Some interesting applications of stochastic differential equations

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ABSTRACT:

We briefly review the basic ideas behind autonomous stochastic differential equations

$$dX(t) = g(X(t))dt + h(X(t))dW(t),$$

with g and h sufficiently smooth functions and $W(t)$ the standard Wiener process. Financial applications are quite well known. We will take a close look to recent developments on some other relevant applications on the growth of populations on randomly fluctuating environments (with applications to nature preservation and to harvesting), on the growth of individuals from birth to maturity under randomly varying conditions (with applications to farming) and on the evolution of mortality rates (with applications to demography, insurance, retirement plans, etc.).

We will also look at the embarrassing circumstance that the two main stochastic calculus, Itô and Stratonovich, lead to apparently different qualitative results regarding important issues like population extinction, which led to a controversy in the literature on which calculus is more appropriate for such applications. We have resolved the controversy by showing that g means different types of "average" growth rate according to the calculus used and the apparent difference was due to the wrong implicit assumption that g represented the same "average". Taking into account the different meaning of g, there is no difference (qualitative or quantitative) between the two calculi.