Nonlinearly perturbed hyperbolic conservation laws

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ABSTRACT

In this presentation we attempt to stress two points of view on hyperbolic conservation laws: *modelization* and *analytical theory*. And, how they are sensitively related. While appliers are concerned with reliability, integrity or failure of solutions, mathematicians are concerned with non uniqueness, selection of physically relevant solutions or entropy criteria. In the modeling process, within simplifications, some "spurious terms" are usually discarded from the equations and so, in order to address uniqueness, a crucial information is lost. We discuss here the relevant dissipative or dispersive effect of some of those small scale terms (zero singular limits). The perturbed equations under consideration have the form

$$\partial_t u + \operatorname{div} f(u) = \epsilon \operatorname{div} b(u, \nabla u) + \delta \operatorname{div} \partial_\xi c(u, \nabla u)\,,$$

which include generalized Korteweg-de Vries-Burgers equation when ξ is a space variable and Benjamin-Bona-Mahony-Burgers equation when ξ is the time variable, or

$$\partial_t u + \operatorname{div} f(u) = \delta \operatorname{div} c(u, \nabla \partial_\xi u)\,,$$

which can present unexpected dissipative properties.

References

[1] Nabil Bedjaoui and Joaquim M.C. Correia, A note on nonlinear KdV-type equations, Bol. Soc. Port. Mat. (2013), no. especial. ISSN: 0872-3672 http://hdl.handle.net/10174/7598

[2] Joaquim M.C. Correia and Rafael S. Sasportes, Nonlinear hyperbolic conservation laws, in "IRF2009, Integrity, Reliability and Failure", eds. J.F. Silva Gomes and Shaker A. Meguid, Edições INEGI, Porto, 2009, ISBN: 978-972-8826-22-2, 978-972-8826-21-5 http://hdl.handle.net/10174/5089

[3] Joaquim M.C. Correia and Philippe G. LeFloch, Nonlinear Diffusive-Dispersive Limits for Multidimensional Conservation Laws, in "Advances in Partial Differential Equations and Related Areas", Beijing, 1997, World Sci. Publ., River Edge, NJ, 1998, pp. 103–123; (disponível em: http://arxiv.org/abs/0810.1880)