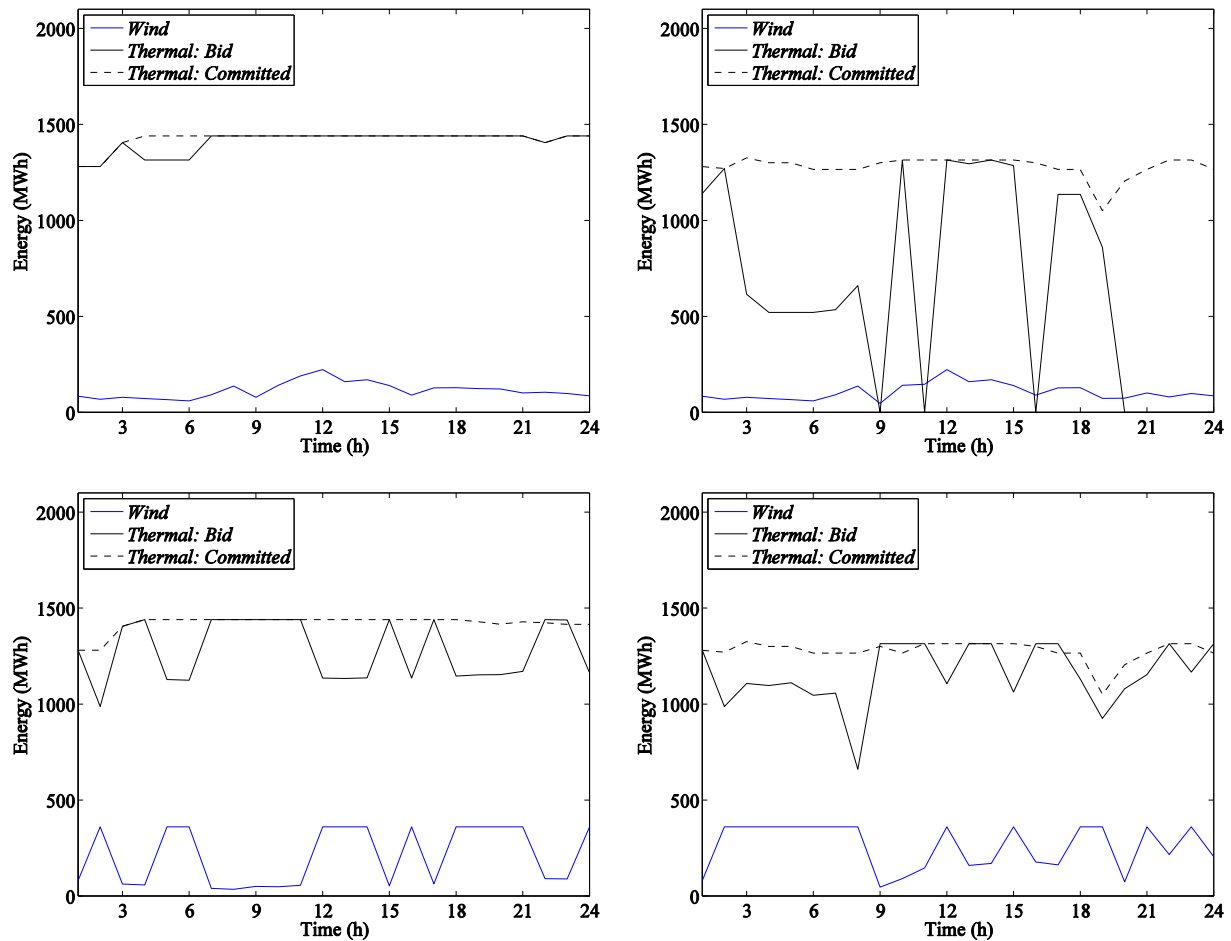


Fig. 7. Energy offer and committed. Top: uncoordinated; bottom: coordinated. Left: scenario 3, right: scenario 10.

**Table 6**

Gain variation in function of wind farm capacity.

Wind farm (MW)	Expected uncoordinated profit (€)	Expected coordinated profit (€)	Gain (%)
720	755247	766543	1.50
1440	993646	1012520	1.90
2160	1232045	1257004	2.03
2880	1470444	1499547	1.98
3600	1708843	1741753	1.93
4320	1947242	1982611	1.82

**Table 7**

Gain variation in function of the thermal units installed power.

Thermal units capacity (MW)	Expected uncoordinated profit (€)	Expected coordinated profit (€)	Gain (%)
1340	585548	591942	1.09
1240	531666	538657	1.31
940	351366	357744	1.82
890	318697	326988	2.60
840	287759	296958	3.20
780	252500	257529	1.99

The stochastic programming is a suitable approach to address parameter uncertainty in modelling via scenarios. Hence, the proposed stochastic MILP approach proved both to be accurate and computationally acceptable, since the computation time scales up

linearly with number of price scenarios, units and hours on the time horizon. Since the bids in the pool-based electricity market are made one day before, this approach is a helpful tool for the decision-maker.



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