BAUMSLAG–SOLITAR GROUP $C^*$-ALGEBRAS FROM INTERVAL MAPS

C. CORREIA RAMOS$^1$, R. EL HARTI$^2$, NUNO MARTINS$^3$, PAULO R. PINTO$^3$

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Abstract. We yield operators $U$ and $V$ on Hilbert spaces that are parameterized by the orbits of certain interval maps that exhibit chaotic behavior and obey the (deformed) Baumslag–Solitar relation

$$UV = e^{2\pi i \alpha} V^n, \quad \alpha \in \mathbb{R}, \; n \in \mathbb{N}.$$ 

We then prove that the scalar $e^{2\pi i \alpha}$ can be removed whilst retaining the isomorphism class of the $C^*$-algebra generated by $U$ and $V$. Finally, we simultaneously unitarize $U$ and $V$ by gluing pairs of orbits of the underlying noninvertible dynamical system and investigate these unitary representations under distinct pairs of orbits.

1. Introduction and preliminaries

In [6, 7, 8, 10] we use symbolic dynamics and yield representations of Cuntz, Cuntz–Krieger, subshift $C^*$-algebras determined by orbits of nonlinear systems – in particular iterated maps of the interval, and Markov systems. These representations has allowed us to get a clearer relationship between the structure of these algebras and the underlying nonlinear dynamics. The studied systems are non-invertible and the symbolic dynamics is based on one-sided sequences. We obtained operators that are partial isometries, generating the referred algebras. In the present paper, we will be able to obtain unitary operators (leading to

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* Corresponding author.

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