Revisiting serial dependence in the stock markets of the G7 countries, Portugal, Spain and Greece

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Revisiting serial dependence in the stock markets of the G7 countries, Portugal, Spain and Greece

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This article uses several tests to analyse serial dependence in financial data, trying to confirm the existence of some kind of nonlinear dependence in stock markets. In an attempt to provide a better explanation of the behaviour of stock markets, we used tests based on mutual information and detrended fluctuation analysis (DFA). Applying these tests to the series of stock market indexes of 10 countries, we concluded for the absence of linear autocorrelation. However, with other tests, we found nonlinear serial dependence that affects the rates of return. With DFA, we found out that most return rate series have long-range dependence, which appears to be more pronounced for Spain, Greece and Portugal. To confirm the inefficiency of those markets, based on our results, we should prove the existence of abnormal profits.

Keywords: serial dependence; stock indexes; mutual information; detrended fluctuation analysis; nonlinearities

JEL Classification: G14; G15

I. Introduction

The study of time series’ dependence is one area of major interest in economics and management. It is important, for example, in the analysis of financial markets, as the existence of dependency, whether temporal or sectional, could lead to any prediction of the series and the possibility of violating the assumption of efficient markets.

The notion of efficient markets hypothesis is critical in this field. In fact, a financial market is considered efficient in its weak form if it is not possible to identify any deterministic pattern in its time series’ behaviour. This means that there is no possibility, through arbitrage, of obtaining systematic abnormal profits using past information (Fama, 1970). In the light of this theory, financial markets have been subject to extensive analysis to check whether there are windows of profit opportunities, considering the fluctuations and dynamics of markets themselves (see, e.g., the review of Pagan, 1990).

The main focus of this article is to analyse the behaviour of stock indices for the G7 countries (Germany, United States, United Kingdom, Italy, Japan, France and Canada) and also for Portugal, Spain and Greece. Initially, we analyse the behaviour of the series of returns through the usual descriptive statistics. This analysis is intended to check the existence of some sort of linear relationship (it is possible to do so using autocorrelation tests). This preliminary assessment is followed by the analysis of nonlinear dependence with tests of nonlinearity complemented by the use of mutual information and detrended fluctuation analysis (DFA). The main results point to the evidence of nonlinear dependence among series, which

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seems to point to some the existence of long range dependences. This finding could be seen as some lack of efficiency on the weak form, but it is important to read the results carefully. Given this, the main conclusion points to the fact that, besides the evidence of serial dependences, the rejection of the efficient market hypothesis is not reliable, since we have no empirical evidence that those dependences could imply systematic gains. The main contribution of this article relies on the exploration of non-linear and nonparametric techniques to the study of nonlinear serial dependence. These techniques have, as major advantage, the absence of assumptions about probability distributions, linearity and stationarity. Given this, we consider that our results are robust, in statistical and econometrical terms, and our conclusions may be more close to the financial reality.

The remaining of this article is organized as follows: Section II presents a survey on the empirical evidence on weak form efficiency. Section III presents some tests and methodologies that are applied in this article, Section IV reports the empirical analysis and its results and Section V concludes.

II. A Survey on the Efficient Market Hypothesis: Some Empirical Findings

The efficient market hypothesis is one of the major issues on the financial literature for the past 30 years. If theoretically there is some constancy on that background, empirically the evidence is not uniform and the results seem, depending on the methodology used, miscellaneous. Given the large number of empirical studies on the HEM it is not our goal to do exhaustive work on them here.

One of the first studies applied to the behaviour of financial markets’ series was developed by Bachelier (1964), who studied the probability distribution of share prices and concluded that prices follow a Gaussian distribution. Kendall (1953) found out that stock prices are randomly determined. The author even made an analogy with the results obtained in a roulette, which implies that return rates are independent and identically distributed (iid). Some other studies, namely those by Osborne (1964), Granger and Morgenstein (1964) or Fama (1963), validated the random walk hypothesis, which seems to indicate that asset prices have no memory and are therefore independent in time. For a long time, Bachelier’s theory (1964) that financial series behave like a random walk was accepted and introduced in many economic models, such as the efficient markets hypothesis proposed by Fama (1970).

However, several studies contradicted this evidence, finding the existence of stylized facts (see, e.g., Cont, 2001). One of these stylized facts is the existence of fat tails in returns distributions, which is related to the fact that the volatility of assets returns is higher than expected by a Gaussian distribution (see, e.g., Osborne, 1964). Cont (2001) also identified other stylized facts that may contribute to reject the evidence of normality in assets returns, including the existence of asymmetries in gains and losses (loss movements are more pronounced); greater than the expected intermittency and variability of returns, with volatility clustering behaviour; leverage effect (negative relation between volatility and profitability); correlation between trading volumes and volatility; and existence of autocorrelation in variance.

The analysis of serial dependence, both linear and nonlinear, has some relevance in the financial literature in recent times. In most cases, empirical studies identified the possibility of autocorrelation. However, generally these linear autocorrelations quickly disappear, although there are authors defending the existence of long-term dependence (see, e.g., Campbell, 1987).

Relevant studies on market efficiency used linear equations to analyse return rate dependence, failing to detect other types of dependency, including nonlinear dependence. This means that, even not rejecting the hypothesis that returns have no autocorrelation, it is possible that markets are not efficient if there is any other kind of dependence, and therefore, analysis of linear dependence is not sufficient (see, e.g., Darbellay, 1998a or Granger et al., 2004). Therefore, to study financial markets, it is necessary to follow general models that are capable of capturing global, and not only linear, dependence.

In this context, mutual information was introduced and its properties were explored as a measure of dependence in time series. This method has some advantages because it considers the whole structure of time series, linear and nonlinear (see, e.g., Darbellay and Wuertz, 2000).

More recently, new methods to analyse long-term dependence in time series have been developed. One of these methods is DFA. DFA was created by Peng et al. (1994), and it studies the behaviour of individual series. In this article, we use DFA to analyse the behaviour of stock markets. Later on, with the presentation of this methodology, there is a more detailed review of literature for studies that use this methodology.

Since our research objects are the stock markets of G7 countries, Portugal, Spain and Greece, we emphasize our analysis on studies about these countries. Borges (2010) presents an analysis on the weak-form market efficiency tests applied to the stock markets of United Kingdom, France, Germany, Spain, Greece and Portugal, using run tests, wild bootstrapping and joint variance ratio. Those results point to some mixed evidence: daily and weekly data display some signs of evidence against HEM, especially for Portugal and Greece, but monthly data follows a random walk in all countries. According to Borges (2010)
and Worthington and Higgs (2004), the results shown by Portugal and Greece may be explained based on the small dimension of those markets and the possibility of speculation. On a different study, Onali and Goddard (2011) use fractal analysis to evaluate the efficiency of European and US equity markets. These authors conclude about the existence of long-range dependence, more pronounced for the smallest markets, and no evidence of long-range dependence for five indexes, including those with the largest capitalizations among the database considered (DJIA and FTSE350). In fact, the majority of studies that include US stock markets agree about the efficiency of those indexes (see, e.g., Narayan, 2006).

The evidence of Asian countries, namely Japan, is not very different. For example, Nagayasu (2003) uses an ARFIMA-FIGARCH model to study the efficiency of the Japanese stock market and concludes about the existence of a long-range dependence. The author considers that the existence of this dependence may suggest some inefficiency of the Japanese stock market.

Using a panel data stationarity test, which incorporates multiple structural breaks, Lee et al. (2010) study 32 developed and 26 developing stock market countries and conclude that the processes are inconsistent with the efficient market hypothesis for all countries, but more for the developing countries and small stock markets.

The existence of nonlinearities in time series can be understood as a failure of efficient market hypothesis in its weak form. In the absence of linear autocorrelation, we could not conclude on the existence of efficient financial markets, since there may be other nonlinear relations that affect the behaviour of a financial asset. We must be also aware that even the verification of some kind of dependence may not necessarily mean inefficiency of markets since the existence of transaction costs can nullify the possibility of making arbitrage transactions (see, e.g., Fama, 1970). In this context, Malkiel (2003) and Timmermann and Granger (2004) realize that stock markets are not perfect, and besides the fact that some predictability tests point to the possibility of predictable patterns and trading opportunities, in fact, markets could stay efficient because the anomalous behaviour of stock prices does not create a portfolio trading opportunity that enables investors to earn extraordinary risk adjusted returns.

III. Methods to Analyse Dependence in Financial Time Series

One critical issue in financial economics is the assessment of time dependence, which is related to the efficient markets hypothesis. There are several tests to analyse time dependence in financial series and some are used in this article: BDS test (Brock et al., 1996), Engle test to analyse ARCH effects (Engle, 1982), McLeod and Li test (McLeod and Li, 1983) and Tsay test (Tsay, 1986). These tests are quite well known in the financial literature and are therefore not described here. Besides these tests, we also analyse the dependence with mutual information and with DFA, which are addressed below.

Mutual Information

Mutual information gives the common information between two (or more) different distributions. Introduced in the literature by Shannon (1948), this concept has been improved and widely used over time. In the context of time series, it is used to analyse dependence over time. Mutual information can be understood as a measure of dependence or correlation. However, we should be careful in its interpretation, as it does not provide indication of causality between variables. Mutual information is given by the following expression:

\[ I(X, Y) = \int \int p_{X,Y}(x,y) \log \frac{p_{X,Y}(x,y)}{p_X(x)p_Y(y)} \]  

(1)

It can take any positive value or may be zero. It will be zero if variables are independent (and therefore have no information in common). According to Granger et al. (2004), this makes mutual information an imperfect measure of dependence, since it does not take absolute values between 0 and 1 only. It is therefore necessary to standardize it to make direct comparisons (see, e.g., Granger and Lin, 1994; Darbellay, 1998b). One possible normalization is

\[ \lambda(X, Y) = \sqrt{1 - e^{-2I(X,Y)}} \]  

(2)

The measure of dependency identified by Equation 2 could vary between 0 and 1 and can be interpreted as a correlation that is based on information theory, taking the value 0 if the variables X and Y are independent (i.e., if the variables do not have information in common). The maximum value is obtained in the case of a perfect relationship between two variables, that is, in a deterministic context.

It is used as an alternative to other tests because it presents several advantages. First, some of the previous tests have some limitations. For example, the Pearson correlation coefficient captures only the existence of linear correlations, but nonlinear correlations may also be present in the data. Thus, mutual information may be used as a measure of overall correlation and not just of linear correlation. For this reason, it is irrelevant if the sign of the relationship is positive or negative. Moreover, measures related to entropy require fewer assumptions and are more flexible.
Mutual information is used to test global dependence of a time series. The null hypothesis is defined as $H_0$: $I(X,Y) = 0$, meaning that variables are independent (or that a given time series has no memory). The alternative hypothesis is given by $H_1$: $I(X,Y) > 0$. The decision of rejecting or not rejecting the null hypothesis is made by comparing the relevant values with the critical ones calculated by Dionisio et al. (2006). This test has the particularity of not needing assumptions on the linearity or normality of time series (see Fernandes and Néri, 2010). We estimate mutual information using the equiquantization method.  

The use of mutual information follows the conclusions of several authors who argued that measures relating to the information theory are very effective (see, e.g., Granger and Lin, 1994; Darbellay and Wuertz, 2000; Menezes et al., 2012).

**Detrended fluctuation analysis**

Exploring the possibility of chaos in financial markets has been a recurrent topic of analysis in several studies. If a time series is described by a random walk, then there is no verification of chaos in it. The Hurst exponent is a statistic used to distinguish between random or not random behaviour of a time series. Initially used by Harold Hurst in determining the randomness in the behaviour of the Nile River (see, e.g., Crato, 1994), it was generalized to other natural phenomena which display a not random (noise) behaviour. In addition, this type of analysis has also been used in economics, particularly in financial economics (see, e.g., Peters, 1996). Basically, a Hurst exponent different from 0.5 contradicts the hypothesis of randomness and hence of efficient markets.

Although this article does not directly use the Hurst exponent, we apply a methodology that indirectly provides the same information: DFA, a technique used to analyse temporal dependence in time series with the advantage of being used in the context of nonstationary time series.

This methodology was developed by Peng et al. (1994) originally to study the behaviour of DNA, but since then it has been used to analyse many different problems, from heartbeats to the behaviour of financial series. The main objective of this technique is to analyse the relationship between values $x_t$ and $x_{t+s}$ at different moments in time.

Consider the data given by $x_k$, with $k = 1, \ldots, T$ equidistant observations. The first step of DFA is to obtain the following values: $x(t) = \sum_{k=1}^{t} x_k - \langle k \rangle$, $\langle k \rangle$ being the average of the observations. The subtraction of $\langle k \rangle$ is not mandatory because it is eliminated in the third step. The second step is the base of DFA and consists of the division of $x(t)$ in $N/s$ mutual exclusive boxes of equal dimension $s$. The third step consists of obtaining the trend $z(t)$ of each segment, with ordinary least squares, and calculating the detrended series given by $x_s(t) = x(t) - z(t)$. The original application assumes a linear trend given by $z(t) = at + b$, but later applications show that is possible to include other polynomial trends (see, e.g., Kantelhardt et al., 2001). For each box, we obtain the values of $x(t)$ and $z(t)$, then we calculate DFA function given by

$$F(s) = \frac{1}{2N} \sum_{k=1}^{2N} [x_s(t)]^2.$$ 

Averaging the value of $F(s)$ for all centred boxes in $s$, we generate the value of fluctuations $\langle F(s) \rangle$, which is a function of $s$. This calculation is repeated for all different values of $s$. Then it is expected a power law behaviour of $\langle F(s) \rangle \sim s^\alpha$.

This $\alpha$ parameter is equivalent to the Hurst exponent used to analyse serial dependence. If $\alpha = 1/2$, this means that time series is represented by a random walk, so the autocorrelation function is zero for any period of time and the process do not have a long memory. If $\alpha \neq 1/2$, it implies the existence of long-term correlations in the considered time interval. There is a positive long-range dependence (the series are persistent) in the case of $1/2 < \alpha < 1$. If, instead, the value is positive but $\alpha < 1/2$, this indicates a negative long-range dependence (anti-persistence) meaning that larger fluctuations are followed by smaller fluctuations (or vice versa). If the value of $\alpha$ is equal to 1, the process is a pink noise. If it is greater than 1, it shows the existence of long-term dependence but cannot be analysed according to a power-law. You can graphically analyse the behaviour of this parameter on log-log graph for values of $\langle F(s) \rangle$ and for the time scale.

Some authors applied this technique to financial markets. Liu et al. (1997) concluded that correlations of returns on S&P500 quickly vanish, but that their absolute values did not show this effect, which shows nonlinearity in returns’ volatility. In turn, Ausloos et al. (1999) analyse foreign exchange markets, comparing the behaviour of exchange rates of the German mark and the Polish zloty, both against the Belgian franc, concluding in favour of temporal dependence in these series. Ausloos (2000) also analysed foreign exchange markets, studying 13 different exchange rates, and found that, in most cases, there was evidence of long-term correlations. Jaroszewicz et al. (2005) developed a pioneer work because they analysed Latin American indexes. In their study, the authors found evidence of correlations of long-term returns primarily in absolute returns and in relation to return rates. The behaviour of the assessed series indicated a slow approximation to the Gaussian distribution. However, as sample size was relatively small, the authors had some caution with their conclusions. Alvarez-Ramirez et al. (2008) applied

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1 For some discussion about the choice of the methods to estimate mutual information, Darbellay (1998a) or Granger et al. (2004)
Revisiting serial dependence

DFA to evaluate the possibility of forecasting ability in oil prices, concluding that, in short periods of time, there was some persistence in the correlations, but for longer horizons (time spans greater than 25 days), such relationship ceased to exist. Analysing 26 different stocks of the NYSE on a given day, Mariani et al. (2009) concluded that 19 out of 26 titles presented evidence of long memory, of which 18 had persistent behaviour and only one was anti-persistent. Muchnik et al. (2009) did not use DFA to analyse returns or volatility directly, but rather to analyse the sequence of maxima and minima in the behaviour of various assets (stocks’ prices and foreign exchange rates). The authors concluded that a long-term correlation existed between these maximum and minimum values, relating such results to volatility clustering.

IV. Results of Serial Dependence in the Stock Markets

Preliminary results

In this study, we use daily data from stock market indexes for 10 different countries: G7 group (Germany, Canada, United States, France, Italy, Japan and the United Kingdom) as well as Portugal, Spain and Greece, three Union European countries with some similar characteristics. Time series consist of 5356 observations, with data from 2 January 1990 to 13 July 2010, and were collected from the DataStream database (5 days, daily closing prices and from Datastream stock indexes). These are standard indexes, which allow making easy comparisons between different countries. Figure 1 shows the evolution of those indexes.

A brief visual analysis suggests that, despite the distinct fluctuation patterns, the majority of the indexes display two periods of pronounced maxima: one around 2000 and the other around 2007. There is, however, an index with a different behaviour: that of Japan. Despite having the same two peaks, it seems to have a more volatile behaviour when compared with the remaining indexes, indicating a decrease in the early sample period, which does not happen in other indexes. In fact, this is the only index that shows a decline in stock market value during this period, with a loss of 66%. All the other indexes display an overall growth pattern, with Italy growing 42% in the period and Greece having the highest growth rate (370%).

Financial time series analysis requires the assessment of stationarity to help decide on the most appropriate methods to be used. Traditionally, the Dickey–Fuller test (or its expanded version) is one of the most widely used tests. However, this test should be used with caution in case there are structural breaks in the series. In such case, the test’s results can be misleading. Therefore, the test of Perron and Vogelsang (1992) is used in this study to assess the stationarity of a series, while checking whether there are structural breaks. According to Perron (1997), these tests are preferable because if one can reject the hypothesis of a unit root in a time series with a structural break, one can also reject it with weaker assumptions.

These tests were applied to the indexes of stock prices. Except for one of the cases (Greece), there seems to be breaks in all series. In addition, and excepting the case of Japan where the series are stationary, all other cases are nonstationary and are integrated of order one.

A statistical analysis of the series of rates of return is now developed in order to assess their characteristics and to try to identify possible sources of stylized facts. The daily returns are calculated as usual:

\[ R_t = \ln P_t - \ln P_{t-1} \]  

where \( R_t \) is the return rate of day \( t \), \( P_t \) and \( P_{t-1} \) are close quotations for different indexes.

Figure 2 shows the evolution of these rates of return in the considered sample period. Although no strong conclusions can be drawn from simple graphical analysis, it seems that the Greek index is the one displaying the higher volatility.

We now calculate some important statistics to analyse the indexes in study. First, Japan is the only index displaying a negative mean in return rates. It is also confirmed what graphic analysis appeared to indicate: Greek index is the one with higher volatility, measured by SD.

We also study the normality of return rates. Through Jarque–Bera test, it is possible to conclude that we always reject null hypothesis of normality and also that the kurtosis of distributions is always very high when compared to a Gaussian distribution. Such results may be explained by the existence of fat tails.

Dependence, linearity and mutual information

We assess the behaviour of the rates of return by testing the existence of autocorrelation (with a Breusch–Godfrey’s test), of heteroscedasticity (with a White’s test) and of autoregressive heteroscedasticity (with an Engle’s test).

In the autocorrelation tests, we never reject the null hypothesis, which means that linear autocorrelation in stock returns apparently does not exist. However, as mentioned earlier, this does not necessarily imply the absence of temporal independence, since nonlinearities can be present in the series. Heteroscedasticity or ARCH effects are some of possible nonlinearities to be detected in

2 Owing to space constraints, results are not shown here, but are available upon request.
financial series. In fact, for all countries, we reject the null hypothesis for these tests, which is evidence supporting the existence of heteroscedasticity and autoregressive heteroscedasticity and indicating the possibility of nonlinearity in stock returns. These results are presented in Table 1.

The McLeod and Li test is an alternative to analyse conditional heteroscedasticity. As it appears that there is no evidence of autocorrelation, there are no problems with the use of this test. We always reject the null hypothesis, which also points for the existence of dependence in time series. Both Engle and McLeod and Li tests provide evidence of nonlinearity in variance. The occurrence of this phenomenon can be related, according to Scalas (2005), to the existence of volatility clusters. Results are presented in Table 2.

The study proceeds with the BDS test, used to check for independence of time series. We always reject the null hypothesis of independence, so the series show evidence of time dependence. The results are presented in Table 3.

The Tsay test also allows us to analyse nonlinearity but in the mean of a time series and not in its variance. In this case, we can conclude that, with the exception of Germany and Spain, all the other series show nonlinearities for its mean. The Tsay test’s results are presented in Table 4.

As previously stated, mutual information can also be used to test independence in a statistical distribution, being preferable to other measures such as the linear correlation.

Fig. 1. Evolution of stock indexes between 2 January 1990 and 13 July 2010
coef
fi
cient, since mutual information is a measure of overall correlation between data and not solely a linear correlation. First, we calculate mutual information for daily return rates, considering the first 10 lags. Results point to the existence of a strong dependence in the data. The only exception is the eighth lag for Japan whose value is not

Fig. 2. Return rates for stock indexes between 2 January 1990 and 13 July 2010

Table 1. Autocorrelation, heteroscedasticity and ARCH effect tests

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>UK</th>
<th>Ger</th>
<th>Jap</th>
<th>Can</th>
<th>Fra</th>
<th>Ita</th>
<th>Gre</th>
<th>Spa</th>
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<tbody>
<tr>
<td>AR model</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ARCH – F</td>
<td>159.56**</td>
<td>193.06**</td>
<td>73.27***</td>
<td>145.78**</td>
<td>210.19***</td>
<td>137.57**</td>
<td>120.27**</td>
<td>67.57**</td>
<td>57.63**</td>
<td>138.27**</td>
</tr>
<tr>
<td>Lags</td>
<td>12</td>
<td>7</td>
<td>13</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>13</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>BG – F</td>
<td>0.008</td>
<td>1.62</td>
<td>2.87</td>
<td>1.92</td>
<td>0</td>
<td>2.23</td>
<td>1.7</td>
<td>1.84</td>
<td>2.54</td>
<td>0.59</td>
</tr>
<tr>
<td>Lags</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>White</td>
<td>222.37**</td>
<td>78.94**</td>
<td>147.96**</td>
<td>218.76**</td>
<td>96.36**</td>
<td>75.66**</td>
<td>144.77**</td>
<td>56.74**</td>
<td>141.98**</td>
<td>185.08**</td>
</tr>
</tbody>
</table>

Notes: ARCH is the Autoregressive conditional heteroscedasticity test; BG is the Breusch–Godfrey test used for autocorrelation test; White is the test for heteroscedasticity. **Denote significance at 1% level.
statistically significant. Excepting the United Kingdom and the north-American markets, mutual information decreases when the number of lags increases.

We also calculate overall correlation coefficients based on mutual information. While linear correlation coefficients are relatively small in most cases, the same does not occur with global correlation coefficients, showing evidence that there are probably nonlinearities that must be considered in the behaviour of the rates of returns. Figure 3 illustrates these differences. Overall, correlation coefficients are represented by lighter grey bars, while linear correlation coefficients are represented by darker bars. To simplify, we use linear correlation coefficients in absolute values.

Despite the fact that global correlation coefficient is a more general measure than the linear correlation coefficient, it is not possible to say that the difference between those measures is the nonlinear part of the correlation. However, it is an indicator than could exist some nonlinear components on return rates.

For first lags, the German index is the one with the highest value for mutual information, followed by the Portuguese, the Greek and the Spanish indexes. This may indicate a strong nonlinear dependence on these indexes. Although in general mutual information values decrease as the number of lags increase, this does not happen in all markets (such as in Canada, the United States or the United Kingdom). Still, coefficients on the 10th lag remain significant, so that the memory of the rates of return is not so short. Some studies, such as Bonanno et al. (2001), indicate that the autocorrelation function is usually monotonically decreasing over time. Our results support this feature, when we consider its linear component.

If we order the indices by the values of the overall correlation coefficients and by linear correlation coefficients, the order is not the same. This may mean that linear and nonlinear components affect each index differently. Orders are not constant, but reinforce the idea that the German index is the one with larger nonlinear correlations, although for linear correlations, its values are the lowest. This allows us to confirm our previous conclusion: there is probably a strong nonlinear dependence for this index.

For all indexes, overall correlation coefficients are higher than linear correlation coefficients which, in some cases, are almost null. In addition, mutual information estimations provide results that are statistically significant for all lags, except for Japan, as previously mentioned. These results indicate, therefore, the possibility of nonlinear dependence in the series under assessment.

We filtered the return rates series in order to isolate possible sources of nonlinear dependence, namely the existence of clusters of volatility. We used an ARCH/GARCH model with one lag for each.
was the United States, where GARCH component has two lags (the number of lags was selected according to AIC).

We continue this study with the BDS and the mutual information tests on the filtered series. BDS test results for the filtered series point to the evidence of nonlinear time dependence, even after filtering. The results are presented in Table 5.

We also calculate mutual information and global correlation coefficient for the filtered series (see Fig. 4). Once again, the results point to the existence of dependence since the null hypothesis is always rejected, excepting one lag for Japan, with 1% of significance.

It is therefore possible to conclude that, eventually, some types of nonlinear correlation remain in the stock indexes returns, even after the filtering process. Not being able to identify the type of nonlinearities present in the data, an analysis with mutual information and with global correlation coefficients allows the identification of the lags, presenting greater evidence of nonlinear dependence. The results indicate the possibility of this type of
Table 5. BDS test applied to filtered return series

<table>
<thead>
<tr>
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<td>0.019**</td>
<td>0.021**</td>
<td>0.016**</td>
<td>0.015**</td>
<td>0.012**</td>
<td>0.024**</td>
<td>0.016**</td>
<td>0.011**</td>
<td>0.029**</td>
<td>0.018**</td>
</tr>
<tr>
<td>3</td>
<td>0.033**</td>
<td>0.033**</td>
<td>0.026**</td>
<td>0.026**</td>
<td>0.020**</td>
<td>0.038**</td>
<td>0.027**</td>
<td>0.018**</td>
<td>0.044**</td>
<td>0.027**</td>
</tr>
<tr>
<td>4</td>
<td>0.036**</td>
<td>0.034**</td>
<td>0.029**</td>
<td>0.028**</td>
<td>0.022**</td>
<td>0.040**</td>
<td>0.029**</td>
<td>0.019**</td>
<td>0.047**</td>
<td>0.029**</td>
</tr>
<tr>
<td>5</td>
<td>0.033**</td>
<td>0.030**</td>
<td>0.026**</td>
<td>0.026**</td>
<td>0.019**</td>
<td>0.037**</td>
<td>0.027**</td>
<td>0.017**</td>
<td>0.043**</td>
<td>0.026**</td>
</tr>
</tbody>
</table>

Notes: k represents dive dimension; SD equal to 0.5. **Denote significance at 1% level.
dependence, both in the series of returns and of filtered returns.

The fact that mutual information and global correlations are significant could be a signal of possibility of predictability and lack of weak efficiency of markets, implying possibility of agents to gain abnormal profits. This could be a signal of inefficiency. However, if the inefficiency level is not pronounced, the existence of transaction costs, information costs or other commissions could make that those windows of profits could be eliminated, making that agents could not beat the market systematically (see, e.g., Frenkel and Levich, 1977; Taylor, 1987). However, we remember that we do not know this nonlinearity can be described, so it could make a weaker conclusion against efficiency.

Although it is not possible to identify the type of nonlinearity present in data, conclusions about mutual information and overall correlation coefficient allows us to think on the presence of nonlinear dependence. This is also the motive to use DFA.

**DFA application to return rates**

DFA is an alternative methodology to analyse temporal dependence in time series, with the advantage of being used in the context of nonstationary time series. It could be used as an alternative to study other types of nonlinearity that traditional models could not capture, like nonlinear dynamics. The fact that mutual information is a more general measure could also be complemented by DFA.

The DFA application is performed using the R software, assuming a linear trend for a total of 65 length boxes, from 4 to 2610. This methodology is applied to the series of returns and of filtered return.

Recovering information for all box lengths, it is possible to calculate the values for the $\alpha$ parameter but also for its SD, and to test the hypotheses $H_0: \alpha = 0.5$ and $H_1: \alpha \neq 0.5$. Traditionally, most works applying DFA are made by physicists, and they just analyse the value of parameter $\alpha$. However, is so much important to analyse its significance. If $\alpha = 0.5$, series could be explained by a random walk.

For the rates of return, the values of $\alpha$ are all very close of 0.5. However, the Japanese index is the one where we do not reject the null hypothesis, which means that for the others, though not very pronounced, there is a long-term dependence in the returns, with persistent characteristics (since the parameter is greater than 0.5). Countries for which the long-term relationship is most marked are Spain, Greece and Portugal, all countries of the Southern Europe, normally associated with less robust economies and also those who have less liquid stock markets.

The results for the filtered returns are very similar, with the difference that in addition to Japan, the coefficient of Canada is also equal to 0.5, which is the same to say that they perform like a random walk. In most cases, as expected, the degree of dependence in filtered returns is lower. However, the fact that results remain above 0.5 means that there are long-term dependencies even after the filtering of data. Results are given in Table 6. In term of interpretation, there are some differences of DFA results when compared with mutual information results. However, it is important to refer that mutual information is a more general measure that could capture other type of nonlinearity that DFA does not.

<table>
<thead>
<tr>
<th>Country</th>
<th>Return rates</th>
<th>Filtered return rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>$\hat{\alpha}$</td>
<td>0.5118**</td>
</tr>
<tr>
<td>UK</td>
<td>0.5102**</td>
<td>0.5094**</td>
</tr>
<tr>
<td>Germany</td>
<td>0.5319**</td>
<td>0.5457**</td>
</tr>
<tr>
<td>Japan</td>
<td>0.5006</td>
<td>0.5152**</td>
</tr>
<tr>
<td>Canada</td>
<td>0.5397**</td>
<td>0.5138</td>
</tr>
<tr>
<td>France</td>
<td>0.5258**</td>
<td>0.5320**</td>
</tr>
<tr>
<td>Italy</td>
<td>0.5296**</td>
<td>0.5436**</td>
</tr>
<tr>
<td>Greece</td>
<td>0.5575**</td>
<td>0.5540**</td>
</tr>
<tr>
<td>Spain</td>
<td>0.5440**</td>
<td>0.5513**</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.5855**</td>
<td>0.5945**</td>
</tr>
</tbody>
</table>

Notes: $\hat{\alpha}$ is the estimated parameter and $s.e.(\hat{\alpha})$ its SD. **Denote significance at 1% level.
dependence only decays slowly or never decays (theoretically speaking). Nonlinear correlation could be caused by some possible chaotic behaviour or by structural changes on the expected return on time. The existence of long memory in returns suggests that linear modelling should be substituted by nonlinear pricing models. As referred by Mandelbrot (1971), when there exists evidence of proximity to a random walk, but the obtained results are qualitatively similar, and in addition to the Japanese index, the Canadian has also supported the random walk. Once again, the German index and the indexes of the Southern European countries in our sample presented higher values.

Although there is evidence of nonlinearity in almost every market there are some different patterns. Southern European markets show more evidence of serial dependence as showed by results of mutual information and DFA. The less liquidity of those markets and some economic fragility of those economies could explain these conclusions.

The difference of results between countries could also be explained by the composition of the Datastream indexes used in this empirical work. In fact, these are standardized indexes, which allow to make comparisons between markets, but the exact composition of them is not the same, as each country has different kind of firms and economic structure.

As previously referred, serial dependence could be understood as a long memory process, meaning that events’ influence persist in time. This persistence cannot be interpreted as evidence of inefficiency, since it would be necessary to prove the existence of abnormal systematic profits to take such conclusion.

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References
Revisiting serial dependence


